Unsteady Rotor-Stator Interaction in a Low Pressure Centrifugal Compressor

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Abstract: - The aim of this paper is to study the unsteady rotor-stator interaction in a low-pressure centrifugal compressor using the finite volume method to solve the Unsteady Reynolds-Averaged Navier-Stokes. In order to understand better, the rotor-stator interaction, the unsteady results are processed using both Adamczyk decomposition [8] and Proper Orthogonal Decomposition (POD) [3, 4]. Both decompositions show the behavior of unsteady rotor-stator interaction but the POD modes also show the numerical errors.

Key-Words: - Unsteady Rotor-Stator Interaction, Adamczyk decomposition, POD, CFD, Compressor, URANS

1 Introduction

The flow in turbomachinery is complex: three-dimensional, viscous, and unsteady, with time scales that vary considerably. This behavior makes difficult the complete flow analysis. Usually, in the industrial practice, the flow is assumed steady in the reference frame linked to the studied row. Furthermore, one can consider that every flow is composed by a main flow and a secondary flow that contains physical phenomena with no null velocity rotor. The secondary flows are characterized by vortexes that lead to three-dimensional behavior of flow and generate the losses due to the entropy increase.

There are some sources of unsteady phenomena, in turbomachinery and C. Dano [1] describes them after their origins. Because the rotor-stator interaction can affect dramatically the turbomachinery performance, we paid it a special attention in this paper. The majority of researchers that studied this interaction from the numerical point of view focused their research on transonic turbomachinery; therefore, there is very few information about the rotor-stator interaction for low velocity turbomachinery. Moreover, a recent study [2] showed important discrepancies between experimental and numerical results for a low-pressure centrifugal stage. Unfortunately, this study did not succeed to identify the effects that caused the major discrepancies between experimental and numerical results. For this reason, we have considered that it is useful to study the rotor-stator interactions in a low-pressure centrifugal stage, using both Adamczyk and proper orthogonal decomposition.

Up to now, the Fourier transform is a common tool for data storage of for the analysis of any periodic signal. Some recent studies [3, 4] clearly showed that POD is a more efficient method to extract the dominant modes involved in unsteady flow field. Unfortunately, these studies applied POD only for one-dimensional decompositions. In order to take the full advantage of POD method, we have applied it for decomposition of full three-dimensional flow field.

2 Nomenclature

e internal energy (J/kg)
f e external acceleration (m/s^2)
F x, F y, F z vectors of convective components of flux
G x, G y, G z vectors of diffusive components of flux
h static enthalpy (J/kg)
I rotalthapy (m^2/s^2)
p static pressure (Pa)
r radius (m)
R gas constant (J/(kg·K))
S vector of source term
T static temperature (K)
t time (s)
u, v, w Cartesian components of velocity (m/s)
V absolute velocity (m/s)
W relative velocity (m/s)
κ thermal conductivity (W/(m·K))
μ dynamic viscosity (kg/(m·s))
μ e eddy viscosity (kg/(m·s))
θ azimuthal (circumferential) angle (rad)
ρ static density (kg/m^3)
τ shear stress tensor (Pa)
Ω angular velocity (rad/s)
Subscript
\( R \)  \( \) rotor
\( t \)  \( \) turbulent

Superscript
\( \text{eff} \)  \( \) effective (laminar + turbulent)

### 3 Governing Equations

For a three-dimensional rotating Cartesian coordinate system, the unsteady Reynolds-averaged Navier-Stokes equations using the Favre averaging (a mass-weighted averaging) could be written in the conservative form as

\[
\frac{\partial Q}{\partial t} + \frac{\partial(F_x - G_x)}{\partial x} + \frac{\partial(F_y - G_y)}{\partial y} + \frac{\partial(F_z - G_z)}{\partial z} = S
\]  \( \quad \) (1)

where

\[
Q = \begin{bmatrix}
\rho \\
\rho u \\
\rho v \\
\rho w \\
\rho \left( e + \frac{W^2}{2} - \frac{\Omega^2 r^2}{2} \right)
\end{bmatrix}, \quad F_x = \begin{bmatrix}
\rho u \\
\rho v \\
\rho w \\
\rho u \rho v \\
\rho u \rho w
\end{bmatrix}, \quad F_y = \begin{bmatrix}
\rho v \\
\rho w \\
\rho v \rho w \\
\rho v \rho w \\
\rho v \rho w \\
\rho v \rho w
\end{bmatrix}
\]  \( \quad \) (2)

\( F_z = \begin{bmatrix}
\rho u \\
\rho v \\
\rho w \\
\rho u \rho v \\
\rho u \rho w
\end{bmatrix}
\]  \( \quad \) (3)

where rothalpy \( I \) is defined by

\[
I = h + \frac{W^2}{2} - \frac{(\Omega r)^2}{2}
\]  \( \quad \) (4)

If we assume that the fluid is Newtonian, the diffusive flux \( G \) may be written as

\[
G_x = \begin{bmatrix}
0 \\
\tau_{x\text{eff}}^{\text{xx}} \\
\tau_{y\text{eff}}^{\text{xy}} \\
\tau_{z\text{eff}}^{\text{xz}} \\
ur_{x\text{eff}}^{\text{xx}} + vr_{x\text{eff}}^{\text{xy}} + wr_{x\text{eff}}^{\text{xz}} + k_{\text{eff}} \frac{\partial T}{\partial x}
\end{bmatrix}
\]  \( \quad \) (5)

\[
G_y = \begin{bmatrix}
0 \\
\tau_{y\text{eff}}^{\text{xx}} \\
\tau_{y\text{eff}}^{\text{xy}} \\
\tau_{y\text{eff}}^{\text{yz}} \\
ur_{y\text{eff}}^{\text{xx}} + vr_{y\text{eff}}^{\text{xy}} + wr_{y\text{eff}}^{\text{yz}} + k_{\text{eff}} \frac{\partial T}{\partial y}
\end{bmatrix}
\]  \( \quad \) (6)

According to the Boussinesq hypothesis, the shear stresses \( \tau_{\text{eff}} \) may be written as

\[
\tau_{x\text{eff}}^{\text{xx}} = \frac{2}{3} (\mu + \mu_t) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)
\]  \( \quad \) (8)

\[
\tau_{y\text{eff}}^{\text{yy}} = \frac{2}{3} (\mu + \mu_t) \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)
\]  \( \quad \) (9)

\[
\tau_{z\text{eff}}^{\text{zz}} = \frac{2}{3} (\mu + \mu_t) \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right)
\]  \( \quad \) (10)

\[
\tau_{x\text{eff}}^{\text{xy}} = \frac{1}{3} (\mu + \mu_t) \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} \right)
\]  \( \quad \) (11)

\[
\tau_{x\text{eff}}^{\text{yz}} = \frac{1}{3} (\mu + \mu_t) \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)
\]  \( \quad \) (12)

\[
\tau_{x\text{eff}}^{\text{zx}} = \frac{1}{3} (\mu + \mu_t) \left( \frac{\partial w}{\partial y} + \frac{\partial u}{\partial z} \right)
\]  \( \quad \) (13)

The Sutherland’s formula could be used to determine the dynamic viscosity \( \mu \) as function of temperature, while the eddy viscosity \( \mu_t \) is computed with a turbulence model.

For gases, the external force \( f_e \) due to the gravitational acceleration is very small, therefore it can be neglected. Moreover, we can assume that the thermal conductivity is the single heat source. If the Cartesian coordinate system is rotating about z axis with constant angular velocity \( \Omega \), source term \( S \) could be written as

\[
S = \begin{bmatrix}
0 \\
\rho (\Omega^2 x + 2\Omega y) \\
0 \\
0
\end{bmatrix}
\]  \( \quad \) (14)

The pressure is obtained from the equation of state,

\[
p = \rho RT
\]  \( \quad \) (15)

### 4 Numerical Simulation

The numerical simulations of the three-dimensional viscous flow were carried out on a centrifugal compressor designed, manufactured and tested by COMOTI, with commercial CFD code FLUENT that is based on finite volume method where each unknown takes an average value on each discretization cell. The
computational domain generated in Gambit was split into eight blocks to facilitate the building of a fully structured mesh as shown in Fig. 1. The mesh for which the results are given, has about 253,000 hexahedral cells for the impeller passage and 127,000 hexahedral cells for the vaned diffuser passage.

In order to decrease the computational time, impressively, the time discretization is made with a backward implicit first order scheme and multigrid technique is used. To take into account the physical properties of flow, the convective fluxes are discretized with the Roe scheme, which is a Godunov-type scheme [5, 6]. Because the turbulence is not a critical issue of this study, we used the Spalart-Allmaras model, which is a one-equation model [7].

At the inlet, a uniform stagnation pressure (96310 Pa) and temperature (300 K) are imposed, turbulent viscosity ratio \( \mu_t/\mu \) is 10 and the flow is normal to inlet. At the outlet, a uniform static pressure (156000 Pa) is imposed. At the left and right sides of computational domain, the rotational periodic boundary conditions are imposed. All the walls have been assumed adiabatic. The shaft speed of impeller is 14915 rpm.

Starting from an arbitrary field \( u \) expressed in an inertial reference frame attached to the stationary row, the first averaging has as objective to extract the axisymmetric field independent by time and azimuthal coordinate. The second averaging is a time averaging in the inertial reference frame and it extracts from the remained field, the flow structures attached to the stationary row while the third averaging also is a time averaging but in the rotating reference frame and it extracts from the remained field, the flow structures attached to the rotating row. Therefore, the third contribution is steady in the rotating reference frame. Finally, after three averaging, the residual field (fourth contribution) represents the unsteady part of initial field \( u \) in the inertial and rotating reference frame associated to stationary and rotating row, respectively. This contribution characterizes purely unsteady phenomena of turbomachinery flow. In order to understand better, the unsteady rotor-stator interaction, the fourth contribution was decomposed with POD technique as shown in the next section.

As it follows, we will give some results for some control points placed in a section at mid height of blade of vaned diffuser, at the middle distance between the blade and the right periodic as shown in Fig. 2. The numbering of these control points is from upstream to downstream.

The first component of Adamczyk decomposition for static pressure and absolute velocity, at considered control points is shown in Fig. 3. One sees that the compression process is smooth while the absolute velocity has big variations especially in the first part of
vaned diffuser where the strong deceleration triggers a huge jet-wake region accompanied by boundary layer separation on suction side of vaned diffuser blade. These phenomena generate huge nonuniformities in the absolute velocity field as shown in Figs. 4 and 5, which induce important total pressure losses. For this reason, the compression process is very slow in the last part of vaned diffuser. Furthermore, the rectangular trailing edge of vaned diffuser blade generates additional nonuniformities, which are shown in Fig. 5 and losses. The homogenization process of flow begins after the trailing edge of vaned diffuser blade and it is accompanied by significant total pressure losses. For this reason, the air compression is very weak downstream of the trailing edge.

The Adamczyk decomposition clearly shows that this classical vaned diffuser with circular arc blades generates a huge jet-wake zone and important pressure losses because the channel is extremely divergent in the first part of vaned diffuser. In order to obtain better compressor performance, it is necessary to renounce single circular arc vaned diffuser.

6 Proper Orthogonal Decomposition

In the field of fluid mechanics, two approaches have been used for the POD. Historically the method of Continuous POD (or the classical method) of Lumley [9] proceeded by the Snapshot POD of Sirovich [10]. More information regarding the application of the proper orthogonal decomposition in the analysis of turbulent flows together with a detailed bibliography is given in [11]. In this paper, we used the Snapshot POD because it is much more efficient from the numerical point of view.

The POD is a method that reconstructs a data set from its projection onto an optimal base. Besides using an optimal base for reconstructing the data, the POD does not use any prior knowledge of the data set. It is because of this that the basis is only data dependent and this is reason that the POD is used also in analyzing the natural patterns of the flow field.

For the reconstruction of the dynamic behavior of a system the POD decomposes the data set in two parts: a time dependent part, \( a_k(t) \), that forms the orthonormal amplitude coefficients and a space dependent part, \( \psi_k(x) \), that forms the orthonormal basis. The reconstructed data set is:

\[
    u(x, t) = \sum_{k=1}^{M} a_k(t) \psi_k(x)
\]

where \( M \) is the number of time instant observations in the data set.

We denote the error of the reconstructed data set as:

\[
    e(x, t) = u(x, t) - \sum_{k=1}^{M} a_k(t) \psi_k(x)
\]

The base from which the data set is reconstructed is said to be optimal in the sense that the average least squares truncation error is minimized for any given number \( (m \leq M) \) of basis functions over all possible sets of orthogonal functions:

\[
    e_m = \left\langle e, e \right\rangle
\]

where the \( \left\langle \cdot, \cdot \right\rangle \) is the ensemble average and \( \left(\cdot, \cdot\right) \) is the standard Euclidian inner product.

It was shown that the minimization condition for error \( e(x, t) \) translates into maximum condition for:

\[
    \lambda = \left\langle u, \psi \right\rangle
\]

This maximization can be proven to take place if the time independent base functions \( \psi(x) \) are obtained from the Fredholm integral equation:

\[
    \sum_{j=1}^{M} \int R_y(x, x') \psi_j(x') dx' = \lambda \psi_j(x)
\]

where \( R_y \) is the correlation kernel. In this way, we transform this into an eigenvalue problem and \( \lambda_k \) is the
eigenvalue corresponding of the eigenvector \( \psi_k \).
Because we can consider the inner product as being the equivalent of an "energy", the value of \( \lambda_k \) is linked to the energy contained in mode \( \psi_k \) and the optimization process involved can be summarized as: the data set is projected onto a basis that maximizes the energy content. While in the classical approach of Lumley [9], the correlation matrix is constructed as a space correlation matrix and solving the eigenvalue problem, we obtain directly the eigenvectors as the spatial modes and then use them in order to obtain the time-dependent coefficients

\[
a_k(t) = \langle u(x,t), \psi_k(x) \rangle
\]

in the Snapshot POD of Sirovich [10], the correlation matrix is a time correlation matrix:

\[
C = \frac{1}{V} \int_V u(x,t) \cdot u(x,t') dV
\]

which is of the size of the square of the number of snapshots. From the time correlation matrix, we get the eigenvalues \( \lambda_k \) and time dependent eigenvectors \( \phi_k(t) \).

The spatial eigenmodes that are time independent, are computed according to the formula:

\[
\psi_k(x) = \frac{1}{\mu_k} \int \phi_k(t) u(x,t) dt
\]

where

\[
\mu_k = \sqrt{\lambda_k}
\]

The processed data are the variations of absolute velocity magnitude and static pressure fields, which represent the fourth term of Adamczyk decomposition according to Eq. 16. These variations were obtained from numerical simulations using the commercial CFD code Fluent. For each period, we took 20 snapshots and the time between adjacent snapshots is of \( \Delta t = 9.5781 \mu s \); therefore, the Snapshot POD of Sirovich yields 20 eigenmodes for each considered field.

The very high efficiency of the proper orthogonal decomposition is clearly underlined by Table 1. The sum of the first two modes represents 90.5% and 95.5% of the total energy, respectively for the variations of static pressure and absolute velocity magnitude fields while the sum of modes 6 to the last mode represents only 0.163% and 0.236% of the total energy, respectively for the variations of static pressure and absolute velocity magnitude fields. Therefore, both variations of static pressure and absolute velocity magnitude fields can be very accurately reconstructed using only the first five modes. Furthermore, these results confirm that the base from which the data set is reconstructed is indeed optimal.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fraction of total energy for variation of static pressure</th>
<th>Fraction of total energy for variation of absolute velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.40E-01</td>
<td>5.49E-01</td>
</tr>
<tr>
<td>2</td>
<td>2.65E-01</td>
<td>4.06E-01</td>
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<tr>
<td>3</td>
<td>6.63E-02</td>
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</tr>
<tr>
<td>4</td>
<td>1.72E-02</td>
<td>1.58E-02</td>
</tr>
<tr>
<td>5</td>
<td>8.97E-03</td>
<td>5.73E-03</td>
</tr>
<tr>
<td>6</td>
<td>1.41E-03</td>
<td>7.57E-04</td>
</tr>
<tr>
<td>7</td>
<td>6.29E-04</td>
<td>4.50E-04</td>
</tr>
<tr>
<td>8</td>
<td>1.60E-04</td>
<td>8.52E-05</td>
</tr>
<tr>
<td>9</td>
<td>4.58E-05</td>
<td>7.30E-05</td>
</tr>
<tr>
<td>10</td>
<td>4.02E-05</td>
<td>5.25E-05</td>
</tr>
</tbody>
</table>

Fig. 6 The first four most energetic modes of variation of absolute velocity magnitude field

The first two most energetic modes of variation of absolute velocity magnitude field contain as much as 95.5% of the total energy. The first mode has 54.9% of the total energy and it represents the interaction between wakes due to the blunt trailing edge of impeller blade and potential effects. According to theory of characteristics, this interaction affects especially the vaned diffuser region and its peak is located near the middle distance between impeller and vaned diffuser as shown in Fig. 6. The second mode contains 40.6% of total energy and it represents the interaction between wakes and potential effects in the vaned diffuser region as well as the propagation of potential effects in the impeller region. The third and fourth modes have 3.7% of the total energy and they contain both physical and numerical information. From the physical point of view, they contain the information regarding the interaction between and potential effects as well as the influence of potential effects in the impeller region. From the numerical point of view, they represent the numerical errors that occur at the interface between rotating region and stationary region and due the rotational periodicity condition. Furthermore, one sees that the value of the third mode is not close to zero at the outlet boundary of
computational domain because we imposed a uniform static pressure on this frontier and this is not too correct according to the theory of characteristics [5, 6].

7 Conclusions
Both Adamczyk and proper orthogonal decomposition have been successfully applied to the decomposition of fully three-dimensional static pressure and absolute velocity magnitude fields obtained from numerical simulations using the commercial CFD code Fluent.

The Adamczyk decomposition clearly shows that the single circular arc vaned diffuser generates a huge jet-wake region and important pressure losses because the channel is highly divergent in the first part of vaned diffuser. In order to obtain better compressor performance, it is necessary to renounce circular arc vaned diffuser.

Both variations of static pressure and absolute velocity magnitude fields can be very accurately reconstructed using only the first five modes; therefore, the proper orthogonal decomposition method is a very efficient method for the data storage of unsteady flows. Moreover, POD technique is able to capture the relevant features of the unsteady rotor-stator interaction, especially, the potential effects and the interaction between wakes due to the blunt trailing edge of impeller blade and potential effects. Furthermore, the POD method clearly show the numerical errors such as those errors that occur at the interface between rotating region and stationary region because the information exchange does not use the characteristic variables, the reflection of numerical waves at rotational periodic and outlet boundaries as well as their magnitude. In order to obtain more accurate results [3, 12], we should impose the phase-lagged condition, which is not yet available in Fluent, on the left and right sides of computational sides, instead of the rotational periodicity condition.

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