Computer-aided analysis of the heat transfer in skin tissue

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Abstract – The thermal therapies are based on the heat transfer in biological tissues. The goal of this paper is to present the computer-aided analysis of the heat transfer in skin tissue using the clinical data and experience of the doctors. Some mathematical models are presented starting from Pennes’ bioheat transfer equation for different boundary conditions in the transient regimes. An exact solution is presented for a one-layer model. For multilayer models, the numerical results are presented for some particular situations using the finite element method.

Keywords: Thermal therapy; Numerical solution; Bioheat transfer equation.

1 Introduction
Clinicians make use of a number of treatments that utilize heat transfer phenomena. With the advanced computers, numerical models were developed to simulate the distribution of temperature in biological tissues. Heat transfer is fundamental and very important process in living tissues in order to maintain an almost constant temperature. The thermal therapies utilize heat transfer phenomena in biological tissues, especially in skin tissue. There are some methods as cryosurgery, hypothermia and hyperthermia that are based on the doctor’s experience, and medical traditional methods as moxibustion. Thus, hypothermia inhibits the metabolism by cooling; hyperthermia heats the tissue locally by electromagnetic waves; moxibustion is a local heating method using combustion of an herb (moxa). But for any method, accurate evaluation of the spatial and temporal distribution of the temperature in the skin is of great importance in the actual thermal therapies based on new technologies and devices.

Research on the prediction of living tissue is a hot research because the temperature control in living tissue is critical for maintaining a healthy state. The heat transfer modelling has developed continuously since 1948, when Pennes proposed a simple heat transfer model called the bio-heat equation in which the blood perfusion rate is treated as a source or sink term to generate or absorb heat in tissue.

In practice, in clinical field it is difficult for a doctor to evaluate accurately the thermal response of the biological tissues because of the complex mechanisms that maintain body temperature, such as blood flow and metabolic heat generation. Therefore, it is very important to provide doctors with useful data concerning the thermal analysis of biological tissues. Two relevant parameters for medical doctors are:

- Steady-state thermal penetration depth
- The time to reach a stable thermal steady-state

Numerical modelling of the heat transfer plays an important role in biomechanics by either solving existing equations or assisting in determining unknown constitutive equations.

Human skin may be divided into different layers as shown in Fig. 1 with a three-layer model. Starting from the surface of the skin, there are the epidermis, the dermis and hypodermis [2]. The skin is the house of many phenomena including heat transfer, blood circulation, sweating, metabolic heat generation and the interaction with the surrounding environment. A study of this organ involves an interdisciplinary analysis.

From a simple visual analysis of the Fig. 1 we conclude that each layer has distinct physical properties, especially in the thermal behaviour of the skin. More, even within the same layer, there is a large non-homogeneity and anisotropy due to presence of the blood vessels. Consequently, a linear mathematical model or an analytical solution of the skin behaviour is not possible. Only a numerical model can lead to an approximate solution, can predict the effects of different factors on the thermal response of the skin tissue.

Also, ones of the subsidiary but important motivations for modelling of heat transfer phenomena and heat-induced stress are:

- Development of biological and biomedical technologies
Design of heating or cooling garments

By use of the temperature fields obtained by numerical simulation, the corresponding thermal damage and thermal stress fields can be calculated.

2 Heat transfer models

In the professional literature for the modelling of the human body and thermal comfort, there is a large variety of mathematical models on the heat transfer in different tissues of the human body, including the influence of the blood flow in the vascular network. These models are developed in some assumptions that are [3]:

- The biological tissue is isotropic and homogeneous
- The physical properties of the tissue are independent of the tissue temperature
- Arterial blood temperature is constant at 37°C
- The metabolic heat generation rate is constant per unit volume and unit time
- The blood perfusion rate is uniformly spatially and temporally and independent of tissue temperature

In the case of bioheat transfer, the computational domain may be selected as a rectangular box. The computational domain can be considered in space of different dimensions (3D, 2D or 1D), using different coordinate systems.

Most theoretical analyses of heat transfer in living tissue are based on the Pennes’ equation, which describes the influence of blood flow on temperature distribution in the tissue in terms of volumetrically distributed heat sinks or sources. The standard Pennes’ bioheat equation is used to determine the

\[
\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \omega_b c_b \left( T_T - T \right) + q_{\text{met}} + q_{\text{ext}} \quad (1)
\]

Here, \( \rho \), \( c \), \( k \) and \( T \) denote density, specific heat, thermal conductivity and temperature of tissue. The density, specific heat, and perfusion rate of blood are denoted by \( \rho_b \), \( c_b \) and \( \omega_b \), respectively. The heat source is denoted by \( q \) and represents the sum of two
components: \( q_{\text{met}} \), which is the metabolic heat generation in the skin tissue, and \( q_{\text{ext}} \), which is the heat source due to external heating. \( T_a \) is the arterial temperature and it is regarded as a constant and equal to 37 °C.

The effect of blood perfusion rate has a significantly large influence on temperature distribution during cooling than during heating. In general, the skin temperature decreases with an increasing blood perfusion rate.

Besides the thermal parameters and metabolic rate of tissue, the skin temperature is also determined by any other factors such as the skin humidity, radiation emissivity of skin, and parameters of surrounding air. These factors can be incorporated into the boundary condition (BC) at skin surface. The boundary condition for the heat transfer occurring at skin surface is generally included in one of the following kinds of conditions [4]:

1. **Dirichlet condition (constant temperature):**
   \[ T_{\text{skin}} = T_{\infty} \] (2)

2. **Neumann condition (specified heat flux):**
   \[ -k \frac{\partial T}{\partial n_{\text{skin}}} = q_s \] (3)

3. **Convective condition:**
   \[ -k \frac{\partial T}{\partial n_{\text{skin}}} = h(T_{\infty} - T) \] (4)

4. **Radiation condition:**
   \[ -k \frac{\partial T}{\partial n_{\text{skin}}} = \varepsilon \sigma (T_{\text{skin}}^4 - T^4) \] (5)

where \( n \) is the outward normal at the boundary of computational domain, \( \varepsilon \) is skin emissivity and \( \sigma \) is Stefan–Boltzmann’s constant in W/(m\(^2\) K\(^4\)).

First kind BC represents heating/cooling at a constant temperature \( T_{\infty} \), second kind BC represents heating/cooling by constant heat flux \( q_s \), third kind BC represents heating/cooling by convective heat transfer, which means heat exchange between the tissue surface and fluid at a constant temperature \( T_{\infty} \), and fourth BC represents heating/cooling by radiative heat transfer. Radiation is the loss of heat in the form of infrared waves. All objects continually radiate energy in accordance with the Stefan–Boltzmann law. When the surrounding is cooler than the body, net radiative heat loss occurs. Under normal conditions, close to half of body heat loss occurs by radiation. In contrast, a net heat gain via radiation occurs when the surrounding is hotter than the body.

At the outer surface of the human body, heat transfer can consist of a linear combination of all kinds of boundary conditions (convection, radiation, and sweat). The generalized boundary condition for the heat transfer occurring at skin surface is generally composed of three parts, i.e. convection, radiation and evaporation. It thus can be written as

\[ -k \frac{\partial T}{\partial n_{\text{skin}}} = h(T_{\infty} - T) + \varepsilon \sigma (T_{\text{skin}}^4 - T^4) + Q_e \] (6)

where \( Q_e \) is the evaporative heat losses due to sweat secretion.

This non-linear boundary condition due to occurrence of the \( \varepsilon \sigma T^4 \) term can be solved through an iteration procedure.

### 2.1 An 1-D model for bioheat transfer

Many authors reported analytical solutions to the bioheat transfer equation problems with space or transient heating on skin surface or inside bodies obtained by the Green’s function method.

A first target example is the skin in contact with a hot plate. The skin is heated at the surface by a heat source with a constant temperature \( T_{\infty} \), while the bottom of the skin is kept at body temperature or it is insulated. We give attention on the cases that heat mainly propagates in the direction \( O_x \) perpendicular to the skin surface (see Fig. 2). As a result, one-dimensional heat transfer can be a good approximation.

The 1-D form of Eq. (1) with constant thermal parameters, \( q_{\text{met}} = \text{constant} \), and \( q_{\text{ext}} = 0 \) is written as

\[ \rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + w_p \rho_p c_p (T_a - T) + q_{\text{met}} \] (7)

A particular form of this equation can be obtained if we introduce a new unknown \( \theta \) as the difference between \( T \) and the initial steady state temperature \( T_i \), that is [5]:

\[ \theta = T - T_i \]

where \( T_i \) is the solution of the equation:
\[ \frac{\partial^2 T}{\partial x^2} + w_b \rho_b c_b \left( T_a - T_i \right) + q_{\text{met}} = 0 \]  
\hspace{1cm} (8)

From Eqs. (6) and (7) the following equation is obtained:

\[ \rho c \frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2} + w_b \rho_b c_b \theta \]  
\hspace{1cm} (9)

with initial condition:

\[ \theta(x,0) = 0 \]  
\hspace{1cm} (10)

As boundary condition, we consider that surface temperature is kept constant at the skin contacts with a large steel plate at a high temperature [5]:

\[ \theta(0,t) = \theta_0; \quad \frac{\partial \theta(L,t)}{\partial x} = 0 \]  
\hspace{1cm} (11)

2.2 Analytical solutions

Based on the Pennes model there is a large professional literature for the analytical solutions. For our target example, the Eq. (7) is solved with the initial condition:

\[ T(x,0) = T_\infty \]

and boundary condition:

\[ T(0,t) = T_{\text{max}} \]

The solution has the form [6]:

\[ \frac{T - T_\infty}{T_{\text{max}} - T_\infty} = 0.5[e^{-\lambda x} \text{erfc}\left( \frac{x}{2\sqrt{\alpha \cdot t}} \right) - \lambda \sqrt{\alpha \cdot t} \] + \[ e^{\lambda x} \text{erfc}\left( \frac{x}{2\sqrt{\alpha \cdot t}} + \lambda \sqrt{\alpha \cdot t} \right) \]  
\hspace{1cm} (12)

where

\[ \lambda = \sqrt{w_b c_b} \ ; \quad \alpha = \frac{k}{\rho c} \]

and \( \text{erfc}(x) \) is the complementary error function.

Similar solutions were reported for the cases with different boundary condition as the Neumann’s condition.

2.3 Numerical results

All computations are performed for the thermal behaviours in the skin with the following properties: \( \rho = 1050 \text{ [kg/m}^3\text{]} \); \( c = 3770 \text{ [J/kg}^\circ\text{C]} \); \( c_b = 3300 \text{ [J/kg}^\circ\text{C]} \); \( w_b = 1 \text{ [kg/m}^3\text{s]} \); \( \rho_b = 1100 \text{ [Kg/m}^3\text{]} \), and \( k = 0.52 \text{[W/m}^\circ\text{C]} \). The distance between the skin and body core is \( L = 0.01208 \text{ m} \) but we define a computational domain more large to include the effect of the temperature in depth. Our domain was \( L = 40 \text{ mm} \) and we used 30 points for the surface plotting. The interval for time was 250 s, for a plate temperature \( T_{\text{max}} \) equal to 90 \( ^\circ\text{C} \). The surface \( T(x, t) \) defined by formula (12) was plotted in the figures 3 and 4 for two different values of \( w_b \). The numerical results were obtained with software package Mathcad [8].

\[ \rho c \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \omega_b \rho_b c_b \left( T_a - T \right) + q_{\text{met}} \]  
\hspace{1cm} (13)
The physical and geometrical properties of the model are presented in Table 1 and are presented in the open literature [7]. In some practical conditions, the Eq. (11) takes simplified forms. For example, blood perfusion can be considered only in the dermis layer. If we neglect the flood flow in epidermis and fat, the bioheat equation may be reduced to a classical heat diffusion equation in these domains. More, in our first study, the metabolic heat generation is negligible since the temperature changes caused by the hot plate are much greater than the metabolic heat generation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Epidermis</th>
<th>Dermis</th>
<th>Fat</th>
</tr>
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<tbody>
<tr>
<td>Thickness [mm]</td>
<td>0.075</td>
<td>1.5</td>
<td>3.42</td>
</tr>
<tr>
<td>Density [Kg/m³]</td>
<td>1190</td>
<td>1116</td>
<td>971</td>
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<tr>
<td>Specific heat [J/Kg.K]</td>
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<td>2700</td>
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<tr>
<td>kₓ=ky</td>
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<td>0.37</td>
<td>0.16</td>
</tr>
<tr>
<td>qᵥ[meat] [W/m³]</td>
<td>368.1</td>
<td>368.1</td>
<td>368.3</td>
</tr>
</tbody>
</table>

The radius of the hot plate is 3 mm.

In Figures 4-7 the results of the numerical simulation are shown using the software Quickfielded [9]. The ambient temperature was considered as being 25 °C. The temperature of the plate was 70 °C and the time of simulation was 15 s. In many therapeutic applications a cooling period must follow after heating. This aspect can be included in the application software based on the finite element method.

The results presented in figures 4-8 were obtained for the natural convection stage at the boundary skin-air. The boundary condition was:

\[-k \frac{\partial T(r,z,t)}{\partial z} |_{z=L} = h[T(r,L,t) - T_\infty] \quad \forall r \in [0, R]\]

where h=7 W/m²K is the convective heat transfer coefficient for natural convection between skin tissue and air, and T_∞=25 °C is the temperature of the ambient temperature.

Obviously, for the forced convection stage the value of h is increased and is around value 12 and the temperature of the cooling fluid is less than natural ambient temperature.

In our target examples we considered the evaporation E at the skin surface as a boundary condition. In Figure 4 the initial temperature map is shown. The ambient temperature was considered as being 25 °C. In Figures 5 and 6 the final temperature map at 70 °C is presented.
condition of Neumann type with the value for $E = 10$ W/m$^2$.

Although we limited our study to a hot plate with an imposed temperature that is kept constant, the case with an imposed heat flux can be considered with a minimal effort. Many therapies using the laser can be modelled with the model presented in this paper. A simple boundary condition must be modified.

6 Conclusions
Clinicians make use of a number of treatments and therapies that are based on the heat transfer phenomena. It was not the goal of this work to present all possible techniques in this area but the common and essential phenomena in these therapies can be included in a general mathematical approach and our work was in this direction. The purpose of the present work was thus to examine the numerical solutions of the bioheat transfer equation to derive assistance for clinicians. Modern clinical treatments and medicines require the understanding of thermal life phenomena and temperature behaviour in living tissues. A computer-aided analysis of the heat transfer in the human body is useful not only for therapeutic techniques but is useful for designing clinical thermal treatment equipments, for evaluation of the skin burn and many other purposes.

Motivations for computer-aided analysis of the heat transfer in skin tissues are multiple. Ones of them are the complex behaviour of the human tissues and the large quantity of data that must be processed by a clinician in a limited time interval. Then, all phenomena in the heat transfer are nonlinear so that an analytical solution may be impossible and an approximated solution can lead to a correct approach for a patient.

We limited our study to some cases but the results can be extended to a large class of therapeutic techniques that represent our future objective. For example, a heat flux on the skin surface can be included by a boundary condition of Neumann type.

In our study we did not considered the skin appendages such as hair/fur and sweat glands that play significant roles in thermoregulation. These appendages can be included by modifying both computational domain and boundary conditions. The hair strands work as an insulation layer and can be included in the geometrical domain.

References

