Possibilistic Aggregations in the Discrete Covering Problem: Application in the Problem of Optimal Choice of Alternatives

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Abstract: In this paper a new criterion is introduced for the discrete covering problem. In this criterion the a priori information represented by a possibility measure and a misbelief distribution on alternatives are condensed by aggregation instruments. Using the Dempster-Shafer belief structure and representations of a possibility measure through associated probabilities, a new criterion for discrete covering problem is constructed based on aggregation by the Most Typical Value (MTV), namely, Monotone Expectation (ME) (or Choquet integral). A bicriterial problem is obtained using a new criterion and the criterion of average price minimization. The example on the application of a new criterion is presented, where the possibility distribution on the optimal choice of the candidates (alternatives) is represented by expert valuations.

Key–Words: Minimal fuzzy covering, possibility measure, associated probabilities, average misbelief criterion, MTV, ME, Dempster–Shafer belief structure

1 Introduction

Fuzzy programming problems have been discussed widely in literature ([2, 5, 8, 13, 14, 15, 17, 19, 20] and so on) and applied in disciplines such as operations research, economic management, business administration, engineering and so on.

An axiomatic approach to discrete Choquet integral is represented by [10] and so on as an aggregation instrument for criteria construction in the optimization problems.

Derived from the Body of Evidence ([6, 9, 12, 22, 23] and so on) we further consider the associated possibility measure [3, 16, 18, 21] and its associated probabilities and connect them to the focal probabilities of the body of evidence. The main aim of these connections is to represent the aggregation instrument ME in such way that can be used for construction of a criterion in fuzzy discrete optimization problem.

In Section 3 we continue an investigation of discrete optimization problems from [13, 14, 17, 19, 20], i.e. minimal covering problems with expert-objective data. The a priori expert information can be represented in different ways. Below we consider the representations of this information in terms of some possibility measure ([6, 9, 12, 23] and so on) based on expert valuations. Using the probability representations of a possibility measure and aggregation by a Monotone Expectation (Choquet integral) we obtain a new criterion for minimal fuzzy covering problem. A bicriterial problem is obtained using a new criterion and the criterion of average price minimization. The obtained bicriterial optimization problem is a specific compromised approach between expert and objective methods of optimization.

2 Preliminary concepts

2.1 Possibility measure and its associated probabilities

The Theory of Evidence (Demster–Shafer Belief Structure) ([9, 12, 22, 23] and so on) is a powerful tool which enables to build: 1) models of decisions and their risks in uncertain environment; 2) optimiza-
tion criteria in general uncertain environment.

The Theory of Evidence is based on two dual fuzzy measures: belief measures and plausibility measures.

Belief and plausibility measures can be characterized by the set function called basic probability assignment:

$$m : \mathcal{P}(X) \rightarrow [0, 1],$$

which is required to satisfy two conditions:

a) $$m(\emptyset) = 0,$$

b) $$\sum_{B \in \mathcal{P}(X)} m(B) = 1.$$  \hspace{1cm} (1)

When the focal elements of a body of evidence \( \langle \mathcal{F}, m \rangle \) are required to be nested: \( \mathcal{F} = \{A_{j_1} \subset A_{j_2} \subset \cdots \subset A_{j_k}\} \) (consonant body of evidence), the theory of evidence is referred to as possibility theory [6, 23].

Assume the finite universe \( X = \{x_1, x_2, \ldots, x_n\} \) is given and let \( A_{j_1} \subset A_{j_2} \subset \cdots \subset A_{j_k} \), where \( A_i = \{x_1, x_2, \ldots, x_{i_j}\} \) for \( i = 1, 2, \ldots, k \) be some consonant body of evidence: \( \mathcal{F} = \{A_{j_1} \subset A_{j_2} \subset \cdots \subset A_{j_k}\} \). Let \( m_j \equiv m(A_{j_k}), l = 1, \ldots, k, \rho_i \equiv \rho(x_i), i = 1, \ldots, n, \rho_1 \equiv 1 \). Then we have the \( k \)-tuple

$$m = \langle m_{j_1}, m_{j_2}, \ldots, m_{j_k} \rangle$$ \hspace{1cm} (3)

and \( n \)-tuple

$$\rho = \langle \rho_1, \rho_2, \ldots, \rho_n \rangle.$$ \hspace{1cm} (4)

It is easy to show that

$$\begin{cases}
\rho_i = \sum_{x_j \in A_{j_k} \in \mathcal{F}} m_{j_k}, \; i = 1, 2, \ldots, n, \\
m_{j_k} = \rho_j - \rho_{j_{k+1}}, \; \rho_{j_{k+1}} \equiv 0, \; l = 1, 2, \ldots, k.
\end{cases}$$ \hspace{1cm} (5)

Let \( \{P_{\sigma}^{(Pos)}\}_{\sigma \in \mathcal{S}_n} \) be the associated probabilities of a possibility measure \( \text{Pos} \). Then we have the following connection between \( \{\rho_i\}, \{m_{j_k}\} \) and \( \{P_{\sigma}^{(Pos)}\}_{\sigma \in \mathcal{S}_n}; \forall \sigma \in \mathcal{S}_n \)

$$P_{\sigma}^{(Pos)}(x_{\sigma(i)}) \overset{def}{=} \text{Pos}(\{x_{\sigma(1)}, \ldots, x_{\sigma(i)}\}) - \text{Pos}(\{x_{\sigma(1)}, \ldots, x_{\sigma(i-1)}\}) =
\begin{cases}
0, \\
\sum_{q \in A_{j_k} \in \mathcal{F}} m_{j_k} - \sum_{q \in A_{j_k} \in \mathcal{F}} m_{j_k},
\end{cases}$$ \hspace{1cm} (6)

On the other hand,

$$m_{j_l} = \rho_{j_l} - \rho_{j_{l+1}} = \bigvee_{\sigma \in \mathcal{S}_n} P_{\sigma}^{(Pos)}(x_{j_l}) - \bigvee_{\sigma \in \mathcal{S}_n} P_{\sigma}^{(Pos)}(x_{j_{l+1}}), \; l = 1, \ldots, k.$$ \hspace{1cm} (7)

2.2 Classical set covering problem

Partitioning, covering and packing problems serve as mathematical models for many theoretical and applied problems such as coloring of graphs, construction of block-diagrams, information search, traffic scheduling, administrative division into zones [1, 4, 7] and so on.

Let us introduce some basic notions [1, 7]. Suppose that we are given the finite set \( R = \{r_1, \ldots, r_m\} \) and the family of its subsets \( S = \{S_1, \ldots, S_n\} \). Let \( S' = \{S_{j_1}, \ldots, S_{j_k}\}, 1 \leq p \leq n, \) be some subfamily of the family \( S \). If each element \( r_i \) is contained in at most (at least) one of the sets \( S'_j \) belonging to \( S' \), then \( S' \) is called a packing (covering) of the set \( R \). A covering which is simultaneously a packing is called a partitioning of the set \( R \). Let \( A = \|a_{ij}\|_{m \times n} \) be an incidence matrix of elements of set \( R \) and subsets \( S_j: a_{ij} = 1 \) if \( r_i \in S_j \), and \( a_{ij} = 0 \) if \( r_i \notin S_j \). Each subfamily \( S' \) of the family \( S \) is represented by means of its characteristic vector which has a component \( x_j = 1 \) if the subset \( S_j \) is contained in \( S' \), and \( x_j = 0 \) otherwise. If to each \( S_j \in S \) we assign a positive price \( c_j \), then partitioning, covering and packing problems take the form

1) \( \min_{A} \sum_{x \in \bar{X}} \sum_{\bar{x} \in \bar{X}} (c, \bar{x}) \); 2) \( \min_{A} \sum_{x \in \bar{X}} \max_{\bar{x} \in \bar{X}} (c, \bar{x}) \); 3) \( \max_{A} \sum_{x \in \bar{X}} \min_{\bar{x} \in \bar{X}} (c, \bar{x}), \) \hspace{1cm} (8)

respectively. Here \( \bar{x} = (c_1, \ldots, c_n) \) is the price vector, \( \bar{x} = (x_1, \ldots, x_n) \) is the vector with components 0 and 1, and \( \bar{x} \) is the vector consisting of 1’s. Note that in many interesting problems \( c_j = 1, \; j = 1, \ldots, n \) (such is, for instance, the problem on finding a minimal dominating set in the graph), but this does not simplify the solution process of these problems.

2.3 Misbelief Distribution in the minimal Fuzzy Covering Problem

Our further discussion concerns a minimal fuzzy covering problem. The problems of fuzzy partitioning and packing can be considered analogously. Some questions of a minimal covering problems with probability-possibility uncertainty is presented in [13, 14, 15, 17, 19, 20].

Now we present a new variant of a solution of the optimal fuzzy covering problem but in general uncertain environment.
Let $\tilde{S} = \{\tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\}$ be some family of fuzzy subsets on $R$. Denote the compatibility level $\mu_{\tilde{S}_j}(r_i) \equiv b_{ij}$ for $r_i \in R$, $j = 1, 2, \ldots, n$. It represents some subjective expert estimation. We assume that $\mu_{\tilde{S}_j}(r_i) > 0$ means that an element $r_i$ is covered by a fuzzy set $\tilde{S}_j$ with some positive level, even if this level is small.

**Definition 1.** A subfamily $\tilde{S}' = \{\tilde{S}_{jk}\} \subset \tilde{S}$, $k = 1, \ldots, p$, $1 \leq p \leq n$, of fuzzy subsets is called a fuzzy covering of the set $R$ if for each $r_i$ there exists a fuzzy subset $\tilde{S}_{jk} \subset \tilde{S}'$ such that $\mu_{\tilde{S}_{jk}}(r_i) > \alpha$, where $0 \leq \alpha \leq 1$ is called the minimal compatibility level, by which an element $r_i$ is covered by a fuzzy set $\tilde{S}_{jk}$.

So we have defined the values of elements of an incidence matrix:

$$a_{ik} = 1 \text{ if } \mu_{\tilde{S}_{jk}}(r_i) > \alpha \text{ and } a_{ik} = 0 \text{ otherwise.}$$

It is clear that if $\alpha = 0$, then we receive a classical case.

If to each $\tilde{S}_j \in \tilde{S}$ we assign a (positive) price $c_i$, then the fuzzy covering problem is formulated as follows: find a fuzzy covering $\tilde{S}'$ of the set $R$ having the least price with the least misbelief in subjective data. Thus under an optimal fuzzy covering we understand a covering defined by means of two criteria: 1) minimization of a covering average price; 2) minimization of average misbelief in fuzzy uncertainty. For aggregation in average misbelief criterion we employ ME.

Thus under an optimal fuzzy covering we understand ME. In this paper we will obtain a bicriterial discrete optimization in average misbelief criterion we employ ME. For aggregation by the ME in the Minimal Fuzzy Covering Problem we introduce expectation of a misbelief of a covering $\tilde{S}$ having the largest average misbelief: $E(\tilde{S}) \equiv \sum_{i=1}^{n} P(\tilde{S}_i) \delta_{\tilde{S}_i(x)}$. Here $E(\tilde{S})$ denotes a Monotone Expectation of a misbelief distribution $\tilde{S}$ with respect to a priori uncertain information represented by a possibility measure $\text{Pos}$.

**3 Aggregation by the ME in the Minimal Fuzzy Covering Problem**

We suppose that we have a priori uncertain information on all admissible covering sets $\mathcal{P}(\tilde{S})$ presented by some possibility measure $\text{Pos} : \mathcal{P}(\tilde{S}) \to [0, 1]$ with the possibility distribution (4).

Suppose also that we have some misbelief distribution

$$\tilde{\delta} = \begin{pmatrix} \tilde{\delta}_1 & \tilde{\delta}_2 & \ldots & \tilde{\delta}_n \end{pmatrix}$$

constructed using the method described in [20]. The Monotone Expectation (ME) as an aggregation instrument of the construction of criteria [10] may be written as

$$ME_{\text{Pos}}(\tilde{\delta}) = \int_{0}^{1} \text{Pos}(\tilde{S}_j \in \tilde{S}/\mu_{\tilde{S}_j}(\tilde{S}_j) \geq \alpha) d\alpha$$

$$= \sum_{i=1}^{n} P_{\sigma}(\tilde{S}_{\sigma(i)}) \cdot \delta_{\sigma(i)} = E_{\text{Pos}}(\tilde{\delta})$$

for each permutation $\sigma = \{\sigma(1), \sigma(2), \ldots, \sigma(n)\} \in S_n(\delta) \subset S_n$, $\sigma(i) \in \{1, 2, \ldots, n\}$, where $S_n(\delta)$ is the subgroup of all permutations, for which

$$\delta_{\sigma(1)} \leq \delta_{\sigma(2)} \leq \cdots \leq \delta_{\sigma(n)}.$$ (10)

Here $ME_{\text{Pos}}(\tilde{\delta})$ denotes a Monotone Expectation of a misbelief distribution $\tilde{\delta}$ with respect to a priori uncertain information expressed by a possibility measure $\text{Pos}$.

Let $\tilde{S}' \subset \tilde{S}$ be some admissible covering on which we may consider the misbelief distribution $\left(\begin{array}{cccc} x_1 & x_2 & \ldots & x_n \\ \delta_1 & \delta_2 & \ldots & \delta_n \end{array}\right)$, where $x_i = \begin{cases} 1 & \text{if } \tilde{S}_i \in \tilde{S}' \\ 0 & \text{otherwise} \end{cases}$.

It is clear that each of the elements $\tilde{S}_{\sigma(i)}$, $i = 1, 2, \ldots, n$, for any permutation $\sigma \in S_n(\delta)$ in the expectation of a misbelief of a covering $\tilde{S}$ (mathematical expectation) $E_{\text{Pos}}(\tilde{\delta})$ has its weight $P_{\sigma}(\tilde{S}_{\sigma(i)}) \delta_{\sigma(i)}\tilde{\delta}$, if only if $x_{\sigma(i)} = 1$. Therefore we introduce expectation of a misbelief of a covering $\tilde{S}'$ for any permutation $\sigma \in S_n(\delta)$:

$$E_{\text{Pos}}(\tilde{\delta}')(\tilde{S}') = \sum_{i=1}^{n} P_{\sigma}(\tilde{S}_{\sigma(i)}) \delta_{\sigma(i)} x_{\sigma(i)}.$$ (11)

Finally, we introduce the definition of a misbelief measure of a fuzzy covering $\tilde{S}'$ as a maximal fuzzy expectation of a misbelief:

$$E(\tilde{S}') = \bigvee_{\sigma \in S_n(\delta)} \left[ \sum_{i=1}^{n} P_{\sigma}(\tilde{S}_{\sigma(i)}) \delta_{\sigma(i)} x_{\sigma(i)} \right].$$ (12)

Note that the value $E(\tilde{S}')$ is a maximal average measure of a misbelief in a fuzzy covering $\tilde{S}'$.

Minimizing the value $E(\tilde{S}')$ with respect to all admissible fuzzy coverings we obtain the criterion

$$\min_{A \in 2^S} \left[ \bigwedge_{\sigma \in S_n(\delta)} \left[ \bigvee_{i=1}^{n} P_{\sigma}(\tilde{S}_{\sigma(i)}) \delta_{\sigma(i)} x_{\sigma(i)} \right] \right].$$ (13)
where \( \mathcal{F} = \{ x_1, x_2, \ldots, x_n \} \subseteq \{0,1\}^n \), \( \mathcal{F} = \{1,1,\ldots,1\} \), \( A \) is an incidence matrix defined in Subsection 2.2.

Finally, the minimal fuzzy covering problem is reduced to a bicriteria problem of the type (minimunminun) [7] for an ordinary covering with the target functions

\[
f_1 = \lor_{\sigma \in S_{n}^{(\delta)}} \left[ \sum_{j=1}^{n} \langle P_{\sigma}^{(\text{Pos})} \rangle(x_{\sigma(i)}) \delta_{\sigma(i)} \rangle \right] x_{\sigma(i)} \Rightarrow \min
\]

(minimization of a maximal average price)

\[
f_2 = E(\tilde{S}') = \lor_{\sigma \in S_{n}^{(\delta)}} \left[ \sum_{j=1}^{n} \langle P_{\sigma}^{(\text{Pos})} \rangle(x_{\sigma(i)}) \delta_{\sigma(i)} \rangle \right] x_{\sigma(i)} \Rightarrow \min
\]

(minimization of a maximal fuzzy expectation of a misbelief) with conditions \( A \tilde{x} \geq \tilde{\sigma} \).

Let \( \text{Pos} \) be a possibility measure on \( \mathcal{P}(\tilde{S}) \) associated with the consonant body of evidence: \( (\mathcal{F}, m) \), \( \mathcal{F} = \{ A_{j_1} \subseteq A_{j_2} \subseteq \cdots \subseteq A_{j_k} \} \), \( m_q = m(A_{j_k}) \), \( q = 1, \ldots, k \).

Using the expression of \( P_{\sigma}^{(\text{Pos})}(x_{\sigma(i)}) \) (formula (6)) we receive

\[
E(\tilde{S}') = \lor_{\sigma \in S_{n}^{(\delta)}} \sum_{i=1}^{n} \langle P_{\sigma}^{(\text{Pos})} \rangle(\tilde{S}_i) \delta_i x_i
\]

\[
= \lor_{\sigma \in S_{n}^{(\delta)}} \sum_{i=1}^{n} \left\{ \left[ \lor_{l=1}^{i} \sum_{q: \tilde{S}_l \subseteq A_{j_q}} m_q \right] \delta_i \right\} x_i. \quad (16)
\]

4 Example: optimal choice of candidates based on experts valuations

If \( X \) is the set of all Boolean vectors satisfying the conditions of the fuzzy covering problem, then by considering the scalar optimization problem

\[
\lambda f_1 + (1 - \lambda) f_2 \rightarrow \min,
\]

\[
(x_1, \ldots, x_n) \in X, \quad \lambda \in (0,1),
\]

where

\[
X = \{ x_{\tilde{e}} \in \{0,1\}^n | \tilde{S}' \subseteq \tilde{S}, \tilde{S}' \text{ is the covering} \}
\]

\[
\equiv \{ \tilde{x} \in \{0,1\}^n | A \tilde{x} \geq \tilde{e} \}
\]

and \( \lambda \) is a weighted parameter, we can find, in the general case, some Pareto optima [7].

As an application let us consider an example based on the problem from [7] called the problem of a choice of translators. Suppose some company needs to hire translators from French, German, Greek, Italian, Spanish, Russian and Chinese into English and there are five applicants A, B, C, D and E. It is assumed that each candidate knows some subset from the above-mentioned set of languages and demands the definite salary. We introduce two new elements of evaluation of suitability of candidates for this job: 1) First element estimates the knowledge of languages by the candidate. Candidates are examined by some group of language experts and the results of examinations are represented in the form of numbers \( b_{ij} \) which determine the level of knowledge of the \( i \)-th language by \( j \)-th candidate. Information on the candidates, these expert estimates of language knowledge and salaries demanded by the candidates are represented in the form of Table 1, where numbers \( a_{ij} \) are given in the upper part of each cell, and numbers \( b_{ij} \) in the lower part (\( a_{ij} = 1 \) if \( \mu_{\tilde{S}}(r_i) = b_{ij} > \alpha = 0,5 \) and \( a_{ij} = 0 \) otherwise; that means that the minimal compatibility level or determined minimal level of language knowledge competitions is equal to 0.5); 2) Often several considerations other than just the language knowledge should be considered as well. These other considerations are wrapped in possibility levels of being hired, which are given by the second group of experts, consisting for example from management or human resource group. The motivations for assigning these possibility levels can vary. Here we give some examples of possible motivations: a) The possibility levels can represent results of some psychological tests, developed by human resource group to measure some personal characteristics of candidates, necessary for this particular job, such as communicability, responsibility level, etc. In this case possibility levels show how well the candidate suits his desired job by his personal features. b) It is very probable that work load of languages are different from each other. Each candidate should have time to perform all translations for the languages he was chosen for during the working day. If the candidate knows too many languages with high work load, then it is less possible that he can do all required translations in time. In this case his possibility to be included in final covering must be less, even if he does not demand high salary and knows all languages perfectly. Bearing in mind any combination of these two motivations or anything else the second group of experts will assign possibility levels to each candidate.

To create a target function which guarantees min-
We construct the permutation of indices $S_n(\tilde{\delta}) = \{(2, 4, 1, 3, 5)\}$. The maximum fuzzy expected misbelief value of some $\tilde{S}^i$ fuzzy covering will be calculated as follows:

$$E(\tilde{S}^i) = \sum P^i_{\sigma(\text{Pos})}(\tilde{\sigma}(\tilde{S}^i)) \cdot \delta_{\sigma(\tilde{S}^i)}^i \cdot x_{\sigma(\tilde{S}^i)}.$$

We construct the class of associated probabilities of the measure-Pos

$$\begin{align*}
P^i_{\text{Pos}}(x_{\sigma(1)}) &= P(x_2) = 0.4, \\
P^i_{\text{Pos}}(x_{\sigma(2)}) &= P(x_4) = 0.2, \\
P^i_{\text{Pos}}(x_{\sigma(3)}) &= P(x_1) = 0.1, \\
P^i_{\text{Pos}}(x_{\sigma(4)}) &= P(x_3) = 0.3, \\
P^i_{\text{Pos}}(x_{\sigma(5)}) &= P(x_5) = 0.0.
\end{align*}$$

The misbelief criterion will be

$$f_2 = E(\tilde{S}^i) = 0.0643 \cdot x_1 + 0.2484 \cdot x_2 + 0.1965 \cdot x_3 + 0.1278 \cdot x_4 + 0. \cdot x_5.$$

Target functions guaranteeing respectively a minimal salary and minimal misbelief have the form

$$\begin{align*}
f_1 &= 2100x_1 + 1900x_2 + 1500x_3 + 1800x_4 + 1300x_5, \\
f_2 &= 0.0643 \cdot x_1 + 0.2484 \cdot x_2 + 0.1965 \cdot x_3 + 0.1278 \cdot x_4 + 0. \cdot x_5.
\end{align*}$$

Let us use the relation

$$\tilde{f}_i = \frac{f_i}{f^*}, \quad i = 1, 2,$$

where $f^*_i = \min f_i, \quad i = 1, 2$.

After passing to dimensionless values we obtain

$$\begin{align*}
\tilde{f}_1 &= 0.420x_1 + 0.380x_2 + 0.300x_3 + 0.360x_4 + 0.260x_5, \\
\tilde{f}_2 &= 0.2056x_1 + 0.7929x_2 + 0.6283x_3 + 0.4090x_4 + 0 \cdot x_5.
\end{align*}$$

Note that for $f_1$ an optimal solution is $x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1, x_5 = 1$, which means that the candidates B, D, E should be hired, and for $f_2$ an optimal solution is $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0, x_5 = 1$, i.e., the preference should be given to the candidates A, B, E. For $\lambda = 0.5$, by scalarized linear convolution, criterion gives the solution A, B, E. To solve the scalar problem on a minimal covering we use an algorithm of search of tree type from [4]. This

<table>
<thead>
<tr>
<th>Language</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td>1</td>
<td>0.9</td>
<td>0</td>
<td>0.55</td>
<td>0.9</td>
</tr>
<tr>
<td>Russian</td>
<td>1</td>
<td>0.8</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>German</td>
<td>1</td>
<td>0.8</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Italian</td>
<td>1</td>
<td>0.8</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Spanish</td>
<td>1</td>
<td>0.8</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Chinese</td>
<td>1</td>
<td>0.8</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Data base of language knowledge and demanded salaries of the candidates A, B, C, D, E.
### Table 2: Positive discriminations

<table>
<thead>
<tr>
<th>Possibility Distribution</th>
<th>$\rho_1 = 0.7$</th>
<th>$\rho_2 = 0.4$</th>
<th>$\rho_3 = 1.0$</th>
<th>$\rho_4 = 0.6$</th>
<th>$\rho_5 = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language \ Translator</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>1</td>
<td>0.670</td>
<td>0</td>
<td>0.660</td>
<td>0.670</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.830</td>
<td>0.837</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.833</td>
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<td>0</td>
<td>0.833</td>
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</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0.840</td>
<td>0</td>
<td>0.826</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.697</td>
<td>0.660</td>
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<td>0.697</td>
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<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.841</td>
<td>0.826</td>
</tr>
<tr>
<td>$\pi_j$</td>
<td>0.333</td>
<td>0.362</td>
<td>0.309</td>
<td>0.339</td>
<td>0.336</td>
</tr>
<tr>
<td>$\mu_{\text{small}}(\pi_j)$</td>
<td>0.667</td>
<td>0.638</td>
<td>0.691</td>
<td>0.661</td>
<td>0.664</td>
</tr>
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</table>

### Table 3: Negative discriminations

<table>
<thead>
<tr>
<th>Possibility Distribution</th>
<th>$\rho_1 = 0.7$</th>
<th>$\rho_2 = 0.4$</th>
<th>$\rho_3 = 1.0$</th>
<th>$\rho_4 = 0.6$</th>
<th>$\rho_5 = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language \ Translator</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>1</td>
<td>0.163</td>
<td>1</td>
<td>0.173</td>
<td>0.163</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.087</td>
<td>0.080</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>$v_j$</td>
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<td>0.603</td>
<td>0.618</td>
<td>0.617</td>
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<tr>
<td>$\mu_{\text{large}}(v_j)$</td>
<td>0.619</td>
<td>0.603</td>
<td>0.618</td>
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</table>
example can be generalized for any contest problem, where optimal choice must be made.

During the research a software has been developed. The software consists of two basic modules: the first module is responsible for reducing minimal fuzzy covering to classical covering problem and the second module is responsible for solving classical covering problem using both exact (search tree type algorithm [4]) and approximate (greedy algorithm) methods.

5 Conclusion

As a result we obtain a bicriterial discrete optimization problem which is solved by the method of linear convolution of criteria. The scalar problem is solved by the search tree algorithm from [7].

The obtained bicriterial optimization problem is a specific compromised approach between the expert (fuzzy) and the objective (probabilistic) method of optimizing decision-making, where both the minimization of average misbelief in alternatives and the minimization of their price are taken into account. The constructed approach (minimal fuzzy covering) to the solution of the discrete optimization problem with data of combined (expert-objective) nature can be thus regarded as more trustworthy from the standpoint of application than the classical optimization methods.

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References:


