Quaternion-Based Albedo Recovery for Shape Reconstruction
B.SATHYA BAMA, M.ANITHA, S.RAJU, V.ABHAIKUMAR
Electronics and Communication Engineering
Thiagarajar College of Engineering
Thiruparankundram, Tamil Nadu, INDIA-625015
sbece@tce.edu, mmanitha@tce.edu, rajuabhai@tce.edu, sbece@tce.edu, http://www.tce.edu/

Abstract: A new quaternion based method is presented which recovers albedo for shape reconstruction from three color images for three directions of white light source. In the conventional approaches, red, green and blue components are considered separately so there is a loss of information. The albedo maps recovered are not accurate enough to recover shape. Property of quaternion is that a color can then be processed as a unit, rather than as three separate channels. Using this three color band components from three input images are combined to form a quaternion. First, the albedo maps are derived from product matrix formed by the input images and illumination matrix. By using the recovered albedo, surface normal of the image is obtained. Comparison of recovered albedo with the conventional Color photometric stereo (CPS) is done by an accuracy parameter to analyze the performance of the proposed work.

Key-Words: Color photometric stereo, Quaternion, Optical polarization filters, Illumination matrix, Jacobi transformation, Hermitian matrix.

1 Introduction
Shape reconstruction of three-dimensional objects is important for many applications that integrate virtual and real data. They form the basis for identification and classification of objects. Study on shape-from-shading [1], [2], has provided a variety of approaches, among which those using a single image appear limited in accuracy due to the ambiguity between surface normal and shade value and due to the existence of shadows in image. That using multiple images is based on the photometric stereo method [3], [4] which uses multiple light sources. In classical photometric stereo [6], [7], the input images are taken with different lighting directions but from the same viewpoint. Moreover, we have discussed the shape measurement method using multiple white light sources [5]. One useful property of albedo is that it implicitly gives the information about the shape. This property of albedo makes it extremely useful in reconstruction of shape.

In color objects, the reflection properties are in general color-dependent. There might also be color dependent inter-reflections between parts of object and between objects. In previously used color photometric stereo, we consider color components separately so there is a loss of information. The albedo maps recovered are not accurate enough to recover shape. Furthermore, albedo maps vary depending on the color temperature of the light source. The component images may have insignificant values in some areas and saturated values in some areas [8]. The use of quaternion to represent color images has been introduced by Sangwine [11] and has allowed the definition of powerful tools for color image processing such as Fourier transforms [11], correlation [12] or edge detection [13]. Property of quaternion is that a color can then be processed as a unit, rather than as three separate channels. The quaternion model for color images encodes the Red, Green and Blue values of each pixel into a single pure quaternion number. A color image can be represented as a pure quaternion as shown below:

\[ S(x,y) = r(x,y)i + g(x,y)j + b(x,y)k \]  

where \( r(x,y) \), \( g(x,y) \) and \( b(x,y) \) are respectively the red green and blue components of the pixel at position \((x,y)\) in the image \( S(x,y) \).

In this paper we introduce quaternion for recovering albedos of color objects for shape reconstruction to overcome the drawbacks in color photometric stereo. Comparison of recovered albedo with previously used CPS by an accuracy parameter is taken to analyze the performance of the proposed work.

The paper is organized as follows: section 2 explains about Color Photometric stereo, its drawbacks and introduction to quaternion, section 3 illustrates the proposed method along with performance measure followed by experimental procedure and discussion of results and section 4 gives the conclusion of the paper.
2 Problem Formulation

A general photometric stereo method for color images used conventionally first recovers the albedo maps and then directly reconstructs shapes from three input images and albedo maps, using Jacobi method.

2.1 Color Photometric Stereo

The general color photometric stereo [10] considers the surface to be Lambertian initially. It uses three input color images of the same surface taken under 3 distinct illumination sources where the following relation holds:

\[
\begin{bmatrix}
\ell_x^m & \ell_y^m & \ell_z^m \\
\ell_x^n & \ell_y^n & \ell_z^n \\
\ell_x^b & \ell_y^b & \ell_z^b \\
\end{bmatrix}
\begin{bmatrix}
I_r^m \\
I_g^m \\
I_b^m \\
I_r^n \\
I_g^n \\
I_b^n \\
I_r^b \\
I_g^b \\
I_b^b \\
\end{bmatrix}
= 
\begin{bmatrix}
n_r \\
n_g \\
n_b \\
\end{bmatrix}
\begin{bmatrix}
c_r \\
c_g \\
c_b \\
\end{bmatrix}
\]

(2)

where \( I^m, I^n, I^b \) are red, green and blue images for the lighting direction, \( l^d=(l_x, l_y, l_z)^T, d=1,2,3 \), \( n=(n_x, n_y, n_z)^T \) is the surface normal and \( c=(c_r, c_g, c_b) \) are the albedos. Equation (2) can be rewritten as

\[
\begin{bmatrix}
n_r & n_g & n_b \\
n_r & n_g & n_b \\
n_r & n_g & n_b \\
\end{bmatrix}
\begin{bmatrix}
j_r^m \\
j_g^m \\
j_b^m \\
j_r^n \\
j_g^n \\
j_b^n \\
j_r^b \\
j_g^b \\
j_b^b \\
\end{bmatrix}
= 
\begin{bmatrix}
j_r \\
j_g \\
j_b \\
\end{bmatrix}
\begin{bmatrix}
j_r^2 + (j_g^2)^2 + (j_b^2)^2 \\
j_r^2 + (j_g^2)^2 + (j_b^2)^2 \\
j_r^2 + (j_g^2)^2 + (j_b^2)^2 \\
\end{bmatrix}
\]

(3)

Then from equation (4) the albedos are obtained as

\[
\begin{align*}
c_r &= \sqrt{(j_r^2)^2 + (j_g^2)^2 + (j_b^2)^2} \\
c_g &= \sqrt{(j_r^2)^2 + (j_g^2)^2 + (j_b^2)^2} \\
c_b &= \sqrt{(j_r^2)^2 + (j_g^2)^2 + (j_b^2)^2}
\end{align*}

(5)

The surface normal is also derived as

\[
\begin{align*}
n_x &= \sqrt{(j_r^2)^2 + (j_g^2)^2 + (j_b^2)^2} \\
n_y &= \sqrt{(j_r^2)^2 + (j_g^2)^2 + (j_b^2)^2} \\
n_z &= \sqrt{(j_r^2)^2 + (j_g^2)^2 + (j_b^2)^2}
\end{align*}

(6)

Even though it is not used to reconstruct shape, it may provide useful information of the object.

2.2 Drawbacks in this approach

The conventional method for recovering albedo for shape reconstruction has few drawbacks listed below:

- In the algorithm, we consider red, green and blue components separately so there is a loss of information. The albedo and shapes reconstructed are not accurate.
- Albedo maps vary depending on the color temperature of the light source.

To overcome these drawbacks in color photometric stereo we go for quaternion based albedo reconstruction which process color as a unit, rather than as three separate channels for shape recovery of color objects.

2.3 Introducing quaternion

Quaternionic algebra was introduced by Irish mathematician Sir William Rowan Hamilton in 1843. Quaternion is a hyper complex number which consists of a single real part and three imaginary parts.

\[
q = a + b \cdot i + c \cdot j + d \cdot k
\]

(7)

Quaternions provide a new way to process color and texture in combination. Quaternions have been used here to represent color in the context of the analysis of color texture. The advantage of using quaternion arithmetic is that a color can be represented and analyzed as a single entity.

3 Problem Solution

The drawbacks encountered using Conventional method is overcome by using quaternion based albedo recovery by considering combination of color bands as a single quaternion thus no loss of information about image is achieved.

3.1 Proposed method

The use of quaternion arithmetic is that a color can be represented and analyzed as a single entity. The quaternion representation for color, as proposed by Sangwine [13] is that it combines a color 3-tuple (RGB or LMS) into a single hyper complex number. Thus a color component can then be processed as a unit, rather than as three separate channels, thereby overcoming the limitations of color photometric stereo. In CPS color bands components for three input images are considered separately and taken as matrices. Here, we combine the same bands into a quaternion and derive albedos.
The flow chart in the Fig.1 explains the recovery of albedo for shape reconstruction using quaternion. Instead of combining same band components from different light directions we can combine different band components under a light direction to a quaternion. The first form of combining yields better results while comparing the two methods and so that is taken for further analysis. By combining quaternion in this form there is no loss of information. Similarly, illumination components for same light direction are combined to form quaternion. Product matrix is obtained by multiplying both. Albedos are recovered from the product matrix and surface normal are derived from recovered albedos finally.

The computations are similar to color photometric stereo except that we work with quaternion instead of matrices. Given three color images for three directions of white light source are combined to quaternion matrix as shown below in equation (8).

\[
H = (H1 \ H2 \ H3)
\]

where

\[
\begin{align*}
H1 &= (0, i_r^d, i_r^d, i_r^d) \\
H2 &= (0, i_g^d, i_g^d, i_g^d) \\
H3 &= (0, i_b^d, i_b^d, i_b^d)
\end{align*}
\]

where \(i_m^d, m=r, g, b\) are red, green and blue images for the lighting direction. The illumination matrix \(I\) is shown in equation (8) where \(I^d = (I_x^d, I_y^d, I_z^d)\), \(d\) is the direction.

We calculate product matrix \(j\) using the above equation.

\[
\begin{pmatrix}
\hat{j}_r^1 & \hat{j}_g^1 & \hat{j}_b^1 \\
\hat{j}_r^2 & \hat{j}_g^2 & \hat{j}_b^2 \\
\hat{j}_r^3 & \hat{j}_g^3 & \hat{j}_b^3
\end{pmatrix} = \begin{pmatrix}
\hat{I}_r^1 \\
\hat{I}_r^2 \\
\hat{I}_r^3
\end{pmatrix} ( H1 \ H2 \ H3 )
\]

where

\[
\begin{pmatrix}
c_r \ \ c_g \ \ c_b \\
c_r \ \ c_g \ \ c_b \\
c_r \ \ c_g \ \ c_b
\end{pmatrix} = \begin{pmatrix}
\hat{j}_r^1 \\
\hat{j}_r^2 \\
\hat{j}_r^3
\end{pmatrix} ( H1 \ H2 \ H3 )
\]

Albedos and surface normal are computed using product matrix in equation (10) same as in color photometric stereo. But instead of matrices the product matrix consists of quaternion here. Thus we are able to recover albedo from color images using quaternion by combining the same color band components into a quaternion. We can obtain better albedos in terms of accuracy in order to compute efficient reconstruction of shape.

### 3.2 Performance measure

Albedos were recovered using both proposed and the conventional color photometric stereo method. Recovered albedos were evaluated in accuracy [10] using the following expression.

\[
e = \frac{\text{Min}_{x,y} \left\{ a(z(x,y) - c) - z_g(x,y) \right\}}{\sum_{x,y} |z_g(x,y)|}
\]

where \(z_g\) and \(z\) are the ground-truth and reconstructed albedos respectively.

### 3.3 Quaternion Jacobi Transformation

The Quaternion Jacobi Transformation is analogous to a complex Jacobi transformation [10]. In contrast, the Jacobi transformation is applied to a Hermitian matrix. The two off-diagonal elements of this matrix are quaternion conjugates, and thus they commute, making the problem analogous to the complex case. This choice of the elements reduces number of multiplications.
needed when the rotation is applied. The above procedure has to be used in future for the purpose of reconstruction of shape of the color objects using quaternion.

### 3.4 Results and Discussion

In order to recover albedo for reconstruction of shape of color objects we require texture database which provides a set of tilt rotations for constant slant angle along with the registered photometric stereo image data. We took images from different databases to ensure the performance of proposed method. The textures used for our experimental work has been downloaded from following websites.

http://www.macs.hw.ac.uk/texturelab/resources/databases/
http://cobweb.ecn.purdue.edu/~aleix/aleix_face_DB.html/

The input six images used for analysis from different texture databases are shown in Fig.2.

![Penguin doll](image1) ![Toy horse](image2) ![Face](image3)

![Poly styrene](image4) ![Alphabet](image5) ![Paper texture](image6)

**Fig.2 Six images used for analysis**

#### 3.4.1 Experimental Procedure

The textures contained in the databases are of different sizes. For our experimental procedure we resize them as 100 x 100 images. The illumination matrix used for obtaining the product matrix is given below.

\[
L = \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} = \begin{bmatrix} \cos \tau \sin \sigma & \sin \tau \sin \sigma \cos \sigma \\ \cos \tau_2 \sin \sigma & \sin \tau_2 \sin \sigma \cos \sigma \\ \cos \tau_3 \sin \sigma & \sin \tau_3 \sin \sigma \cos \sigma \end{bmatrix}
\]

We consider input samples with various tilt angles 0°, 30°, 60°, 90°, 120°, 270° under constant slant angles 30°, 45°, 50°. Illumination matrix is used to obtain the product matrix in our proposed work. The images are in jpeg or bitmap format. Software used is Matlab 7 with quaternion toolbox.

The corresponding illumination matrix for various input images is given in table 1.

<table>
<thead>
<tr>
<th>Image</th>
<th>Illumination matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penguin doll</td>
<td>((10, 6.5, 35), (-10, 6.5, 35), (0, -12, 35))</td>
</tr>
<tr>
<td>Toy horse</td>
<td>((11, 6.1, 30), (-11, 5.8, 30), (-13, 7, 30))</td>
</tr>
<tr>
<td>Face</td>
<td>((0.5, 0, 0.87), (0, 0.5, 0.87), (-0.5, 0, 0.87))</td>
</tr>
<tr>
<td>Poly styrene</td>
<td>((0.77, 0, 0.64), (0, 0.77, 0.64), (-0.77, 0, 0.64))</td>
</tr>
<tr>
<td>Alphabet</td>
<td>((0.71, 0.71), (-0.71, 0.71), (0, 0.71, 0.71))</td>
</tr>
<tr>
<td>Paper</td>
<td>((0.61, 0, 0.87), (0.35, 0.61, 0.71), (-0.35, 0.61, 0.71))</td>
</tr>
</tbody>
</table>

**Table 1 List of illumination matrix for various images**

The color band components are separated for three images for three different directions of light source. Consider penguin doll as the input image for our explanation. The three input images for three directions of white light obtained from database are shown in fig.3. It is noted that there exist shadows in the images especially for first two light sources.

![Penguin doll](image7) ![Penguin doll](image8) ![Penguin doll](image9)

**Fig.3 Input images for three directions**

The illumination matrix for penguin doll is given by \(l_x=(10, 6.5, 35), l_y=(-10, 6.5, 35), l_z=(0, -12, 35)\). Using the obtained product matrix three color components of albedo maps are recovered and surface normal is derived from albedo in both color photometric stereo method and proposed method using quaternion. The red, blue, green and color components of albedo and the three components of surface normal are shown in the Fig.4. The red component albedo map has insignificant value in the eyes and tie. The color information in color albedo map is not efficient. The surface normals obtained are not efficient except the \(n_x\) component.
Fig. 4 Results using Color Photometric Stereo
Recovered albedo maps and surface normal derived using proposed approach are shown in Fig. 5. The color albedo map is significant than the one obtained using CPS. The yellow part and tie part are clearly visible in the reconstructed albedo of penguin doll.

Fig. 5 Results using Proposed Method-penguin doll
The albedos obtained using toy horse input image for both CPS and proposed method are shown in Fig. 6.

Fig. 6 Results using Proposed Method-toy horse
The albedo maps and surface normal components obtained using proposed method are better compared to color photometric stereo which is to be further used for reconstruction of shape. However, the simulation of albedo and surface normal requires more time.

3.4.3 Comparative Performance
The accuracy of the obtained albedo maps are calculated using equation 12 and the observations are tabulated in the Table 2.

<table>
<thead>
<tr>
<th>Object</th>
<th>Color photometric stereo</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penguin doll</td>
<td>71.73</td>
<td>93.89</td>
</tr>
<tr>
<td>Toy horse</td>
<td>84.39</td>
<td>85.49</td>
</tr>
<tr>
<td>Face</td>
<td>67.25</td>
<td>71.12</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>75.43</td>
<td>92.25</td>
</tr>
<tr>
<td>Alphabet</td>
<td>68.61</td>
<td>75.35</td>
</tr>
<tr>
<td>Paper</td>
<td>75.12</td>
<td>77.59</td>
</tr>
</tbody>
</table>

Table 2 Comparison Table for Proposed and Conventional Method
From the table, recovery of albedo using proposed method yields better accuracy compared to conventional method for different input shapes. There is a tradeoff between accuracy and runtime in recovery of albedo and surface normal. The comparison graph for both the methods is given in Fig. 7.

Fig. 7 Comparison graph

4 Conclusion
This paper investigates albedo recovery for shape reconstruction of the color objects illuminated in three
different directions of white light source. The obtained albedo is compared with the conventional method for better accuracy. Albedo reconstructed using quaternion are better in terms of accuracy compared to color photometric stereo. And also surface normals obtained are significant than conventional method. But it requires more time for simulation. So to get better accuracy we have to sacrifice runtime. Future work is that by applying quaternion Jacobi transformation over recovered albedos we can get better and accurate results in reconstruction of shape.

References:
[18] Osamu Ikeda, Ye Duan, “An Accurate Shape Reconstruction from Photometric Stereo Using Four Approximations of Surface Normal”, *Faculty of Engineering, Takushoku University*.