

Abstract: For wiredrawing force computation the literature presents numerous mathematical expressions. It cannot be stated that a certain formula determines more precisely than the other the wiredrawing force. Only experimental results obtained in real working conditions can for sure establish this aspect. This paper analyses the results obtained by using different formulas. By comparing the results obtained experimentally and theoretical results, the conclusion is that expression (15) can be used with good results to approximate the micro-fibre wiredrawing force.

Key Words: micro-fibre wiredrawing force computation

1 Introduction

The field of study books present numerous mathematical expressions used to determine the wiredrawing force [1], these expressions having a different content and varying from author to author. In the following expression:

\[
F = \sigma_T \left[ \left( 1 + \frac{\mu g \alpha}{\mu_f} \right) \left( 1 - \left( \frac{d}{d_0} \right)^{\frac{\mu_f}{\mu}} \right) + \frac{\pi d^3}{4} \right]
\]  

or in the expression

\[
F_w = \sigma \left[ 4 \mu \frac{l}{d} + (1 + \mu g \alpha) \frac{d_0^2 - d^2}{d^2} \right] \frac{\pi d^3}{4}
\]

and in other expressions which are no longer presented it cannot be stated that a certain formula determines more precisely than other the wiredrawing force.

Only experimental results obtained in real working conditions can for sure establish this aspect.
elementary material volume $MNN'M'$ from the deformation area (Fig. 1).

Fig. 1 Unitary stress in the wiredrawing deformation cone

Solving the forces from the $MNN'M'$ element in horizontal direction, we obtain:

$$
\left(\sigma_x + d\sigma_x\right)\pi \left(r + dr\right)^2 + \\
\frac{2\pi d r}{\sin \alpha} \cdot \sigma_x \left(\sin \alpha + \mu \cos \alpha\right) - \sigma_x \pi r^2 = 0
$$

$$
(4)
$$

Or

$$
-r d\sigma_x = 2 d r \left[\sigma_n (1 + \mu \cot \alpha) + \sigma_x\right]
$$

$$
(5)
$$

Assuming that $\sigma_n$ is a principal loading acting in a direction perpendicular on the wire axis, this being true for small angles, we can consider:

$$
\sigma_n = \sigma_m - \sigma_x
$$

$$
(6)
$$

And

$$
\mu \cot \alpha = \delta
$$

$$
(7)
$$

Replacing (6) and (7) in (5) one obtains:

$$
\frac{2 d r}{r} = \frac{d \sigma_x}{\delta \sigma_x - \sigma_x (1 + \delta)}
$$

$$
(8)
$$

After integration, we obtain:

$$
\delta \sigma_x - \sigma_x (1 + \delta) = c \cdot r^{2 \delta}
$$

$$
(9)
$$

where $c$ is a constant which is determined from the boundary conditions:

- for $r = \frac{d}{2}$, $\sigma_x = \sigma_t$, equation (9) becomes:

$$
\delta \sigma_x - \sigma_x (1 + \delta) = r^{2 \delta} \cdot \frac{\delta \sigma_x - \sigma_x (1 + \delta)}{\left(\frac{d}{2}\right)^{2 \delta}}
$$

$$
(10)
$$

- for $r=d/2$, $\sigma_x = 0$ and using the substitution $R = \frac{d^2}{d \sigma_x}$, equation (10) allows to determine the unitary stress at the exit from the deformation area:

$$
\sigma_i = \sigma_n \left(1 + \frac{1}{\delta}\right) \left(1 - \frac{1}{R^2}\right)
$$

$$
(11)
$$

In order to estimate the forces needed for the wiredrawing through the calibration area, one can write the equilibrium equation of the forces which action on the elementary volume surfaces from the calibration area (Fig. 1).

$$
\frac{\pi d^2}{4} (\sigma_{xw} + d \sigma_{xw}) + \pi d \mu \sigma_{xw} \cdot dx = \frac{\pi d^2}{4}.
$$

$$
(12)
$$

After reducing the similar terms, the expression above becomes:

$$
\frac{4 \mu c e^x}{d} = c (\sigma_{xw} - \sigma_m)
$$

$$
(13)
$$

Or

$$
\frac{4 \mu c e^x}{d} = c (\sigma_{xw} - \sigma_m)
$$

$$
(14)
$$

$c$ constant is determined using the boundary conditions:

$$
x = h_i; \\
\sigma_{xw} = \sigma_i,
$$

$$
c = \frac{4 \mu c e^x}{d} \left(\sigma_i - \sigma_m\right)^{-1}
$$

The total unitary stress $\sigma_T$ which appears on the wire section at the exit from the calibration area is obtained for $x=0$.

With these conditions from (14) we obtain:

$$
\sigma_T = \left(\sigma_i - \sigma_m\right) \cdot \frac{4 \mu c e^x}{d} + \sigma_m
$$

$$
(15)
$$
Taking into account the value of the unitary stress value from expression (11), the relation (15) becomes:

\[
\sigma_T = \sigma_m \left[ 1 + \frac{1}{\delta} e^{-2\mu} - \left( 1 + \frac{1}{\delta} \right) e^{-2\mu} \cdot \frac{1}{R^2} \right] \tag{16}
\]

For dimensions under 300 \( \mu \)m the calibration area length \( h_c = 0.5 \ d \) (Table 5), the working cone angle is \( 2\alpha = 16^\circ \) (Phase I) and \( \mu \) has a constant value for a given processing.

Taking into account the above considerations, we can consider as being constant the following expressions:

\[
1 + \frac{1}{\delta} e^{-2\mu} = k_1 \tag{17}
\]

\[
\left( 1 + \frac{1}{\delta} \right) e^{-2\mu} = k_2 \tag{18}
\]

and relation (16) becomes:

\[
\sigma_T = \sigma_m \left( k_1 - k_2 \frac{1}{R^2} \right) \tag{19}
\]

With this last expression, the traction force necessary for a wiredrawing process is:

\[
F = \sigma_m \left( k_1 - k_2 \frac{1}{R^2} \right) \frac{\pi d^2}{4} \tag{20}
\]

Analyzing this formula one can deduce that it offers an overall image regarding the main factors which influence the wire processing, namely [3]:

- metal deformation average resistance
- working
- cone opening angle
- friction coefficient expression between the contact surfaces
- deformation degree.

In order to verify expression (20) we followed more steps in the given conditions:
- material: AlSi1Mg0.5;
- dry wiredrawing;
- friction coefficient \( \mu = 0.12 \);

### 3 Results

<table>
<thead>
<tr>
<th>No.</th>
<th>Wire Diameter -( \mu )m-</th>
<th>Admissible resistance ( \sigma )N/mm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64</td>
<td>140.01</td>
</tr>
<tr>
<td>2</td>
<td>61</td>
<td>150.27</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>153.25</td>
</tr>
</tbody>
</table>

**Table 1**

Admissible resistance values after wiredrawing

<table>
<thead>
<tr>
<th>No.</th>
<th>Traction Force (experimental) ( N )</th>
<th>Traction Force (theoretical) ( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \phi 64 \ \mu \m 0.15 ) ( 3 )</td>
<td>( \phi 64 \ \mu \m 0.147 )</td>
</tr>
<tr>
<td>2</td>
<td>( \phi 61 \ \mu \m 0.16 ) ( 0 )</td>
<td>( \phi 55 \ \mu \m 0.176 )</td>
</tr>
</tbody>
</table>

**Table 2**

**VERSION II**

- wire diameter reduction from 74 \( \mu \)m to 61 \( \mu \)m in one step.

<table>
<thead>
<tr>
<th>Crt. No.</th>
<th>Wire diameter ( \mu )m</th>
<th>Admissible resistance ( \sigma )N/mm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74</td>
<td>113.45</td>
</tr>
<tr>
<td>2</td>
<td>61</td>
<td>160.55</td>
</tr>
</tbody>
</table>

**Table 3**

**VERSION I:**

- wire diameter reduction from 64 \( \mu \)m to 55 \( \mu \)m following the steps:

\( 64\mu m \Rightarrow 61 \mu m \Rightarrow 55\mu m \)
Table 4

<table>
<thead>
<tr>
<th>Crt. No.</th>
<th>Traction Force (experimental) N</th>
<th>Traction force (theoretical) N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.304</td>
<td>0.277</td>
</tr>
</tbody>
</table>

VERSION III

- wire diameter reduction from 68 µm to 61 µm in one step.

Table 5

<table>
<thead>
<tr>
<th>Crt. No.</th>
<th>Traction Force (experimental) N</th>
<th>Traction force (theoretical) N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.206</td>
<td>0.212</td>
</tr>
</tbody>
</table>

4 Conclusion

Analyzing the obtained results we can state that there is closeness between the results obtained experimentally and theoretical results. The conclusion is that expression (15) can be used with good results to approximate the micro-fibre wiredrawing force [4].

References: