Abstract: In this paper the modelling and analysis of three-phase four-leg inverter are presented. Also, this paper presents two strategies of control and their limitations. The frequency characteristics of the system are useful for determining its control strategies. Initially a PI compensator is used. Finally, the control design strategy using 4zeros/5poles is chosen as optimum strategy. The system operation is verified for load variation from 1% to 100% rated load. The closed loop performances of the inverter are verified using the Matlab/Simulink software package.

Key-Words: - Three-phase four-leg, Uninterruptible Power Supply, Power distribution system, control strategy.

1 Introduction
The main feature of a three phase inverter, with an additional neutral leg, is the ability to deal with unbalance load in a standalone power supply system [1].
industrial automation, military equipment, which require high performance uninterruptible power supply (UPS). Figure 1 presents the scheme of the switched inverter model with four conductors [2]. The fourth conductor (N) permits the control of the neutral current. For the three-phase inverters, if the load requires us to connect the null, we utilize the null point of the capacitor filter of the dc connection [2], [3]. In this case, the unbalanced loads or the non-linear single phase loads generate null currents and distortions when passing through zero. When using inverters with four terminals (A, B, C and N), the control over the null is obtained and the zero distortions caused by the inverter control system are reduced.

![Fig.1](image1)

**Fig.1** The scheme of the switched inverter model with four conductors

In figure 2 ([2]) is presented the three-phase four-leg inverter used in the shipboard dc PDS (Power Distribution System) to provide secondary AC power distribution. It can be used to supply utility power for combat equipments, radar and other critical electronic loads [2].

![Fig.2](image2)

**Fig. 2** The diagram of Power Distribution System (PDS)

The PDS comprises of a front-end boost rectifier feeding the dc link and various loads connected to the dc link which includes a three-phase four-leg inverter and a dc to dc power converter.

2 Power stage modelling of the three-phase four-leg inverter

Generally, in the literature, three levels of models are available for inverter system: detailed model, behavioral model and reduced order model. The detailed model is based on equations derived from the system structure and electrical circuit [4]. An example of a detailed model for a three-phase four-leg inverter would be a model that exactly describes all voltage and current waveforms in circuit produced by the switching action of the semiconductor switches. A detailed model requires a very long time for numerical simulation. So, a detailed model often includes discrete and discontinuous functions and it cannot be used for control loop design based on linearization and frequency domain analysis techniques. For this reason, a detailed model is used only in simulations when a detailed study of the system operation is required. The behavioral model is derived from the detailed model by time averaging of high frequency periodical waveforms such as switching waveforms [4].

The behavioral model of inverter is called “average model”. The reduced order model is obtained from the behavioral model by its linearization at an equilibrium point. So, it is often called “linearized model”. A large signal model is reduced to a small signal model, which is available in the vicinity of this operating point only. The Matlab/Simulink software package can performs automatically this linearization using the “linmod” command. The average model of the three-phase four-leg inverter in steady-states coordinates is shown in figure 3 ([5]).

![Fig.3](image3)

**Fig. 3** The average model of inverter

The model is obtained replacing the switches with controlled voltage and current sources.
The inverter output voltage and input current can be represented as [5]

\[
\begin{bmatrix}
V_{d, o}

V_{q, o}

V_{0, o}
\end{bmatrix}
= \begin{bmatrix}
d_{an}
d_{bn}
d_{cn}
\end{bmatrix} \cdot \begin{bmatrix}
I_a

I_b

I_c
\end{bmatrix},
\]

(1)

\[
I_p = [d_{an} \quad d_{bn} \quad d_{cn}] \cdot \begin{bmatrix}
I_a

I_b

I_c
\end{bmatrix},
\]

(2)

where \(V_{in} \ (i=a, b, c)\) are the inverter outputs voltages (line-neutral), \(I_{in} \ I_b \ I_c\) are line currents, and \(d_{in} \ (i=a, b, c)\) are the line-to-neutral duty cycles.

The parameters on three-phase four-leg inverter are [2]:

\[
\begin{align*}
V_{L-N} & = 277 \text{ Vrms, } L = 333 \mu\text{H, } C = 100\mu\text{F, } V_{DC} = 800 \text{ V, } P_0 = 150 \text{ kW, } f_i = 20 \text{ kHz, } f = 60 \text{ Hz.}
\end{align*}
\]

The load of the inverter is represented as a disturbance and is considered resistive. The output varies from 100% to 1% rated power, so load varies from 1.53 \(\Omega\) to 153 \(\Omega\).

The average large signal model is simulated using the Matlab/Simulink software package. In order to obtain the small signal model, the average model is processed with “linmod” Matlab command around an operating point, in the small perturbations hypothesis; the small signal model is necessary in the control design phase.

The Bode diagrams for system open loop control, in the light load case, are presented in figure 4. Figure 5 depicts the same characteristics for heavy load case.
\[ \omega_0 = \frac{1}{\sqrt{L \cdot C}}. \quad (10) \]

\( \omega_0 \) is the resonant frequency (approximately 872 Hz), and \( Q \) is the circuit quality factor. According to the equation (9), \( Q \) is a function of the resistive load, so, \( Q \) is larger when \( R \) is larger.

It can be observed that the amplitude-frequency characteristic depicted in figure 5, presents a peak between \( 10^3 \) rad/sec and \( 10^4 \) rad/sec. The controller is designed to satisfy the system performance for both heavy and light load cases.

3 Control loop design

In figure 6 is presented the closed loop model of the inverter implemented in Matlab/Simulink. The output filter capacitor voltages are sensed and transformed from stationary to rotating coordinates. These are used as feedback signals for d-q-0 channels voltage compensators.

The voltage loop gain \( T_v \) is

\[ T_v(s) = \frac{V_d}{d_d} \cdot H_v(s). \quad (11) \]

The three-phase four-leg inverter is a multi loop and highly coupled multi-variable system [7]. The d-q-0 channels compensators are designed independently with the other 2 channels assumed as open (q and 0 channels).

In the first stage a PI compensator was used (design I) [6]

\[ H_v(s) = K_p + \frac{K_i}{s} = \frac{K_i}{s} \left[ 1 + \frac{s}{K_i / K_p} \right], \quad (12) \]

where the coefficients are chosen with the numerical values \( K_p=1.25 \times 10^{-5} \) and \( K_i=0.07 \).

The Bode diagrams for the system using the PI controller in the heavy load case are presented in figure 7. Also, it can be observed that the amplitude-frequency characteristic, presents a peak between \( 10^3 \) rad/sec and \( 10^4 \) rad/sec.

Closing the loop gain below the resonance yields low system bandwidth and this results in a poor transient response. Another possibility is to close the system loop after resonance [8], [9].

The finally chosen compensator architecture is 4zeros/5poles (design II), with zeros placed at \( z_1 = 780 \) Hz, \( z_2 = 800 \) Hz, \( z_3 = 820 \) Hz, \( z_4 = 840 \) Hz, and poles \( p_1 = 1 \) kHz, \( p_2 = 2.5 \) kHz, \( p_3 = 10 \) kHz, \( p_4 = 10 \) Hz, \( p_5 = 0 \) Hz.

The Bode diagrams for this control architecture are presented in figure 8. The Bode diagrams for the same control architecture but with heavy load are shown in figure 9.
Figure 10 shows the plots in time of $V_d$ and $V_q$ with light load and figure 11 - with heavy load. The compensator is designed in rotating coordinates and hence it controls $V_d$ and $V_q$. The final output voltage is derived from them after applying the transformation $T$.

In figure 12 are presented the plots in time of $V_a$, $V_b$ and $V_c$. The $V_o$ turns out to be 0 in these simulations as the load is taken as the balanced load.

Control design II is superior to design I as the cross over frequency in case II is much higher than in case I. Dynamic simulations of inverter employing strategy I shows that the settling time for an output load voltage is higher.

4 Conclusions

In this paper, the modeling and control of three-phase four-leg inverter is presented. Two control strategies are presented in this paper. The performances of inverter are verified through simulation using the Matlab/Simulink software package.

As is shown in figure 5, for system open loop control in heavy load case there is a large peak. Also, the presence
of a similar peak can be observed on the amplitude-frequency characteristic for the system using the PI controller in the heavy load case (figure 7).

So, control design II (4zeros/5poles compensator) is superior to design I (PI compensator) as the cross over frequency in case II is higher than case I (see figures 8 and 9). Dynamic simulations of inverter employing strategy I shows that the settling time for an output load voltage is higher.

References: