Dynamic Features of a Planetary Speed Increaser
Usable in Small Hydropower Plants

JALIU CODRUTA, SAULESCU RADU, DIACONESCU DORIN,
NEAGOE MIRCEA, CLIMESCU OLIVER
Product Design and Robotics Department
University Transilvania of Brasov
Address Eroilor 29, Brasov
ROMANIA
cjaliu@unitbv.ro http://dpr.unitbv.ro/

Abstract: - The planetary chain transmission analyzed in this paper as a speed increaser for small hydropower plants is proposed by the authors. The dynamic model of the chain speed increaser is presented in the paper, based on Lagrange and Newton-Euler methods. The paper approaches the appropriate kinematical and dynamic features of the transmission. The dynamic response is obtained by means of Matlab-Simulink software.

Key-Words: - planetary speed increaser, dynamics, numerical simulation, small hydro.

1 Introduction
Usually, the speed increasers from the small hydropower plants multiply the input angular speed from 3 to 5 times [3,7]. But in the case of using electric generators with medium or high nominal speeds, the plant must include gearboxes with multiplication ratios over 5. The paper analyzes the case of a speed increaser with a multiplication ratio of 12, which can be used either in small hydro plants or wind turbines. The analyzed transmission is an innovative planetary chain increaser, which was proposed by the authors. The paper presents its dynamic modeling, as the first step in the design of the small hydropower plant control system. The paper approaches the appropriate kinematical and dynamic features of the speed increaser, based both on Lagrange and Newton-Euler methods. The dynamic response of the aggregate composed of a turbine – speed increaser - generator is obtained by means of Matlab-Simulink software.

2 Kinematical and Dynamic Aspects
The first step in dynamical modelling consists in defining the structural and kinematical aspects of the transmission. Thus, the planetary transmission contains: a fixed sun gear (3,3'), a satellite gear (2), a semi-coupling with pins (1) and a carrier (H). The internal kinematical ratio $i_0$, the transmission ratio $i_{H1}$, the interior efficiency $\eta_0$ and the transmission efficiency $\eta$ can be calculated by means of relations (1) – (3) [4,5,6]:

$$i_0 = i_{13} = \frac{\omega_{03}}{\omega_{33}} = i_{12} \cdot i_{23} = \left( +1 + \frac{z_3}{z_2} \right) (1)$$

Fig. 1. The scheme of the chain planetary increaser: a) the structural scheme, b) the dynamic scheme for the input element, c) the dynamic scheme for the output element and d) the block scheme of the considered machine (DC motor = water turbine, brake = electric generator).
\[ i_{3H}^3 = \frac{\omega_{i3}}{\omega_{H3}} = \frac{\omega_{iH} - \omega_{2H}}{-\omega_{3H}} = 1 - i_0 \]  

(2)

\[ \eta_0 = \eta_{i3} = \eta_{i2} = \frac{\eta_{i13} - \eta_{i12}}{\eta_{i13}} = 0.995 - 0.96 = 0.9552 \]

\[ \eta = \eta_{i3} = -\frac{-\omega_{iH}T_{iH}}{\omega_{3H}T_{i1}} = 1 - i_0 \]

(3)

where: \( i_{3H}^i \) is the transmission ratio from element \( x \) to element \( y \), while element \( z \) is considered blocked; \( \omega_{xy} \) represents the relative speed between elements \( x \) and \( y \), and \( x \) from rel. (3) is given by

\[ x = \text{sgn}(\omega_{iH}T_{i1}) = -\text{sgn}(\omega_{iH}T_{i1}) = -\text{sgn}(\frac{i_0}{1 - i_0}) \]  

[2].

The transmission can be used in two structural cases: \( z_1 < z_3 \) and \( z_3 > z_2 \). The multiplication ratio and the efficiency of the speed increaser in the two cases are represented in Fig. 2. The diagrams from Fig. 2,a highlight the fact that, for a multiplication ratio \( i = 12 \), the speed increaser with \( z_1 < z_2 \) has a smaller overall dimension (\( z_2 = 24, z_3 = 22 \)) than in the case \( z_1 > z_2 \) (\( z_2 = 24, z_3 = 26 \)). The efficiency of the speed increaser with \( z_1 < z_2 \) is higher than in the second case (see Fig. 2,b).

Fig. 2. Simulations for the planetary speed increaser in the cases \( z_3 < z_2 \) and \( z_3 > z_2 \) of: a) the multiplication ratio and b) the efficiency.

Imposing the value of the transmission multiplication ratio at \( i = 12 \), the transmission ratio is done by relation (4):

\[ i = \frac{1}{i_{iH}^i} = 12 \Rightarrow i_{3H}^3 = 1 - i_0 = 0.8333 \]  

(4)

The angular speeds and accelerations transmission functions (rel. 5) can be established based on relation (2), while the moments transmission function (rel. 6) based on relation (3):

\[ i_{3H}^3 = \frac{\omega_{i3}}{\omega_{H3}} \Rightarrow \]

\[ \omega_{H3} = \frac{\omega_{i3}^3}{i_{iH}^i} = 12\omega_{i3}; \]

\[ \eta_{i3} = \frac{-\omega_{H3}T_{iH}}{\omega_{i3}T_{i1}} \Rightarrow T_{iH} = T_{i1} \cdot i_{iH}^3 \cdot \eta_{iH}^3; \]

- in the case of neglecting friction, the moments transmission function is given by the following relation:

\[ T_{iH} = T_{i1} \cdot i_{iH}^3 \cdot \eta_{iH}^3 = T_{i1} \cdot 0.8333 \cdot 0.4925 = 0.4104T_{i1}. \]

3 Premises for Dynamic Modeling

The speed increaser dynamic modeling relies on the following premises:

- in the dynamic modeling, the inertial effects due to the satellite gears rotation are neglected (their masses being considered in the axial inertial moment of the afferent carrier shaft), while the inertial effects of the mobile central elements are considered integrated into the shafts that materialize the external links of the planetary gears; under this premise, the static correlations between the external torques of each planetary gear are valid, while the dynamic correlations interfere only for the shafts that materialize the planetary gears external links. The mechanical inertia momentum of the two shafts (see Fig. 1) are:

\[ J_1 = 0.035; \]

\[ J_H = 0.02 \text{ [Kg} \cdot \text{m}^2 \text{]} \]  

(7)

- the rubbing effect is considered by means of the efficiency \( \eta \);

- the real machine (water turbine – speed increaser – generator) is replaced on the experimental stand by a machine of type: DC Motor – increaser – brake, in which the motor and the brake have the following mechanical characteristics:

\[ T_m = -0.13a_m + 10; \]

\[ T_b = -a_b \text{ [Nm]} \]  

(8)
in the numerical simulations, the following values for 
the kinematical and dynamic parameters are 
considered:

- the satellite and sun gears teeth numbers are
  \( z_2 = 24 \), \( z_3 = 22 \),
- the efficiencies of the pin coupling and chain 
  transmission are \( \eta_{12} = 0.995 \), \( \eta_{23} = 0.96 \).

4 The Dynamic Model

In the dynamic modeling, the main objective is to 
determine the transmission functions for the angular 
speeds, accelerations and moments, relative to time:

\[
\omega_i(t), \omega_{13}(t), \omega_{133}(t), \omega_b(t) \\
\varepsilon_i(t), \varepsilon_{13}(t), \varepsilon_{133}(t), \varepsilon_b(t) \\
T_m = T_a(t), T_i = T_i(t), T_{13} = T_{13}(t), T_b = T_b(t)
\]

The dynamic modeling is made by means of Fig. 1, b,c and d, for the following cases:

- The motion equation is modeled by neglecting friction,
- The motion equation is modeled by considering 
  friction.

4.1 Case I, friction is neglected

In this case the Lagrange method is being used:

\[
\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{\theta}} \right) - \left( \frac{\partial E_c}{\partial \dot{\phi}} \right) = Q 
\]

According to Fig 1, the kinetic energy and generalized 
force have the following forms:

\[
E_c = \frac{1}{2} J_i \omega_i^2 + J_H \cdot \omega_b^2 \\
Q = T_m \cdot \dot{\omega}_i + T_b 
\]

in which \( J_i \), represents the mechanical inertia momentum 
of the input element 1, respectively \( J_H \) for the output 
element H.

Deriving \( E_c \) relative to time, it is obtained:

\[
\frac{d}{dt} \left( \frac{\partial E_c}{\partial \omega_i} \right) = \varepsilon_i J_1 + \varepsilon_H J_H; \\
\frac{d}{dt} \left( \frac{\partial E_c}{\partial \varepsilon_i} \right) = 0
\]

From relations (11) and (12) it results:

\[
e_i J_1 + \varepsilon_H J_H = T_m \cdot \omega_{i1} + T_b
\]

By replacing the known parameters into relation (13), 
the dynamic equation when neglecting friction 
outcomes:

\[
\varepsilon_H J_1 + J_1 \cdot (1 - i_b)^3 + \\
+ \omega_H \left[ 1 + 0.1273 \cdot (1 - i_b)^3 \right] - 10 \cdot (1 - i_b) = 0
\]

After numerical replacements, the following motion 
equation results:

\[
0.20200208 + \omega_H \cdot 1.000884 - 0.8333 = 0
\]

4.1 Case II, friction is considered

In this case, according to Fig. 1, b, c and d, it can 
be written the following system of equations, using 
the Newton-Euler method:

\[
T_i + T_H + T_3 = 0; \\
T_i \cdot i_1^3 \cdot \eta_{11}^3 + T_H = 0; \\
J_i \cdot \dot{\varepsilon}_i = T_m - T_i; \\
J_H \cdot \dot{\varepsilon}_H = T_b - T_H
\]

By solving system (16) and by taking into account 
friction, the equation used for modeling the machine 
dynamic system is obtained:

\[
\varepsilon_H J_1 + J_1 \cdot (1 - i_b)^3 \cdot \eta_{11}^3 + \\
+ \omega_H \left[ 1 + 0.1273 \cdot (1 - i_b)^3 \right] \cdot \eta_{11}^3 - 10 \cdot (1 - i_b) \cdot \eta_{11}^3 = 0
\]

After numerical replacements, the following motion 
equation results:

\[
\varepsilon_H \cdot 0.20200102 + \omega_H \cdot 1.000433 - 0.408317 = 0
\]
5 Numerical Simulations

The values of the output and input angular speeds in state-state regime ($\varepsilon_H = 0$) are obtained from relations (15) and (18):

- when friction is neglected:
  \[ \omega_H = 1.2011, \quad \omega_1 = 0.1 \]
- when friction is considered:
  \[ \omega_H = 2.450138, \quad \omega_1 = 0.2041 \]

The Simulink scheme that models the motion equation of the planetary speed increaser is presented in Fig. 3. The machine dynamic response, considering the speeds, accelerations and moments, as functions of time, of the motor, speed increaser and brake are drawn in the Fig. 4.

Fig. 4 Dynamic response of the aggregate: motor angular speed (a), input angular speed (b), output angular speed (c), brake angular speed (d), motor angular acceleration (e), input angular acceleration (f)
4 Conclusion

1) In certain conditions, the considered transmission can function in two constructive situations: \( z_3 < z_2 \) and \( z_3 > z_2 \), the first one having higher values of the efficiency and a smaller overall dimension.

2) The theoretical aspects, from which the dynamic response was obtained, are presented in the paper; the mechanical momentums of inertia and the motor mechanical characteristic (belonging to the turbine) and brake mechanical characteristic (belonging to the generator) were considered.

3) As the dynamic modeling is being accomplished in order to compare the theoretical results with the experimental ones, obtained on the stand, the turbine is being replaced with a DC engine, and the generator is being replaced with a brake. The considered inertia mechanical momentums (see Fig. 1) have as an input element 1, the engine rotor, the entering shaft with it’s appropriate rolls, while for the output element H, the exit shaft, brake rotor and the inertial effect of the satellite considered as a concentrated mass.

4) The analyzed planetary chain transmission increases the input speed 12 times and decreases the input moment 5.88 times (see Fig. 4).
5) The system consisting of motor, speed increaser, brake starts practically, in the both cases, in about 0.14 s, after which enters in the steady-state regime.

6) The dynamic model is useful in the design of the control system for the small hydro stations and wind turbines. The system control program can be established by considering certain environmental conditions/seasons and by replacing the motor and the brake from the dynamic modeling with a turbine and a generator.

7) Based on the dynamic modeling, the authors will accomplish the design, manufacturing and testing of the speed increaser for an off-grid hydropower station in the framework of a research project.

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6 CORRESPONDING ADDRESS
Professor dr. eng. Codruta Jaliu
Transilvania University of Brasov,
Product Design and Robotics Department
Eroilor 29, 500036 Brasov, Romania
Phone/Fax: +40 268 472496,
E-mail: cjaliu@unitbv.ro

References: