Research about Correlation between the Performance of Plate Heat Exchanger and the Number of Plates

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Abstract: - Plate heat exchangers are very important equipments used in industrial applications. The paper presents an analysis related to the influence of the number of plates on the performance of a heat exchanger. 3D models are made for eight cases and using finite element method are performed numerical simulations of fluid flow distribution. Numerical results are presented for 1 pass -1pass counter-flow plate heat exchanger.

Key-Words: - plate heat exchanger, finite element method, mesh, fluid flow, number of plates, heat exchanger effectiveness, log mean temperature difference

1 Introduction

Plate heat exchangers are very common equipments used in industrial applications. These are used to exchange heat between two fluids, cooling and heating processes, heat recovery. Plate heat exchangers have a lot of advantages, including a high heat exchange area per unit volume and good heat transfer performance [1].

An important number of numerical studies applying finite element method has been made to research fluid flow and heat transfer into plate heat exchangers [2], [3]. It was studied the optimal configuration of a heat exchanger, too [4], [5].

The paper presents some aspects about the influence of number of plates on heat transfer in a plate heat exchanger. Eight constructive cases are considered, obtained by changing the number of thermal plates. Meanwhile, the paper represents a continuation of research in [6], [7] and tackle them from a new perspective.

Geometric models are realized using SolidWorks. To realize the proposed studies, applications based on computer simulation of fluid flow are made using COSMOS/Flow.

2 Presentation of studied plate heat exchangers

The studied heat exchangers are a pack of few stainless steel flat plates with gaskets. The hot water flows are in one direction in alternating chambers while the cold water flows are in counter flow in the other alternating chambers. The number of passes is 1 and the thermal agents are directed into their proper chambers either by a suitable gasket made from rubber EPDM [7]. The width of channel between plates is 0,004 m. The geometric dimensions of the thermal plate are represented in table 1.

Dimension	Notation	Value	Unit
Diameter of the inlet tube	d	0,002	m
Effective plate length	L	0,43	m
Effective plate width	l	0,13	m
Stainless steel plate thickness	δ_p	0,002	m

Table 1.Geometric dimensions of the plate

Heat exchangers are used to warm the cold water, considered a secondary thermal agent, with hot water, a primary thermal agent.We are interested to get as high the outlet cold water temperature, with higher heat exchanger effectiveness as well.

The influence of the number of plates on heat transfer is investigated considering a variable number of plates : 3,4,5,6,7,8,9,10 plates. It is important to note that the heat exchangers are 1pass-1pass counter-flow plate heat exchangers. Figure 1 shows the functional scheme of a heat exchanger with 4 plates –an even number of plates and the functional scheme for the case with odd number of plates, 5 plates respectively. It is observed that for a counter-flow plate heat exchanger with an odd number of plates, the number of flow channels is identical for hot and cold water. For a counter-flow exchanger with an even number of plates,

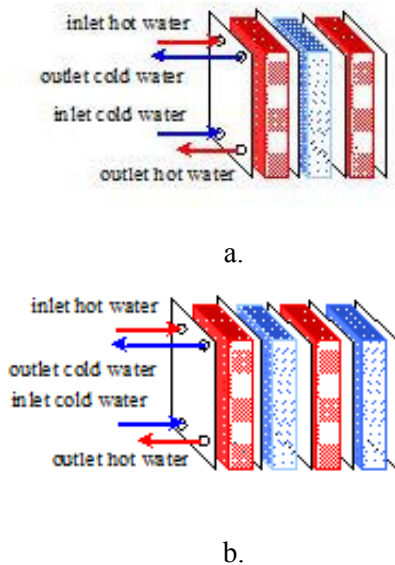in the studied cases, the number of flow channels for hot water is one greater than for cold water.

a.

b.

Fig.1.The functional scheme of plate heat exchanger
a.4 plates, b.5 plates

3D geometric models of the heat exchangers are created using SolidWorks program. Figure 2 shows the model of plate heat exchanger with 10 plates.
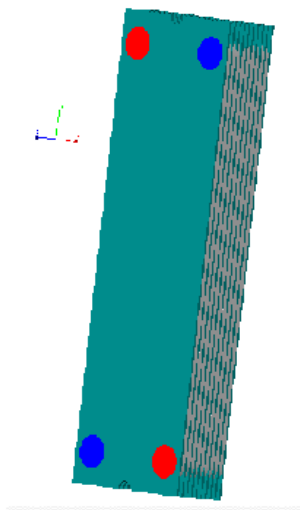

Fig.2. 3D model of counter-flow plate heat exchanger with 10 plates

3 Mathematical modeling of plate heat exchangers

Mathematical modeling is based on the governing equations which include the continuity equations, the Navier-Stokes equations and the energy equation. These equations are intimately coupled and non-linear making a general analytic solution almost impossible. The governing pdes can be written as [7], [8]:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \qquad (1)$$

x-,y-,z-, momentum equations:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} =$$
$$= \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[2\eta \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] + S_\omega + S_{DR}$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} =$$
$$= \rho g_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left[2\eta \frac{\partial v}{\partial y}\right] + \frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\eta\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] + S_\omega + S_{DR}$$

$$\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} =$$
$$= \rho g_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left[2\eta \frac{\partial w}{\partial z}\right] + \frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] + S_\omega + S_{DR} \qquad (2)$$

The two source terms in the momentum equations, S_ω and S_{DR}, are for rotating coordinates and distributed resistances, respectively. The distributed resistance term can be written in general as [8]:

$$S_{DR} = -\left(K_i + \frac{f}{d}\right)\frac{\rho V_i^2}{2} - C\eta V_i, \qquad (3)$$

where i refer to the global coordinate direction (u, v, w momentum equation), f- friction factor, d- hydraulic diameter, C – permeability and the other factors are descript in table 2. Note that the K-factor term can operate on a single momentum equation at a time because each direction has its own unique K-factor. The other two resistance types operate equally on each momentum equation [6], [8].

The other source term is for rotating flow. This term can be written in general as [8]:

$$S_\omega = -2\rho \omega_i \times V_i - \rho \omega_i \times \omega_i \times r_i, \qquad (4)$$

where i refer to the global coordinate direction, ω is the rotational speed and r is the distance from the axis of rotation.

For incompressible and subsonic compressible flow, the energy equation is written in terms of static temperature [8]:

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} + \rho c_p v \frac{\partial T}{\partial y} + \rho c_p w \frac{\partial T}{\partial z} =$$
$$= \frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + q_V \qquad (5)$$

The volumetric heat source term from equation (5) is considered zero for this model.

Table 2 presents the variable of the equations:

Variable	Description
c_p	specific heat at constant pressure
k	thermal conductivity
p	pressure
q_V	volumetric heat source
T	temperature
t	time
u	velocity component in x-direction
v	velocity component in y-direction
w	velocity component in z-direction
ρ	density
η	dynamic viscosity

Table 2. Variable of the governing equation

The equations describe the fluid flow and heat transfer under steady-state conditions for Cartesian geometries. For the turbulent flow, the solution of these equations would require a great deal of finite elements (on the order of $10^6 - 10^8$) even for a simple geometry as well as near infinitesimal time steps. COSMOS/Flow solves the *time-averaged* governing equations.

The time-averaged equations are obtained by assuming that the dependent variables can be represented as a superposition of a mean value and a fluctuating value, where the fluctuation is about the mean [8]. For example, the velocity component in y-direction can be written:

$$V = V + v', \ [\text{m/s}] \qquad (6)$$

where V [m/s] – the mean velocity, v' [m/s] – the fluctuation about the mean. This representation is introduced into the governing equations and the equations themselves are averaged over time.

4 The realized analysis, using finite element analysis

Researchers found a lot of ways to solve the equations in fluid flow domain. A reliable numerical technique often used is finite element method (FEM). This is used to *discretize* the flow domain, thereby transforming the governing partial differential equations into a set of algebraic equations whose solution represents an approximation to the exact analytical solution [7].

In this paper Cosmos/Flow program is used for solving heat and momentum transfer equations in a moving fluid [8]. The numerical formulation is derived from SIMPLER solution scheme, introduced by Patanker [8].

In figure 3 is shown the used analysis scheme. A set of simplified hypothesis are introduced for realize the mathematical models:

- Hot water and cold water are Newtonian fluids;
- No phase change occurs, the fluids are unmixed;
- Turbulent flow is fully developed;
- Working fluids are incompressible,
- Steady state conditions;
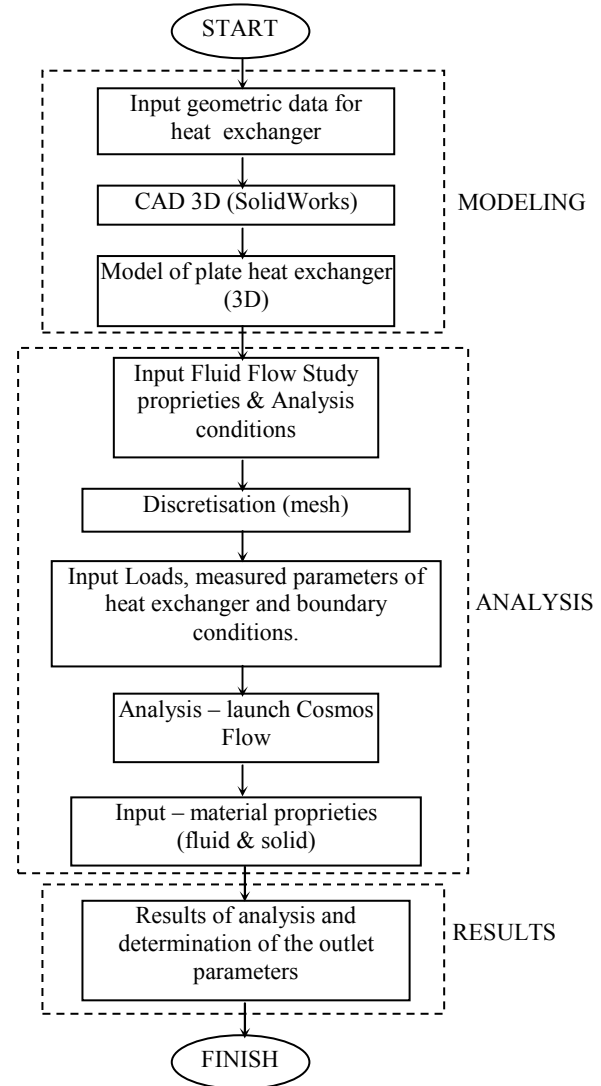- Coefficient of heat retention equal with 1.

Fig.3. Analysis scheme

In COSMOS/Flow it is open the 3D model and it is defined the case study, by specifying the name, analysis type, the disparagement mode. The study is fluid flow type and the analysis is thermal (result type: permanent or transient, the iterative resolving technique: Newton-Raphson scheme, initial temperature and calculus tolerance) [7].

After those, it applied mesh, the disparagement mode, which is very important for the final results. The models are divided in a multitude of little parts with simple geometrical forms, defined as finite parts, and connected

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in common points called nods. From analysis point of view, FEM accounts the different behavior of each part depending of the imposed conditions, the global behavior of the model being calculated from the totality of calculated information for all the finite elements of the model [8]. The mesh quality depends on the active mesh options. In this case, the quality of mesh is high (10-node tetrahedral). Mesh type is solid mesh. The number of elements and nodes associated with the studied geometry are given in table 3.

Number of plates	Nodes	Elements	Element size [mm]
3	110610	72159	5,2306
4	147265	99046	5,3894
5	169261	115136	5,8804
6	186844	127891	6,6008
7	218811	151027	6,6714
8	238019	164897	7,005
9	259729	180407	7,3094
10	286491	199604	7,5904

Table 3. Detail of mesh

For all eight cases are proposed the next boundary conditions:

No.	Fluid	Description	Unit	Value
1	Hot Water inlet	Temperature T_1,in	^{o}C	53
2		Velocity, v_1	m/s	0,303
4	Cold Water inlet	Temperature T_2,in	^{o}C	8
5		Velocity, v_2	m/s	0,26
6	Hot water outlet	Static pressure	N/m^2	0
7	Cold water outlet	Static pressure	N/m^2	0
8	Air	Convection on exterior plate,α	W/m^2K	5
9	Air	Convection on exterior plate,α	W/m^2K	5

Table 4. Inlet data for analysis

For incompressible flows, the most robust condition for the pressure equation is to specify a value at the outlet. Since only relative pressures are calculated by COSMOS/Flow, a value of 0 is recommended [8]. The numerical simulation of turbulent flow is modeled by $k-\varepsilon$ turbulence model. ε represents the turbulent energy dissipation. To calculate the boundary layer, either "wall functions" are used, overriding the calculation of k and ε in the wall adjacent nodes, or integration is performed to the surface, using a "low turbulent Reynolds (low-Re) k-ε" model [5].

In figure 4 is shown the disparagement mode for the model of heat exchanger with 7 plates. It is very important to realize the mesh with great accurately. In the case of high quality mesh, time for analyze running is very big.
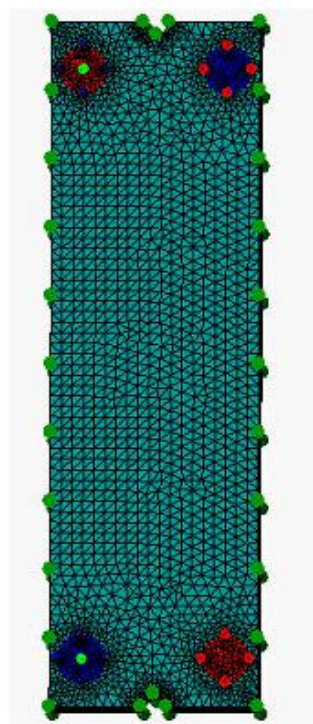

Fig.4. Mesh

5 Results

After the analysis was processed it can be visualized the results, under graphical form or numerical value. Because the governing equations are non-linear, they must be solved iteratively. A Picard or successive substitution is used [8]. In this method estimates of the solution variables are substituted in the governing equations. The equations are solved for new values which are the used as the estimates for the next pass. The convergence criterion is the level at which the specified variable's residual norm must reach. With each pass, the residuals should become smaller if the solution is converging.

Analysis runs for 100 iterations, in turbulence conditions, for all eight cases. Profiles are obtained for the following parameters: $u, v, w, T, k, \varepsilon$.

In figure 5 is presented the distribution of the nodal temperature, after 100 iterations, for the heat exchanger with eight plates.
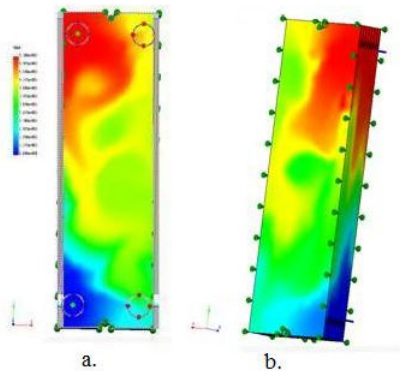

Fig.5. Distribution of the temperature: a. front view, b. rear view

Convergence control of a solution variable is accomplished by reduction the solution progression rate so that the change of divergence is minimized. COSMOS/Flow has the Graphical Convergence Monitor, where are presented the numerical data like the average, the average minimum, average maximum values for each degree of freedom over the completed range of iterations.

The values obtained for average temperature at the outlet of hot water and outlet of cold water are presented in table 5.

Number of plates N_p	Hot water outlet $T_{1,out}$ [°C]	Cold water outlet $T_{2,out}$ [°C]
3	31,8	32,6
4	25,2	40,2
5	27,8	37,4
6	22,7	43,2
7	23,4	42,4
8	22	44,1
9	22,08	43,9
10	21,4	44,7

Table 5. Determined values of average temperatures

As expected, it is observed that with increasing number of plates, cold water temperature at outlet, increase, too. Faster growth is obtained for heat exchangers with even number plates. In these cases, there is an extra hot-water channel, under the original construction considerations.

Knowing for both agents, flow volumes, $V_1' = 1,905*10^{-4}$ m^3/s for hot water and $V_2' = 1,633*10^{-4}$ m^3/s for cold water and knowing thermodynamic proprieties, we can determine flow stream heat capacity rates for all cases. Formulas for heat capacity rates are:

$$C_1 = V'_1 \times c_{p,1} \times \rho_1 \text{ [W/K]} \tag{7}$$

$$C_2 = V'_2 \times c_{p,2} \times \rho_2 \text{ [W/K]} \tag{8}$$

where c_p [J/kgK] – heat capacity at constant pressure, ρ [kg/m^3] – density.

Minimum heat capacity rate is:

$$C_{min} = min(C_1, C_2). \text{ [W/K]} \tag{9}$$

Number of plates N_p	Hot water outlet C_1[W/K]	Cold water outlet C_2 [W/K]	C_{min} [W/K]
3	789,198	681,828	681,828
4	790,152	680,883	680,883
5	789,778	681,227	681,227
6	790,509	680,519	680,519
7	790,408	680,616	680,616
8	790,608	680,410	680,410
9	790,597	680,435	680,435
10	790,693	680,338	680,338

Table 6. Heat capacity rates

We observe that $C_1 > C_2$.

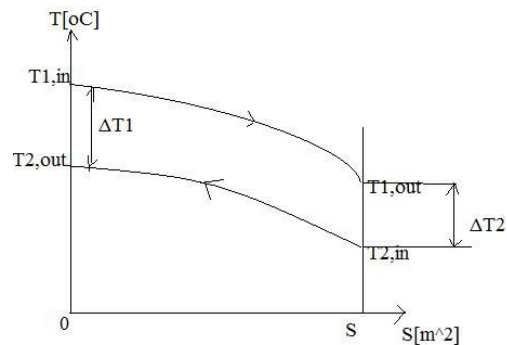The hot and cold water temperature distributions in the counter-flow heat exchanger are shown in figure 6.

Fig. 6. The hot and cold fluid temperature distributions in the counter flow heat exchanger

LMTD – log mean temperature difference is computed under assumption of counter flow condition with next relation [1]:

$$LMTD = \frac{\Delta T_{max} - \Delta T_{min}}{\ln\left(\frac{\Delta T_{max}}{\Delta T_{min}}\right)} \tag{10}$$

where $\Delta T_{max} = \max(\Delta T1, \Delta T2)$, $\Delta T_{min} = \min(\Delta T1, \Delta T2)$; $\Delta T1, \Delta T2$ from figure 6.

Heat exchanger effectiveness ε is determined with relation (11).

$$\varepsilon = \frac{C_1 \times (T_{1,in} - T_{1,out})}{C_{min} \times (T_{1,in} - T_{2,in})} = \frac{C_2 \times (T_{2,out} - T_{2,in})}{C_{min} \times (T_{1,in} - T_{2,in})} \tag{11}$$

We study separately heat exchangers with even number of plates and heat exchangers with odd number of plates, and after that, we compare the results.

N_p	3	5	7	9
LMTD	22,056	17,616	12,851	11,409
ε	0,547	0,653	0,764	0,798

Table 7. Results for heat exchanger with an odd number of plates

N_p	4	6	8	10
LMTD	14,892	12,085	11,258	10,647
ε	0,716	0,782	0,802	0,816

Table 8. Results for heat exchanger with an even number of plates

In these cases, the efficiency of exchangers with an even number of plates is greater than that of those with odd number plates. One explanation is that exchangers with an even number of plates has the number of flow channels for hot water is one greater than for cold water. The efficiency changes depending on the number of plates are shown in figure 7.

It is noted that even versus odd number of plates have a strong influence for $N_p < 8$ and for $N_p = 9$, $N_p = 10$ a negligible influence (the increase in ε is insignificant). The same applies to cold water temperature at outlet. The paper [4] presented similar findings, extended for $N_p = 40$ plates.

6 Conclusions

The conducted research about the counter-flow plate heat exchanger, using finite element method, represents an important tool for study the fluid flow distribution and heat transfer in a plate heat exchanger. In the sometime, the model is perfectible and offers good information about the behavior of counter-flow plate heat exchanger in different conditions of operation and it can be integrated in a more complex system [7].

Numerical results are presented for 1 pass-1 pass counter-flow heat exchangers. These results are shown in terms of cold water temperature at the outlet, hot water temperature at the outlet, exchanger effectiveness and log –mean temperature difference as functions of the heat capacity ratio and the number of plates.
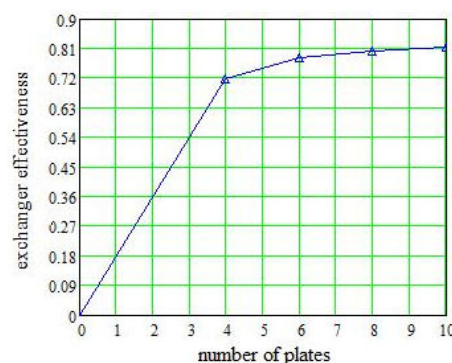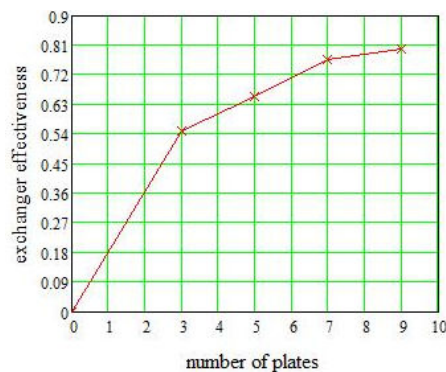

Fig. 7. The influences of the number of plates for 1pass-1 pass counter-flow heat exchanger effectiveness: a. even number of plates, b. odd number of plates

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