Discrete state space channel modeling and channel estimation using Kalman filter for OFDMA systems

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Abstract: In this paper, a communication system using Orthogonal Frequency Division Multiple Access (OFDMA) is implemented. An iterative Kalman filtering algorithm for estimation of the time-variant Rayleigh fast fading channel is proposed. The Rayleigh channel is approximated to be a Jakes process which is modelled using an autoregressive model. An autoregressive (AR) channel model is used to provide the state space estimates necessary for Kalman filter based channel estimation. The Kalman algorithm, using state space concepts, computes the channel matrix which can then be used to estimate the baseband signal transmitted. Since this algorithm uses both pilot sequences and the underlying channel model to estimate the channel, they are more bandwidth efficient compared to only data-based algorithms. The error plots validate the performance of the algorithm.

Key-words: OFDM, OFDMA, Rayleigh fading, Autoregressive model, State space, Kalman filter

1. Introduction

The upcoming 4G wireless technologies like WiMax, Long Term Evolution, Ultra Mobile Broadband, and Wireless Broadband are aimed at higher data rates. They all share one thing in common, apart from being Internet Protocol based, are all based on OFDM multiple access.

Orthogonal Frequency Division Multiplexing (OFDM) [1] modulates data onto a number of orthogonal subcarriers using a digital modulation scheme such as Quadrature Amplitude Modulation (QAM). In OFDM, the data is divided into a number of narrow bands, thereby increasing the symbol period. This enables it to cope with severe attenuation problems like impulse noise and frequency selective fast fades, without the need for complex equalization filters. Moreover, Intersymbol interference (ISI) is reduced considerably by adding a cyclic prefix. OFDMA extends the functionality of Orthogonal Frequency Division Multiplexing (OFDM) by providing multiple access features in time or in frequency domain. If Time Division Multiple Access (TDMA) is used, different users are allotted different time slots, and if Frequency Division Multiple Access (FDMA) is used, different users are allotted different frequencies. In OFDMA, since the subcarriers allotted will be independent for each user, the subcarriers in a deep fade for one user may not be for another. Thus subcarrier allocation is of prime importance in an OFDMA system [2]. By adaptively allocating subcarriers for each user, by calculating instantaneous channel information, the system efficiency can be improved. This adaptive allocation is done by the scheduler present in the base station.

The main problems encountered in an OFDMA system are frequency and timing offsets. Frequency offset introduces attenuation and phase shift, but it is deterministic. Moreover, once estimated it can be removed. In practice the sampling clocks at the receiver and the transmitter are not synchronized, and this degrades the performance of the system. The timing offset introduces a phase shift to the desired symbol depending on whether a cyclic prefix is used or not. Frequency offset, timing mismatch, and time varying fading causes loss of orthogonality of the subcarriers. A nonlinearity introduced in the system is the Peak to Average Power Ratio (PAPR) [3]. It increases when the various modulated subcarriers superimpose and constructively interfere. This brings complexity issues in the ADC and DAC design, thereby reducing the efficiency of the RF power amplifier.

Channel estimation is one of the key blocks of most wireless receivers. There are two types of channel estimation techniques used in OFDMA systems [4].
Blind channel estimation makes use of certain deterministic or statistical parameters of the received signal whereas, the semi-blind channel estimation techniques use the previous estimates of the channel and pilot symbols to estimate the channel. Further, semi-blind estimation techniques can be divided into data aided, and decision directed channel estimation (DDCE). In data aided channel estimation known symbols can be transmitted on consecutive OFDM symbols, or can be interspersed between the data. The former is called training sequence based channel estimation, and the latter, pilot based channel estimation. Both lead to inefficient usage of spectrum [5]. The Kalman filter helps in achieving better estimation keeping the pilot sequence length constant. This is achieved by using the underlying channel model also to estimate the channel. Thus it overcomes the major disadvantage of using training sequences or pilot symbols.

2. OFDMA System Architecture
Consider the downlink OFDMA system with K users as shown in figure 1. The data for the K users are fed into the system in a serial manner at the base station transmitter. The total bandwidth of the channel B Hz is divided into N number of subcarriers that are dynamically allocated to the K users. The subcarriers are allotted so that they are mutually exclusive, thus simplifying the receiver design. This allocation is done by assuming the knowledge of the channel state information at each user. This information can be obtained by estimating the SNR at each terminal and then sending this information back to the base station via the uplink channels [2]. The information collected for allocation is used not only at the base station, but also at each receiver for subcarrier selection for detection. The output of the encoder is then fed into the IFFT. The IFFT achieves modulation of the parallel streams of data onto different subcarriers. A cyclic prefix formed by the end part of each OFDM symbol is then inserted. This preserves the orthogonality of the subcarriers.

The wireless channel between the transmitter and each of the receivers considered to be a Rayleigh fading channel. Rayleigh channel is used mostly in urban environments where the number of scatterers are very high. The Jakes model is an approximation of the Rayleigh fading model based on summing sinusoids. It models the Rayleigh channel using a finite number of scatterers.

At receiver K, after removing the cyclic prefix, and performing FFT, the data symbols of each user are obtained from the corresponding subcarriers. This is followed by decoding each symbol which gives the transmitted symbols.

3. Channel estimation using Kalman
Each subchannel allocated to a user in an OFDMA system is modeled by using a complex polynomial which consists of the complex gain of each subcarrier in the subchannel. This complex gain is iteratively estimated using the Kalman filter for which it uses two types of estimates-channel model based and data based [6].

3.1 Channel Model based estimator
The Jakes model is represented as an auto-regressive process where the current value of the process is a weighted sum of previous values plus plant noise. The weights for the AR Jakes model can be calculated, by solving the Yule-Walker set of equations(1) using the autocorrelation R of the autoregressive process and the autoregressive coefficient $r_{ss}$.

$$\phi = R^{-1}r_{ss}$$ (1)
The AR coefficients thus obtained helps to model the channel in a state space form which is used for channel estimation by the Kalman algorithm.

### 3.2 Data based estimator formulation

A pilot sequence of length $M$ is transmitted with every OFDMA symbol send. This sequence is known both at the transmitter and the receiver. It is assumed that the channel is invariant along the length of the symbol. The signal received, after transmitting the pilot sequence will be the convolution of the transmitted pilot sequence and the impulse response of the channel as in (2). Thus using the received and transmitted signal, an estimate of the channel is found.

$$Y = x^*h + n$$  \hspace{1cm} (2)

where $x$ is the transmitted pilot sequence, $Y$ is the signal received after passing through the channel and $h$ is the impulse response of the channel.

### 3.3 The Kalman filter

The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. In contrast to batch estimation techniques, no history of observations and/or estimates is required. This reduces the hardware in terms of memory requirements.

The many approaches to the basic observer design problem, according to which given a set of observations the behaviour of the system needs to be characterised, are typically based on two state-space models [7]. The process model models the transformation of the process state. In this model, the channel based estimate is used which formulates the state transition matrix $A$ using the AR coefficients obtained.

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$$  \hspace{1cm} (3)

In addition to this there is a measurement model that describes the relation between the process state and the measurement or observations. This computes the data based estimate by using known sequences $z_k$.

$$z_k = Hx_k + v_k$$  \hspace{1cm} (4)

where $v_k$ and $w_k$ are the measurement and process noise respectively. They are random fluctuations which account for the uncertainty in measurement. Assume the process noise $w_k$ is white with a covariance matrix $Q$. Further, assume that the measurement noise $v_k$ is white with a covariance matrix $R$ that is uncorrelated with the process noise.

$$p(w) \sim N(0, Q)$$
$$p(v) \sim N(0, R)$$  \hspace{1cm} (5)

We follow the following notations. $\hat{x}_k$ is defined as the apriori estimate of the system at time step $k$ given knowledge of the system prior to step $k$. $\hat{x}_k$ is the aposteriori measurement of the state knowing $z_k$. $x$ is a $n \times 1$ matrix, and $z$ is a $m \times 1$ matrix. The $n \times n$ matrix $A$ in the difference equation equation (3) relates the state at the previous step $k-1$, to the state at the current step $k$. The $n \times l$ matrix $B$ relates the optional control input $u$ to the state $x$. The $m \times n$ matrix $H$ in the measurement equation (4) relates the state $x$ to the measurement $z$. We need to formulate an estimation algorithm such that the expected value of our state estimate should be equal to the state or as close to it as possible. The estimation algorithm should minimise the squared of the error.

We define the apriori and aposteriori errors as

$$e_k^a = x_k - \hat{x}_k$$
$$e_k = x_k - \hat{x}_k$$  \hspace{1cm} (6)

Then apriori and aposteriori error covariances are given as

$$P_k^a = E[e_k^a e_k^{aT}]$$  \hspace{1cm} (7)
$$P_k^z = E[e_k e_k^{T}]$$  \hspace{1cm} (8)

The equations contained in the Kalman filtering algorithm are essentially of two types, the time update equations, and the measurement update equations [8]. The time update equations can be thought of prediction equations, and the measurement update equations as correction type. The time update equations are responsible for projecting forward in time the current state and error covariance to obtain the apriori estimate of the system for the next time step. The measurement equations are responsible for feedback, i.e. incorporating the new observation into the apriori estimate of the state, so as to obtain an improved aposteriori estimate. Thus the Kalman algorithm is also called the predictor-corrector algorithm [9]. The steps involved in this algorithm can be condensed as follows:

**Time update equations:**

$$\hat{x}_{k-1} = A\hat{x}_{k-1} + Bu_{k-1}$$  \hspace{1cm} (9)
$$P_{k-1} = AP_{k-1}A^T + Q$$  \hspace{1cm} (10)
Measurement update equations:
\[
K_k = P_k^r H^T (HP_k^r H^T + R)^{-1} \tag{11}
\]
\[
\hat{x}_k = \hat{x}_{k-1} + K(z_k - H\hat{x}_k) \tag{12}
\]
\[
P_k = (I - K_k H)P_k^r \tag{13}
\]

Here \(K_k\) is the Kalman gain, and it is a \(nxm\) matrix. The term \(z_k - H\hat{x}_k\) in equation (12) is called innovation or measurement residual. It gives the amount of new information that must be incorporated into the apriori estimate of the state, given the new observation obtained.

After each time and measurement update pair, the process is repeated with the previous aposteriori estimates used to project or predict the new apriori estimates. This recursive nature is one of the very appealing features of the Kalman filter since it makes practical implementations much more feasible.

Thus using the Kalman filter, the complex gains of each subchannel is estimated. This is further used to recover the transmitted data symbols using any detection method such as QR detection [10].

4. Results

The Kalman algorithm has been implemented for the following specifications. The pilot sequence of 8 bits included with every data block of 200 bits. The number of iterations used were 1000.

<table>
<thead>
<tr>
<th>Data Rate</th>
<th>1 Kbps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency</td>
<td>1MHz</td>
</tr>
<tr>
<td>Cyclic prefix</td>
<td>10% of the Data rate</td>
</tr>
<tr>
<td>FFT size</td>
<td>128</td>
</tr>
<tr>
<td>Modulation Scheme</td>
<td>16-QAM</td>
</tr>
<tr>
<td>Number of users</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1. Simulation parameters

The modulated 16-QAM waveform is represented as shown in Figure 2.

The complex gains estimate by the Kalman filter for each iteration is shown in Fig.3

The error is then computed by taking the difference between the actual and estimated complex tap gains. The error plot is shown in Figure 4.
Fig 4. Variation of error with the number of iterations

The above plot shows that the error reduces with the number of iterations thus making the estimate more accurate.

5. Conclusion

Thus using appropriate simulations it has been proved that the iterative Kalman algorithm produces estimates which closely resemble the actual estimates. The use of autoregressive model to model the channel also reduces the number of iterations to get an optimum estimate by formulating the matrices appropriately. The performance of the algorithm can be further improved by using MMSE analysis after the estimate is obtained which reduces the prediction error adaptively.

References


