The stability of operating parameters for fire-extinguishing rocket motor

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Abstract: - The aim of this paper consists in developing a model for realistic calculation, but at the same time not a very complicated one, in order to determine the operating parameters of a rocket motor with solid propellant (RMSP). The model results will be compared with experimental results and the quality of the model will be evaluated. The study of operating stability RMSP will be made accordingly to Liapunov theory, considering the system of parametric equations perturbed around the balance parameters. The methodology dealing with the stability problem consists in obtaining the linear equations and the verification of the eigenvalues of the stability matrix. The results are analyzed for a functional rocket motor at low pressure, which has the combustion chamber made of cardboard, motor used for fire-extinguishing rocket. The novelty of the work lies in the technique to tackle the stability problem for the operation of rocket motors at low pressure, many of them representing specific applications for civil destination.

Key-Words: - Rocket, Motor, Solid propellant, Stability, Liapunov theory, Fire-extinguishing

NOMENCLATURE

 $\lambda\,$ - Ratio between velocity in exit plane and velocity in throat area;

 ρ - Gas density in burning chamber;

 ψ - Ratio between propellant mass consumed and total propellant mass;

 $\boldsymbol{\phi}$ - Erosion factor;

 σ - Ratio between instantaneous burning surface and initial burning surface;

k - Gas specific heats ratio;

- A_t Throat area;
- A_e Exit area;
- F Motor thrust;

 $I_{\scriptscriptstyle \Sigma}\,$ - Total impulse;

- Q_{c} -Heat quantity educts by burning reaction;
- *p* Gas pressure in burning chamber;
- p_{e} Gas pressure in exit area;
- p_{H} Atmospheric pressure;
- *R* Gas constant in burning chamber;
- *T* Gas temperature in burning chamber;
- *u* Burning rate;
- u_{1n} Linear burning rate in normal conditions;
- V Volume of the burning chamber;
- w_e -Gas velocity in exit plane;
- w_t -Gas velocity in throat plane;
- *S* Instantaneous burning surface;
- S_T Instantaneous propellant cross surface;

1 Introduction

Using missiles into civilian area involve a series of specific measures for compliance with environmental restrictions like a greater degree of safety in operation, and person's protection. An example of such an application is the fire extinguishing rocket, which has a motor made of cardboard, ecological, non-hazardous but with low operating pressure. This type of technical problem causes the need for a scientific approach to support the technological effort of achieving such a missile motor capable of stable operating at low pressure, which is the subject to approach in this work.

Determining the functional parameters and analyzing the stability are one of the main challenges in designing rocket motor solid propellant - RMSP.

The problems of combustion stability can be addressed by different ways both experimental and theoretical, a series of methods and models being shown in the works [1], [3]. Note that some papers propose a different approach of stability for linear and non-linear phenomena. Unlike this, in our work the approach will be unitary, being focused on a particular and difficult case, that of low pressure combustion.

In our study we will develop a non linear model for calculus of the functional parameters of RMSP, followed by the analysis of the evolution of balance stability regarded as the basic movement. Stability analysis for the perturbed equations of the RMSP will be made according to Liapunov theory, by placing them in the linear form.

Resuming, our work has two purposes:

- Scientific one – to check the possibility of applying Liapunov theory [2] to analyze the stability of the balance parameters of RMSP at low pressure.

- Technical one – to design the rocket motor for the fireextinguishing rocket

2 RMSP internal ballistic model

An important parameter in an internal ballistic model for a rocket motor is the burning rate of propellant. In the case of RMSP, the burning rate is called regression rate and it is given by the relation indicated in paper [1]

$$u = \varphi(x)ap^m, \tag{1}$$

where the erosion factor has been denoted with $\varphi(x)$ and the coefficient *a* can be expressed by:

$$a = u_{oN} p_H^{-m} e^{D(T_{in} - T_N)}$$
(2)

where $e^{D(T_{in}-T_N)}$ shows the influence of the variation of the initial propellant temperature and p_H means atmospheric pressure. Exponent D, parameter m and regression rate u_{1N} are determined experimentally, under

normal propellant temperature (T_N) .

To assess the erosive phenomenon we use the parameter named in [1] "Pobedonosetov" parameter:

$$x = (S - S_T) / (S_{cam} - S_T),$$
(3)

which allows us to determine the erosion factor:

$$\rho(x) = \begin{cases}
1+3, 2 \times 10^{-3} (x-100) & \text{for } x > 100; \\
1 & \text{for } x \le 100
\end{cases} \tag{4}$$

In order to obtain surface burning area, we define the parameter:

$$\Psi = \left(V - V_0\right) / V_p \tag{5}$$

For burning area and propellant cross-section the quadratic fitting can be used:

$$\sigma(\psi) = \begin{cases} a_2 \psi^2 + a_1 \psi + 1 & \text{for } \psi < 1; \\ 0 & \text{for } \psi \ge 1 \end{cases} ; (6)$$

$$\sigma_T(\psi) = \begin{cases} b_2 \psi^2 + b_1 \psi + 1 & for \ \psi < 1; \\ 0 & for \ \psi \ge 1 \end{cases} ; \quad (7)$$

This results in:

$$S(\psi) = S_0 \sigma(\psi); \qquad (8)$$

$$S_T(\psi) = S_{T0}\sigma_T(\psi). \tag{9}$$

Altogether, by simple geometrical reasoning, volume variation in time is given by:

$$\dot{V} = S(\psi)\phi(x)ap^{m} , \qquad (10)$$

relation which represents volume equation.

Using the continuity equation, the variation of the mass in the burning chamber is the difference between the mass produced in time unit by burning the propellant and the mass that exits the motor through the nozzle in time unit:

$$\frac{\partial(\rho V)}{\partial t} = \dot{m}_{in} - \dot{m}_{out}, \qquad (11)$$

where V is the volume of the burning chamber, and ρ is gas density inside the burning chamber, \dot{m}_{in} is the input mass generated by the combustion of propellant inside the motor chamber and \dot{m}_{out} is the output mass ejected through the nozzle of the rocket motor. The input mass per time unit is given by the propellant input:

$$\dot{m}_{in} = \dot{m}_p, \qquad (12)$$

and the output mass in time unit is expressed by the exit through the nozzle:

$$\dot{m}_{out} = \Lambda A_t \sqrt{p\rho} , \qquad (13)$$

where A_t is the throat area, p is chamber pressure, ρ is gas density, and

$$\Lambda = \sqrt{k (2/(k+1))^{(k+1)/(k-1)}} .$$
(14)

Taking into account that the propellant consuming mass in time unit is:

$$\dot{m}_p = \rho_p \dot{V}, \qquad (15)$$

developing relation (11) we obtain density equation:

$$\dot{\rho} = \left(\rho_p - \rho\right) \frac{V}{V} - \frac{\Lambda A_t}{V} \sqrt{p\rho} \quad . \tag{16}$$

From equation (10) the density equation becomes:

$$\dot{\rho} = (\rho_p - \rho) S \varphi a V^{-1} p^m - \Lambda A_t V^{-1} p^{1/2} \rho^{1/2}$$
(17)

Beside the volume equation (10) and density equation (17), the third equation expressing the change in temperature or pressure of the combustion products we need.

We consider the input energy for the system is given by the heat quantity Q_c inserted by burning reaction of m_p solid propellant.

Also, we take into account that the specific heat at constant volume C_v can be obtain from the relation:

$$C_V = R/(k-1). \tag{18}$$

where k is the ratio of specific heats and R is the gas constant in burning chamber.

To build the temperature equation, we start from the following relationship of energy balance:

$$dU = dU_1 + dU_2 + dU_3 + dU_4, \qquad (19)$$

where the reaction energy of the propellant, given by:

$$\mathrm{d}U = Q_C \,\mathrm{d}\,m_p \,\,, \tag{20}$$

is converted into:

- internal energy growth due to additional gas from the combustion chamber:

$$dU_{1} = C_{V}TV d\rho = R(k-1)^{-1}TV d\rho; \quad (21)$$

- energy in gas from the combustion chamber increased due to temperature variation:

$$dU_{2} = \rho V C_{V} dT = R(k-1)^{-1} \rho V dT ; \quad (22)$$

- kinetic energy due to gas flow:

$$dU_{3} = k(k-1)^{-1} RT dm_{out}.$$
 (23)

- loss of energy due to the disposal of heat through the chamber walls:

$$\mathrm{d}U_4 = q\,\mathrm{d}t\,,\qquad(24)$$

where q is the amount of heat transferred to the combustion chamber in time unit (heat flow) [J/s].

If we take the derivative of (19) with respect to time and then simplify it, we obtain:

$$Q_C \dot{m}_p / C_V = TV \dot{\rho} + \rho V \dot{T} + kT \dot{m}_{out} + q / C_V \quad (25)$$

hence we obtain the temperature equation:

$$(k-1)Q_C \frac{\rho_p}{p} \frac{\dot{V}}{V} = \frac{\dot{\rho}}{\rho} + \frac{\dot{T}}{T} + \frac{k\Lambda A_t}{V} \sqrt{\frac{p}{\rho} + \frac{(k-1)q}{pV}}$$
(26)

Taking into account that the state equation can be written in form:

$$\dot{p}/p = \dot{\rho}/\rho + \dot{T}/T$$
, (27)

we transform the temperature equation (26) into the pressure equation:

$$\dot{p} = (k-1)\rho_p Q_C S \varphi a V^{-1} p^m - -k\Lambda A_t V^{-1} \rho^{-1/2} p^{3/2} - (k-1)q V^{-1}.$$
(28)

Having differential equations (17) and (28) solved, for temperature we can use the state relation:

$$T = p/(R\rho). \tag{29}$$

Regular paper, for the rate between throat area A_t and exit area of the nozzle A_e propose the relation:

$$\frac{A_{t}}{A_{e}} = \sqrt{\frac{2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{1-k}} \widetilde{p}_{e}^{\frac{2}{k}} \left(1 - \widetilde{p}_{e}^{\frac{k-1}{k}}\right)}, \quad (30)$$

with the relative pressure is given by:

$$\widetilde{p}_e = p_e / p , \qquad (31)$$

where p_e is gas pressure in exit area.

If we take into account that the gas velocity in the exit plane is:

$$w_e = \sqrt{2\frac{k}{k-1}RT\left(1-\widetilde{p}_e^{\frac{k-1}{k}}\right)},\qquad(32)$$

and the gas velocity in throat plane is:

$$w_t = \sqrt{2\frac{k}{k+1}RT} \quad , \tag{33}$$

the velocity report becomes:

$$\lambda = w_e / w_t = \sqrt{\frac{k+1}{k-1} \left(1 - \widetilde{p}_e^{\frac{k-1}{k}}\right)} .$$
 (34)

From (30) and (34) we can obtain the rate surfaces formula:

$$\frac{A_{t}}{A_{e}} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \lambda \left[1 - \frac{k-1}{k+1}\lambda^{2}\right]^{\frac{1}{k-1}}, \quad (35)$$

The relation (35) leads to the transcendental equation:

$$\lambda = \frac{A_t}{A_e} \left(\frac{k+1}{2}\right)^{\frac{1}{1-k}} \left[1 - \frac{k-1}{k+1}\lambda^2\right]^{\frac{1}{1-k}}.$$
 (36)

We can observe that the right member of the relation (36), satisfies the inequality:

$$\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda} > 1, \qquad (37)$$

which means that relation (36) considerate like iterative, does not converge. In this case, we put this relation in Newton-Raphson form:

$$\lambda_{i+1} = \lambda_i - \frac{\lambda_i - f(\lambda_i)}{1 - \frac{\mathrm{d} f(\lambda)}{\mathrm{d} \lambda} \Big|_{\lambda_i}}.$$
 (38)

Assuming constant ratio of specific heats throughout the expansion process, one finds the thrust force relation indicated in paper [1]:

$$F = A_e p_H \left[\sigma_c \frac{p}{p_H} \frac{A_t}{A_e} k \left(\frac{2}{k+1} \right)^{\frac{1}{k-1}} \lambda - 1 \right]$$
(39)

where p_H is atmospheric pressure, and σ_c is overall loss of thrust by nozzle. The simplest nozzle is the conical one with a divergence cone half angle of 10-18 degrees. For such nozzles, part of the force of exhaust gases is orientated transversally and thus does not produce any thrust at all. In order to correct this phenomenon one can use a correction factor related to the divergence cone half angle. Also other loses can appear, all of these can be taking into account using coefficient σ_c .

3 Balance parameters

The studying of stability in operating a RMSP will be made according to Liapunov theory, considering the system of parametric equations perturbed.

This means that one has to consider the system of parametric equations perturbed around the balance parameters. This involves a disturbance applied shortly on the evolution of balance, which will produce a deviation of the state variables. Developing in series the perturbed parametric equations in relation to status variables and taking into account the first order terms of the detention, we will get linear equations which can be use to analyze the stability in the first approximation, as we proceed in most dynamic non linear problems.

Thus, for defining the evolution of balance, we consider:

$$\dot{p} = 0 \ \dot{\rho} = 0; V = Sap^{m} = ct$$
 (40)

Using these, from relations (16) and (28) we obtain:

$$(0 - \alpha)\dot{V} - \Delta A n^{1/2} \alpha^{1/2} = 0$$
(41)

$$(k-1)\rho_p Q_C \dot{V} - k\Lambda A_t \rho^{-1/2} p^{3/2} - (k-1)q = 0,$$
(42)

moreover:

$$\rho = \rho_p - \frac{\Lambda A_t}{\dot{V}} \sqrt{p\rho} \quad ; \tag{43}$$

$$p = \frac{k-1}{k\Lambda A_t} \left(\rho_p Q_C \dot{V} - q \right) \sqrt{\frac{\rho}{p}} \quad , \quad (44)$$

from which we obtain:

$$p = k_1 \sqrt{\frac{\rho}{p}}; \qquad \rho = k_2 - k_3 \sqrt{p\rho}, \qquad (45)$$

where we denote:

$$k_{1} = \frac{k-1}{k\Lambda A_{t}} \left(\rho_{p} Q_{C} \dot{V} - q \right); \ k_{2} = \rho_{p}; k_{3} = \frac{\Lambda A_{t}}{\dot{V}}.$$
(46)

Finally the balance equations become:

$$\rho = p^3 / k_1^2; \quad p^3 + k_3 k_1 p^2 - k_1^2 k_2 = 0. \quad (47)$$

The pressure equation can be arranged in transcendental

form:

where

$$a = k_3 k_1; \ b = k_1^2 k_2$$
 (49)

(48)

This can be solved using iterative Newton-Raphson method:

 $p = bp^{-2} - a,$

$$p_{i+1} = p_i - \frac{p_i - bp_i^{-2} + a}{1 + 2bp_i^{-3}} .$$
 (50)

In order to help our analysis we will use dimensionless parameter ψ defined by relation (5).

4 Linear equations

In the context of the balance parameters established above, the operating equations (10), (17) and (28) can be put in linear form:

$$\Delta V = a_V^V \Delta V + a_V^p \Delta p;$$

$$\Delta \dot{\rho} = a_\rho^V \Delta V + a_\rho^p \Delta \rho + a_\rho^p \Delta p;$$

$$\Delta \dot{p} = a_p^V \Delta V + a_\rho^p \Delta \rho + a_p^p \Delta p,$$
(51)

where, neglecting erosion factor, the coefficients of the equations are:

$$a_{V}^{V} = E\dot{V}V^{-1}; \ a_{V}^{p} = m\dot{V}p^{-1}; a_{\rho}^{V} = \left[\left(\rho_{p} - \rho \right)\dot{V}(E-1) + \Lambda A_{t} p^{1/2} \rho^{1/2} \right] V^{-2} a_{\rho}^{p} = -\left[\dot{V} + 0.5\Lambda A_{t} p^{1/2} \rho^{-1/2} \right] V^{-1}; a_{\rho}^{p} = \left[m \left(\rho_{p} - \rho \right) p^{-1} \dot{V} - 0.5\Lambda A_{t} p^{-1/2} \rho^{1/2} \right] V^{-1}; a_{p}^{V} = \left[(k-1)\rho_{p}Q_{c}\dot{V}(E-1) + + k\Lambda A_{t} \rho^{-1/2} p^{3/2} + (k-1)q \right] V^{-2}; a_{p}^{\rho} = 0.5k\Lambda A_{t} \rho^{-3/2} p^{3/2} V^{-1}; a_{p}^{p} = \left[(k-1)\rho_{p}Q_{c}m\dot{V}p^{-1} - 1.5k\Lambda A_{t} \rho^{-1/2} p^{1/2} \right] V^{-1},$$
(52)

where:

$$E = \frac{\sigma'(\psi)}{\sigma(\psi)} \frac{V}{V_p} = \frac{2a_2\psi + a_1}{a_2\psi^2 + a_1\psi + 1} \frac{V}{V_p}$$

Finally we can put the linear system in regular form:

$$\mathbf{x} = \mathbf{A}\mathbf{x}$$
, (53)
where the state vector is:

$$\mathbf{x} = \begin{bmatrix} V & \rho & p \end{bmatrix}^T , \qquad (54)$$

the stability matrix is:

$$\mathbf{A} = \begin{bmatrix} a_V^V & 0 & a_V^p \\ a_\rho^V & a_\rho^\rho & a_\rho^p \\ a_P^V & a_\rho^\rho & a_p^p \end{bmatrix}, \quad (55)$$

From the previously relation one can observe that all the stability coefficients a_i^j are dependent by volume.

5 Input data

For exemplifying the method, we will build a study model out of motor test.

5.1 Propellant geometry

First we describe the geometry of propellant which is a cylinder, with a cylindrical hole inside, non insulated, so burning simultaneously on all surfaces (fig. 1).

Denoting instantaneous sizes:

R - Outside radius of the cylinder; r - inside radius of the cylinder; l - cylinder length,

the burning area, terminal area and propellant volume are given by:

$$S = 2\pi(R+r)(R-r+l); S_T = \pi(R+r)(R-r);$$

$$V = S_T l = \pi(R+r)(R-r)l.$$
 (56)

If we denote x the linear burning distance, which at the time t is given by integration of burning rate:

$$x = \int_0^t u \,\mathrm{d}t \;, \tag{57}$$

the main geometric quantities are rewritten as it follows:

 $R = R_0 - x; \ r = r_0 + x; \ l = l_0 - 2x,$ (58) from which the combustion areas and volume become: $S = S_0 - 4\pi(R_0 + r_0)x; S_T = S_{T0} - 2\pi(R_0 + r_0)x;$ $V = V_0 - 2[S_{T0} + \pi(R_0 + r_0)l_0]x + 4\pi(R_0 + r_0)x^2,$ (59)

where we denoted with index "0" the initial values for length, surfaces and volume.

After processing we obtain:

$$\sigma = \frac{S}{S_0} = 1 - \frac{2x}{R_0 - r_0 + l_0}; \quad \sigma_T = \frac{S_T}{S_{T0}} = 1 - \frac{2x}{R_0 - r_0};$$

$$\psi = 1 - \frac{V}{V_0} = \frac{2x}{l_0} + \frac{2x}{R_0 - r_0} - \frac{4x^2}{(R_0 - r_0)l_0}$$
(60)

For the application the main geometrical quantities are:

$$R_0 = 33 \, mm$$
; $r_0 = 7 \, mm$; $l_0 = 319 \, mm$.

In this case, the initial areas are:



Fig. 1 Propellant geometry

Developing the relations (60) in a numerical form related on the parameter x result the dependences between the no dimensional areas σ , σ_T and the burn parameter ψ . By quadratic fitting we obtain:

$$\sigma(\psi) \cong 0.999951 - 0.069120\psi - 0.00614672\psi^2;$$

$$\sigma_T(\psi) \cong 0.999352 - 0.917169\psi - 0.0815622\psi^2$$
(61)

5.2 Motor geometry

For the test considered, the motor geometry elements are: -Combustion chamber cross surface: $A_{com} = 3739 \, mm^2$;

- Flow area at the throat:	$A_t = 490 mm^2$;
Flow area at the throat:	$A_t = 490 mm^2$;

-Flow area at nozzle exit plane: $A_e = 1206 \ mm^2$.

-Burning chamber volume: $V_{cam} = 1924555 \text{ mm}^3$

5.3 Propellant and process features

The features for the used propellant are:

- Propellant mass: $m_p = 1.834 kg$;

- Propellant density: $\rho_p = 1790 \text{ Kg} / m^3$;

- Adiabatic gas coefficient of the combustion products k = 1.4;

- Gas constant: R = 336.7 J/Kg/K;

- Linear burning rate in normal conditions: $u_{1n} = 4.6 \ mm/s$;

- Pressure exponent of burning law: m = 0.18; -Coefficient of variation of burning rate with temperature: $D = 0.0038 K^{-1}$;

- Heat quantity educts by burning reaction of 1 kilogram propellant: $Q_c = 4.9 \times 10^6 J / Kg$; - The quantity of heat transferred to the combustion chamber in time unit (heat flow) q = 1000 J / s;

- Ratio between igniter gas mass and the propellant mass $\gamma = 0.0011$;

- Overall coefficient of thrust loss by nozzle: $\sigma_c = 0.71$;

6 Results

Figure 2 presents the comparison between pressure produced by the relationship (28) and the experimental pressure of the test motor, and figure 3, shows the influence of initial propellant temperature for pressure. Further on we will analyze the balance parameters and

the dynamic stability of the operating RMSP.

Henceforth, setting the basic trend, we can evaluate, using the matrix (55), the parametric stability of the operating motor. To do this in figure 4 there are given the real part of eigenvalues for the matrix corresponding to the stable balance parameters.



Fig. 3 Influence of initial propellant temperature for pressure



Fig. 4 Real part of eigenvalues for stable RMSP

7 Conclusions

As we resumed in the introductive part, our work followed two purposes:

Scientific one – to check the possibility of applying Liapunov theory to analyze the stability of the balance parameters of RMSP at low pressure. With this reason we obtained:

- A flexible parametric expression of the propellant surface which allows to use different propellant geometries without major modification of the input data structure;

- A good concordance between parametric non-linear equations of the RMSP and the experimental results as we can see in figure 1 where is shown the comparative pressure diagram;

- An algorithm to define the balance parameters and stability matrix;

Technical one – to design the rocket motor for the fireextinguishing rocket, which was successfully accomplished, as we can see in figure 5.



Fig. 5 RMSP for fire-extinguishing rocket

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