A Mathematical Model and Implementation of the Test System for Measurement of the Gyro Stabilization Error

Slobodan OBRADOVIC, Milan TUBA, Nicolae POPOVICIU

Faculty of Computer Science
Megatrend University of Belgrade
Bulevar umetnosti 29, N. Belgrade
SERBIA
sobradovic@ptt.rs, tubamilan@ptt.rs, nicolae.popoviciu@yahoo.com

Faculty of Mathematics-Informatics
Hyperion University of Bucharest
Calea Calarasilor nr. 169
ROMANIA

Abstract: Modern armored vehicles allow firing during motion. In order to reduce the influence of angular displacements (caused by the vehicle moving) of the cannon’s tube (and hindsight), axis stabilization using rate-gyroscopes is provided. A particular problem is to check the performance, that is to measure the error of the achieved stabilization. This paper describes a new, simple and cost-effective simulation method for the stabilization quality testing which enables the complete laboratory simulation of driving over the optional terrain in the real conditions, as well as the undisturbed examination in the extreme temperature conditions.

Key-Words: Modeling, Gyroscope stabilization, Control system

1 Introduction
In contemporary wars gunfiring from the vehicle in motion has become the basic way of action for operational armored vehicles (conveyors and tanks). Because of the rolling of the body of the vehicle (caused by moving on the ground) and the change in the target coordinates (because the target is in motion, as well), the shooting accuracy compared to the shooting of passive targets from the fixed position is substantially decreased. In order to reduce the influence of angular displacements (caused by the vehicle moving) of the cannon’s tube (and hindsight), axis stabilization using rate-gyroscopes is provided. It is enough to use two rate gyroscopes to measure angular velocities by the azimuth and elevation [1,2,3].

During manufacturing and testing it is necessary to examine stabilization quality. It is usually done with the whole vehicle on the road or on the special platform. We propose a new method where during testing, instead of the whole vehicle, only gun tube is moved. This greatly simplifies testing equipment with the same quality of the results since the same servos move the same masses during the testing. This method of simulating vehicle movement by rotating the gyroscope block with respect to the cannon tube can also be used for marksman training device.

Similar research on armored vehicles was done in [4],[5]. Related research on different devices (robots, cranes, cameras etc.) and some theoretical aspects are described in [6,7,8,9,10].

2 Mathematical Model
The following definitions and reference frames (coordinate systems) on the armored vehicle are established and used in this paper (see Fig. 1):

- \( x_0, y_0, z_0 \) absolute non-rotating frame referenced to the horizon and to the North.
- \( x_p, y_p, z_p \) frame firmly connected to the body of the vehicle (conveyor or tank).

![Figure 1. Frames](image-url)
azimuth (direction) of the cannon’s axis, with \( z' = z_{ct} \).

- \( x_{ct}, y_{ct}, z_{ct} \) frame firmly connected to the tube of the cannon in such a manner that the longitudinal axis of the cannon’s tube coincide with the axis \( x_{ct} \) (that is the top of the tube determines the ort \( i_{ct} \), of axis \( x_{ct} \)). The axis \( x_{ct} \) “watches” the target, and it is at the same time the axis that should be stabilized in space. At the same time it is \( y_{kp} = y_{ct} \).

- \( x, y, z \) frame firmly connected to the block of the gyroscope, which gives the angular velocity of the virtual azimuth of the turret and virtual elevation of the cannon’s axis.

The gyroscopes are built in the hindmost part of the cannon and \( \omega = \omega_{kp}, \gamma = \gamma_{ct} \) and \( \upsilon = \upsilon_{kp} = \upsilon_{ct} \), where \( \varphi, \upsilon \) and \( \gamma \) represent angular disturbances, which could be represented by three angles: \( \varphi - \) revolving, \( \upsilon - \) rearing and \( \gamma - \) milling. These orthographic frames are not independent, as a result of the facts that \( z_{kp} = z_{kp}, y_{kp} = y_{ct}, x = x_{ct}, y = y_{ct} \) and \( z = z_{kp} = z_{ct} \).

### 2.1 The Gyro Stabilization Fundamentals

The basic philosophy of the cannon’s tube axis stabilization consists of the following facts: the longitudinal axis of the cannon should stay still, that is without angular moving. As \( \vec{i}_{ct} \) is ort \( x_{ct} \) and if \( \Omega_{ct} \) denotes angular velocity of turning of the cannon’s tube, the condition for stabilization can be expressed with the vector equation:

\[
\frac{d\vec{i}_{ct}}{dt} = 0
\]  
(1)

This frame rotates with respect to the fixed (absolute) frame with the angular velocity: 
\( \vec{\Omega}_{ct} = \hat{\Omega}_{ct} \cdot \vec{1}_{ct} + \dot{\Omega}_{ct} \cdot i_{ct} + \ddot{\Omega}_{ct} \cdot \vec{k}_{ct} \). By differentiating the vector \( \vec{R} \), which determinates the position of the point in the moving frame \( 0X_{ct}Y_{ct}Z_{ct} \) (with coordinates \( x, y, z \)), we get its absolute line velocity \( \vec{V} \). Applied to the case of the peak of the cannon’s tube moving, i.e. \( x_{ct} \), we get:

\[
\vec{V} = \frac{\vec{dR}}{dt} = \frac{\vec{dr}}{dt} + \hat{\Omega} \times \vec{R} \]

\[
\frac{d\vec{i}_{ct}}{dt} = \frac{\vec{di}_{ct}}{dt} = \hat{\Omega} \times \vec{i}_{ct} \]  
(2)

The first component on the right side of the equation represents the propriety rate of the point (peak of the tube) in the system \( 0X_{ct}Y_{ct}Z_{ct} \). Line displacements of the vehicle’s body practically do not affect gunfiring efficiency, since these rates are minor in comparison to projectile’s starting speed (1000 or up to 2000 km/h). The second component comprises the property that system rotate with respect to the fixed absolute frame \( 0X_{ct}Y_{ct}Z_{ct} \). Since the axis \( X_{ct} \) is fixed related to the frame to which it belongs, that component of the equation equals zero.

The equation for the ort stabilization \( \vec{i}_{ct} \) is now:

\[
\frac{d\vec{i}_{ct}}{dt} = \vec{0} = 0
\]

\[
\frac{d\vec{\xi}_{ct}}{dt} = \hat{\Omega} \times \vec{1}_{ct} = \begin{vmatrix}
\vec{i}_{ct} & \vec{j}_{ct} & \vec{k}_{ct} \\
\Omega_{xct} & \Omega_{xct} & \Omega_{yct} \\
1 & 0 & 0
\end{vmatrix} = \Omega_{xct} \cdot \vec{j}_{ct} + \Omega_{yct} \cdot \vec{k}_{ct} = 0
\]  
(3)

As it is well known from the theoretical mechanics [1], [2], the axis of the cannon’s tube will be stabilized in the space if the condition holds:

\[
\Omega_{xct} = 0 \text{ and } \Omega_{yct} = 0.
\]  
(4)

Hence, those conditions (4) represent basic scalar equations, which must be fulfilled for the peak of the cannon’s tube to stay fixed in the absolute non-rotating frame, although the conveyer is moving. It should be kept in mind that the tube could be turned around its own longitudinal axis (around itself), which means that in general: \( \Omega_{x} \neq 0 \).

In order to achieve the cannon’s tube stabilization it is enough to use two rate gyroscopes to measure angular velocities \( \Omega_{xct} \) and \( \Omega_{yct} \) [1], [2]. Since gyroscopes are firmly connected to the cannon, they measure absolute angular velocities of the turning the turret and the cannon’s tube, leading:

\[
\Omega_{x} = \Omega_{xct} \text{ and } \Omega_{y} = \Omega_{yct}.
\]  
(5)

Then the conditions for stabilization may be expressed by the following equations:

\[
\Omega_{x} = 0 \text{ and } \Omega_{y} = 0.
\]  
(5a)

Namely, due to moving of the vehicle on the ground, the vacillation of the body of the vehicle appears, some angular disturbances appear, which could be represented by three angles of turning: \( \varphi - \) revolving, \( \upsilon - \) rearing and \( \gamma - \) milling. These disturbances are, by means of the drive system of the turret, being (by the direction) transferred to the turret and further to the tube of the cannon, in the form of angles of disturbances \( \varphi' \), \( \upsilon' \) and \( \gamma' \). It is certain that, without taking the type of the drive system into consideration, it must be: \( \varphi' \leq \varphi \), \( \upsilon' \leq \upsilon \) and \( \gamma' \leq \gamma \). We can give the following conclusion [3]: regarding the stabilization it is better to use the drive devices which are not self braking because then it is \( \varphi' << \varphi \), \( \upsilon' << \upsilon \) and \( \gamma' << \gamma \). Regarding the influence
of the forces of cramping during the fire it is better to use self braking drive devices.

This speculation will not be less general if we suppose the following: $\phi' = \phi$, $\upsilon' = \upsilon$ and $\gamma' = \gamma$, but by doing so the mathematical description of the problem will be substantially simplified (because we can now completely omit the frame 0X1Z1 tied to the body of the vehicle). In further speculations we will use the fact that the angle of rolling is minor, i.e. $\gamma \approx 0$, $\gamma < 10^0$, hence $\cos(\gamma) = 1$, since the elevation angles during the shooting at the earth targets are not large.

Because of the fact that the plane for determining the angle (angular velocity) of the rotation is not horizontal, and the plane for determining the angle of the rearing is not vertical, we need to define the angle of virtual azimuth $\alpha$ (the rotation angle of the turret caused by the direction drive system) and the virtual elevation $\varepsilon$ (the angle of turning of the cannon’s tube caused by the elevation drive system) instead of the angles of rotation and rearing $\dot{\alpha}$ and $\dot{\varepsilon}$ are appropriate angular velocities of turning. What are gyroscopes actually measuring by the $z$ and $y$-axes, i.e. what are the values of $\Omega_z$ and $\Omega_y$ when the drive devices are active, that is during the stabilization mode activated to fulfill the conditions of stabilization (equations (4), (5a))? We get:

\[
\begin{align*}
\Omega_y &= \dot{\phi} \cos \delta \cos \gamma + \dot{\delta} \sin \gamma \\
\Omega_x &= \dot{\delta} \cos \gamma - \dot{\phi} \cos \delta \sin \gamma
\end{align*}
\]

Angular velocities $\Omega_x$ and $\Omega_y$ are being measured using rate gyroscopes whose sensitivities are oriented along the appropriate axes $y$ and $z$. According to these measured parameters it is possible to determine the angular velocities of the individual turning in the horizontal and vertical planes:

\[
\begin{align*}
\dot{\phi} &= (\Omega_y \cos \gamma - \Omega_x \sin \gamma) \frac{1}{\cos \delta} \\
\dot{\delta} &= \Omega_y \sin \gamma + \Omega_x \cos \gamma \\
\dot{\phi} &= \Omega_x \frac{1}{\cos \delta} \\
\dot{\delta} &= \Omega_x
\end{align*}
\]

The fact that only the movement ring of the vehicle is considered indicates that the rolling angle cannot have the value larger than several degrees, and since the angle $\gamma$ in the conditions of stabilization will be held in the area of minor values, it is possible to establish the following approximations: $\cos \gamma = 1$ and $\sin \gamma = \gamma$. In the majority of systems the correction $1/\cos \upsilon$ is not taken into the consideration because the gunfiring is taking place under minor elevation angles (when tanks are shooting ground targets [3], [4]).

This approximation (7) is absolutely reasonable since we practically have minor variation in these two angles, i.e. $\varepsilon \approx \upsilon$. Simple solution to this problem is the separation of the gyroscope’s blocks into two independent gyroscopes: with the gyroscope for measuring the displacement caused by the turning of the turret (and the vehicle) by the direction, built in the turret itself instead of being built in the hindmost part of the cannon, as it is normal nowadays. With this we completely eliminate the influence of the target elevation angle to the error signal of the angular velocity by the direction, with the influence of the angle of getting up of the conveyor only

### 2.2 The Block Scheme of the System in the Stabilization Mode

In order to get the cannon’s tube fixed in space it is necessary that the servo system of the turret (by the direction) and the cannon (by the elevation) moves the turret and the cannon in the opposite directions from the direction of the disturbance action, but for the same angle and with angular velocity that is equal to the angular velocity of the disturbance. In the stabilization mode there are two more types of the motion: moving of the turret in relation to the body of the conveyor with the angular velocity $\dot{\alpha}$ and moving of the cannon’s tube with respect to the turret with the angular velocity $\dot{\varepsilon}$. The angular velocities measured by the gyroscope are:

\[
\begin{align*}
\Omega_y &= (\dot{\phi} + \dot{\alpha}) \cos \delta \cos \gamma + \dot{\delta} \sin \gamma \\
\Omega_x &= (\dot{\alpha} + \dot{\varepsilon}) \cos \gamma - (\dot{\phi} + \dot{\alpha}) \cos \delta \sin \gamma \\
\Omega_y &= \dot{\phi} + \dot{\alpha} \\
\Omega_x &= \dot{\delta} + \dot{\varepsilon}
\end{align*}
\]

Hence, the gyroscopes (sensors) in the stabilization mode actually measure the error of stabilization, that is the signals achieved from the gyroscope equal the difference between the angular velocity of the turning of the body of the conveyor related to the ground (disturbance) and the angular velocity of the turning of the turret related to the body of the conveyor by the direction ($\Omega_y = \dot{\phi} + \dot{\alpha}$). The block scheme of the system in the stabilization mode is shown in the Fig. 2, where components are:

1. Target tracking error correction made by the marksman
2. Integrator (calculates the stabilization error)
3. Servo regulator (generates the control signal for the drive system)
4. Drive system with local feedbacks for the angular velocity control
5. Gear (non-linear element)
6. Control object (the turret or the cannon)
7. Sensor, rate gyroscope (and free gyroscope for measurement only)
8. Disturbance due to the vehicle movement.

The signals achieved from the elevation gyroscope equal the difference between the angular velocity of the turning of the body of the conveyor (and the turret) related to the ground and the angular velocity of turning of the cannon’s tube related to the turret by elevation $(\Omega_z = \dot{\theta} + \dot{\epsilon})$. Then the error of the stabilization is:

$$\Delta \varphi = \int \Omega_{e\varphi} dt \quad \text{and} \quad \Delta \theta = \int \Omega_{e\theta} dt \quad (9)$$

Since our goal is to prevent the moving of the cannon’s tube in space, it means that the errors must be as small as possible, i.e. the ideal case would be with $\Delta \varphi = 0$ and $\Delta \theta = 0$. To calculate the error of stabilization we must integrate the signals from the rate gyroscopes, so the first part in the servo controller for the cannon’s stabilization would be the ideal integrator (second block on the Fig. 2.). The output signal from the integrator would be the measure of the quality of the achieved stabilization, and at the same time the input to the servo controller

5 The Proposed Test System
The particular problem is the quantitative test of the stabilization system, which is obligatory for the warfare resources; not only in as real as possible conditions regarding the terrain, but also in the extreme temperature conditions (between $-30 \, ^\circ\text{C}$ and $+50 \, ^\circ\text{C}$). So far the following methods have been used:

1. The ride on the terrain or on the special made sinusoidal path (for tanks), and
2. The ride simulation using the platform with three degrees of freedom.

The disadvantages of the first method are that the whole vehicle has to be assembled first and then driven on the appropriate terrain which is rather expensive. It is also difficult to investigate extreme temperature conditions. The second method is also very expensive since it requires a platform with high power moving systems.

We propose a much simpler system that does not compromise the quality of results. Discussing the stabilization of the cannon’s tube or the hindsight axis, we are actually discussing the stabilization of the axis of the sensor sensitivity, i.e. rate gyroscopes. Hence the essential idea of the stabilization and the relations that must be fulfilled are expressed not with the equations (4), but with the equations (5a).

Common for both known methods is that the measurement gyroscopes are placed on the rear part of the cannon and that the axes of the gyroscopes are firmly connected to the coordinate axes connected to the tube of the cannon, so it is $x=x_{ct}$, $y=y_{ct}$ and $z=z_{ct}$. Hence, it is $\Omega_x = \Omega_{xct}$ and $\Omega_y = \Omega_{yct}$. The essence of the proposed method is to untie the axis of the gyroscope from the frame connected to the tube of the cannon. The gyroscope block is connected to the tube by the device simulating servo system, so it can be independently moved related to the tube (and the turret). In this case the equations (4) and (5) are no longer valid, that is, it would be: $\Omega_{xct} \neq 0$, $\Omega_{yct} \neq 0$, $\Omega_x \neq \Omega_{xct}$ and $\Omega_y \neq \Omega_{yct}$, but the conditions for the stabilization of the sensor axes would still be the same, (5a), i.e. $\Omega_x = 0$ and $\Omega_y = 0$.

The fundamental idea is the following: instead of bringing the information about the displacement (caused by the movement of the vehicle or the simulation platform) to sensors by means of the drive plants of the turret and the cannon’s tube, displacement is brought to the sensors directly using the simulating servo systems for moving the gyroscope block related to the cannon’s tube. That means that gyroscopes can move (turn) related to the cannon’s tube, and they are not firmly connected to them (Fig. 3.).

![Figure 2](image2.png)
**Figure 2.** Block diagram of the stabilization model system.

![Figure 3](image3.png)
**Figure 3.** Vehicle movement simulation by means of instrumental servo
The servo system for moving the cannon (and the turret by the direction), now by means of the sensors, get the stabilization error signal and their turning in the opposite direction begins (because of the negative reverse connection, feedback), so the axes of the sensitivity stay stable in the space so it is \( \Omega_x \neq \Omega_{\text{ext}} \) and \( \Omega_y \neq \Omega_{\text{ext}} \), but the conditions for the stabilization of the sensor axes are fulfilled, i.e. it would be \( \Omega_x = 0 \) and \( \Omega_y = 0 \), and not necessarily \( \Omega_{\text{ext}} = 0 \) and \( \Omega_{\text{ext}} = 0 \) (or \( \Omega_{\text{ext}} \neq 0 \) and \( \Omega_{\text{ext}} \neq 0 \)).

So, the gyroscopes move, related to the tube, with some angular velocities \( \dot{\alpha} \) and \( \dot{\delta} \), which cause the error signals \( \Omega_x \neq 0 \) and \( \Omega_y \neq 0 \). These signals are now being processed using the controller in the servo system of the drive plant of the cannon, and under the action of the control signal, the servo of the cannon moves the cannon in the opposite direction with the angular velocities \( \dot{\alpha} \) and \( \dot{\delta} \). Since the servo system of the cannon gets the same signals from the same sensors and the same values as in the case of the ride over the terrain, then the servo of the cannon would move in the same way as in the real conditions, so the angular velocities \( \dot{\alpha} \) and \( \dot{\delta} \) would be \( \dot{\alpha} = -\dot{\phi} \) and \( \dot{\delta} = -\dot{\vartheta} \) such that the axes of the gyroscopes would stay fixed in space \( \Omega_x = 0 \) and \( \Omega_y = 0 \).

In this case frame connected to the gyroscopes can by the two axes freely turn related to the tube of the cannon, while the third axis \( x \) is firmly connected to the axis of the cannon’s tube, i.e. \( x = x_{\text{ct}} \). Hence, the gyroscopes can freely turn related to the cannon’s tube in the vertical and horizontal plane with some angular velocities \( \dot{\alpha} \) and \( \dot{\delta} \) for the angles \( \alpha \) and \( \delta \). The angular velocity measured by the rate gyroscopes is now obtained based on the equations identical to equations (8) and (9).

Since the gyroscopes always measure the angular velocities in the absolute frame, the origin of the displacement is irrelevant; only the angular velocities (and perhaps the angles of inclination) are relevant.

When the servo system is in the stabilization mode, the signals from the gyroscopes (proportional to the value of the angular velocity) are led to the input of the controller and according to them, control signals for the drive plant of the turret by the direction and for the cannon by the elevation are generated. The drive plants move the turret and the cannon’s tube in the directions opposite to the direction of the disturbance (because of the negative feedback), with the rates \( \dot{\alpha} \) and \( \dot{\delta} \) for the angles \( \alpha \) and \( \delta \).

Since the block scheme of the servo system hasn’t been changed, the response of the system to the signal from the gyroscope will be the same; hence the turret and the cannon’s tube move so that the gyroscopic block stays motionless in the space. So, the angular velocities of the turning of the turret and the cannon’s tube will be:

\[
\begin{align*}
\Omega_y &= (\dot{\vartheta} + \dot{\phi}) \cos \gamma + \dot{\phi} \sin \gamma \\
\Omega_x &= (\dot{\vartheta} + \dot{\phi}) \cos \gamma - (\dot{\phi} + \dot{\alpha}) \cos \delta \sin \gamma
\end{align*}
\]

(10)

With already mentioned approximation and with the minor values of the angles \( \gamma \) and \( \vartheta \), we get the following relations:

\[
\begin{align*}
\Omega_y &= \dot{\vartheta} + \dot{\phi} \\
\Omega_x &= \dot{\delta} + \dot{\epsilon}
\end{align*}
\]

(11)

That means that now, in the stabilization mode, the rate gyroscopes (sensors) actually measure the stabilization error, that is the signals obtained from the gyroscopes are equal to the difference between the angular velocity of turning of the gyroscopic block related to the ground (the disturbance) and the angular velocity of turning of the turret related to the ground by the direction \( \Omega_x = \dot{\delta} + \dot{\epsilon} \) and the angular velocity of turning of the gyroscopic block related to the ground and the angular velocity of turning of the cannon’s tube related to the turret by the elevation \( \Omega_y = \dot{\vartheta} + \dot{\phi} \). The stabilization error is now also being calculated according to the equations (9). In order that the gyroscopic block stays fixed in the space the errors must be as small as possible. The ideal case would be with \( \Delta \varphi = 0 \) and \( \Delta \vartheta = 0 \), that leads in:

\[
\begin{align*}
\Omega_x &= 0 \quad \text{and} \quad \Omega_y = 0 \quad \text{i.e.}
\dot{\alpha} &= -\dot{\varphi} \quad \text{and} \quad \dot{\epsilon} = -\dot{\vartheta}
\end{align*}
\]

(12)

So, the essential of the proposed stabilization method is the stabilization of the sensors, which are detecting the movement, i.e. the angular velocity. The gyroscopic block is being stabilized, and not the cannon’s tube. However, the servo system and the laws of managing are the same in the case of the stabilization test riding over the terrain, so the results of the stabilization of the gyroscopic block with this method and the cannon’s tube are the same. As the consequence of the gyroscopic stabilization, the object on which it is situated is stabilized as well, and that does not change the essential idea of the stabilization.

It is necessary for this measurement method to construct a mini platform with three instrumental servos, dedicated to enable the movement of the gyroscopic block itself (with its weight about a few
kilos). The advantage of this method is that it is not necessary to construct the platform, but instead to build just one instrumental servo, possibly with just one degree of freedom.

7 Conclusion
In order to reduce the influence of angular displacements (caused by the vehicle moving), stabilization of the axis of the cannon’s tube is provided using two rate gyroscopes. The proposed simulation method enables the test and measurement of the stabilization error using the instrumental servo system, which simulates the moving of the vehicle on the terrain. The advantages of the proposed method of using servo system is its low price, low power and the possibility for testing and measuring the stabilization error in the early phase of the production of the servo system for the stabilization. This method is also suitable for testing and examination even in the extreme temperature conditions.

The method of simulating vehicle movement by rotating the gyroscope block with respect to the cannon tube (and turret) can also be used for marksman training device. It can be the basis for devising several types of training devices which enable various models of training marksmen to shoot different target types (fixed and moving). The training could be divided into several steps to ensure gradual training, evaluation of the achieved level of training and marksman selection.

The combat units can be used for training (or just the turret with the cannon), without having to modify them. It is not necessary for the combat units to move in the field (which is too expensive) or to simulate the vehicle movement by platform (which requires very complex and expensive equipment). It is sufficient to have a simulation servo system (simulation platform), and each unit has its own marksman training center. Not only are disturbances simulated, but the real influences of forces which act upon the marksman during the vehicle movement.

Acknowledgment: This research is supported by Ministry of Science, Republic of Serbia, Project 144007.

References: