Nonlinear Command Simulation for Pump Functioning Powered by Electrical Drives

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Abstract: - As compared to hydraulic, pneumatic, or thermal motor devices, electrical drives have certain advantages owed to the ease of generating, transporting and distributing the electrical energy. This fact made them become the first choice in most of the cases emerging in actual activity. The present paper simulates the behavior of electrical drives powered by continuous current motors.

Keywords: - nonlinear command, continuous current motor, modelation and simulation.

1 Introduction
Classification of electrical drives systems will be done according to the degree of complexity in interpreting information, which actually denotes the evolution of the concept of electrical drive in time. From this perspective electrical drives systems fall into three categories:
- systems with sequential elements (non-automatic),
- systems with automatic regulation of certain parameters
- systems with process computers.

Quality in revolutions regulation comes from several quality parameters, among which:
1. Regulation range – defined by the ratio between maximum and minimum number of revolutions achieved by the suggested drive. It usually ranges between (1....1000), however there are more sophisticated drives with a wider range (1....20000).
2. The direction of regulation – how the number of revolutions varies as compared to the nominal ones. If the regulation system allows revolutions variation only under the nominal ones, or exclusively above the nominal ones, there is an inferior or superior mono-zone regulation. If the system allows regulation in both directions, we speak about dual-zone regulation.
3. Fine regulation – is the ratio of two adjacent revolutions, within the regulating range achieved; in this respect, some methods allow continuous regulation of the number of revolutions (the ratio above tends to 1), others allow regulation in steps.
4. Regulation output – motor system output – speed regulation device. Certain methods use regulation resistances, and the regulation output is low, hence they can be considered only for reduced power.
5. Regulation nature – the modality in which regulation is performed. Certain technological processes require that revolutions regulation should be performed at constant torque (lifting machines), others at constant power (tool drives).

2 The Mathematical Model for the Continuous Current Motor
In order to determine the mathematical model for the continuous current motor, the starting point is the voltage equation for the induced circuit:

\[
\begin{align*}
\dot{u} &= R \cdot i + L \cdot \frac{di}{dt} + e \\
\dot{e} &= k_e \cdot \phi \cdot \omega \\
m &= k_m \cdot \phi \cdot i \\
\dot{m} &= m - m_o = J \cdot \frac{d\omega}{dt}
\end{align*}
\]

Equation system (1) is non-linear, owing to the multiplication results of type \((\varphi \cdot i)\) and \((\varphi \cdot \omega)\) respectively, as well as to the non-linearities generated by the magnetization curve of the motor. Since the use of non-linear mathematical models implies complicated mathematical formalities, the
system will be linearized around a functioning point, neglecting infinitely small variations. In order to do this, the excitation flux is considered constant, so that multiplication results \( k_e \omega = K_e \) and \( km \omega = Km \) should be constant and computed by the following relations:

\[
K_e = \frac{U_e - R_sI_n}{2 \pi n_a} \text{[V/rot/min]}
\]

\[
K_m = \frac{K_e}{1,03} \text{[Kg\cdot m/A]}
\]

With these notations and applying Laplace transformation in initial null conditions, the MCC transfer function is obtained under the form:

1. Transfer function as compared to the entry:

\[
H_{\text{MCC}}(s) = \frac{\Omega(s)}{U(s)} = \frac{1}{k_e} \frac{1}{T_m \cdot s^2 + T_m \cdot s + 1}(2)
\]

2. Transfer function as compared to the perturbation:

\[
H'_{\text{MCC}}(s) = \frac{\Omega'(s)}{M'(s)} = \frac{T_m}{T_m \cdot s^2 + T_m \cdot s + 1}(3)
\]

### 2.1 Automatic Revolutions Regulation Systems in Continuous Current

#### 2.1.1 Irreversible automatic revolutions regulation system for the continuous current motor, with simultaneous limitation of current

This is a regulation system with multiple loops or cascading, and it is preferred in electrical drives, as compared to independent regulators systems (of revolutions and current, each with its own reference parameter) or as compared to the ones with only 1 regulator for the main variable (number of revolutions) and with limits for the secondary variable (the current).

![Fig. 2: Irreversible cascade regulation scheme for the continuous current motor.](image)

#### 2.1.2 Reversible automatic revolutions regulation system for the continuous current motor with simultaneous limitation of current

The system has a single revolutions regulator \( R_n \) and two current regulators \( R_{i1}, R_{i2} \), one for each possible direction of rotor current. The two current regulators cannot function simultaneously because of the two diodes.

![Fig. 3: Structural scheme of regulation.](image)

#### 2.1.3 Automatic revolutions regulation system for the continuous current motor, by weakening the excitation flux

When increasing the revolutions regulation range of the electrical motor is required, automatic systems are used as they allow modifications both in the input voltage and the excitation current \( I_e \). Such systems are achieved via combining the systems allowing voltage regulation with systems allowing excitation current regulation.

![Fig. 4: Irreversible cascade regulation scheme for the continuous current motor.](image)

### 3 Automatic Regulation Systems for Drives in Alternative Current

Asynchronous machines, usually three-phasic, have wide applications in electrical drives, as a result of the advantages they hold as compared to other drives, namely: simple and robust construction, safety in operation, low cost, direct plugging to the alternative current grids [1]-[5].
3.1 Three-phasic Asynchronous Motors

Revolutions Regulation

According to the analytical expression of the three-phasic asynchronous motor number of revolutions [7]:

\[ n = n_1 \cdot (1 - s) = \frac{f_1}{p} \cdot (1 - s) \text{ [} \text{rot/s} \text{]} \quad (4) \]

it becomes obvious that the number of revolutions can be altered by: modifying input frequency \( f_1 \), modifying the number of pole pairs \( p \), or modifying slide \( s \).

### 3.1.1 Modifying the number of pole pairs \( p \)

Modifying the number of pole pairs \( p \) leads to a discrete alteration in the rotation speed of the motor. Changing the number of pole pairs is done either by modifying connections in stator spires, or by fitting the motor with spires having different numbers of poles, or by combining these two methods. Modifying the number of pole pairs is done only in asynchronous motors with a cage rotor (in shortcircuit), as the cage is enabled to automatically adapt its number of pole pairs to the number of pole pairs of the stator. Modifying the number of pole pairs by 1/2 ratio can be performed relatively easily by modifying the connections of stator spires (the best known being the Dahlander), thus obtaining the two-speed synchronism asynchronous motor. This motor is provided with statoric coiling consisting in two halves for each phase (for the first phase the halves are \( U_1U_2 \) and \( U_3U_4 \)). The halves on each phase can be serially connected (Fig. 5.b,c) or in parallel (Fig. 6b, c.).

![Fig. 5: Serial connection for stator coiling.](image)

![Fig. 6: Parallel connection for stator coiling.](image)

### 3.1.2 Slide modification

When studying alterations opportunity for the number of revolutions via slide modification, the starting point is the approximate relation written for slide in the form [6]:

\[ s \simeq \text{const} \cdot \frac{R_2}{U_1} \cdot \frac{M_r}{\Phi_m} \quad (5) \]

This last relation is indicative of the fact that slide varies inversely commensurated with the square of input voltage, and commensurated with rotor resistance. As voltage \( U_1 \) can only be decreased as compared to the nominal value, and resistance can only be increased as compared to the value of phase resistance, the conclusion is that slide modification in the motor can be done only by increasing it, therefore only by decreasing revolutions number, obtaining mono-zone revolutions regulation. Moreover, considering Joule losses in the rotor circuit as commensurate with slide \( (P_J = sP) \), the consequence is that for a given load torque \( M_r \), regulation output decreases with slide increase.

### 3.1.3 Modifying the frequency of input voltage

Regulating revolutions number by modifying the frequency of input voltage is achieved powering the motor from a frequency converter, which can be an inverter or a cyclo-converter. Frequency cannot vary independently from the voltage input. Indeed, neglecting the voltage on phase impedance of the motor stator, the following is true:

\[ U_1 \equiv E_i = 4,44f_w k_w \Phi_m \cdot \Phi_m = \text{const} \cdot f_1 \Phi_m \quad (6) \]

where \( U_1 \) is the effective phase voltage applied to the motor. So as not to impact on motor performance (torque, current operation to empty, nominal current), the magnetic flux \( \Phi_m \) must stay constant, inasmuch as possible. It is therefore deduced that the ratio \( U_1/f_1 = \text{const} \), meaning the voltage must be commensurate with frequency. This relation is adopted when frequency \( f_1 \) decreases up to the nominal value. When the frequency exceeds the nominal value, voltage \( U_1 \) remains constant \( (U_1 = U_1n) \), as a result of isolation and increase in iron losses, so that, with frequency increase over the nominal value there is a decrease in the magnetic flux \( \Phi_m \).

### 4 Non-linear Command in the Continuous Current Motor

The mathematical model of a process with concentrated parameters consists in a set of differential equations we are going to present as state equations. A linear model with constant coefficients and a reasonable degree of complexity can be determined for numerous processes and can be controlled via classical automation theories. When the process is non-linear – however, with sufficiently regular non-linearities – a tangent linear model can be determined in each point, which...
enables command computation when the functioning domain is restricted (the case of regulation).

Nevertheless, there are situations when it is impossible to accurately represent process behavior in a linear model for the whole functioning domain. The following possibilities exist for such cases:

- maintain the linear command law and check its robustness regarding the model structure and parameters;
- designing and implementing command algorithms able to allow for system non-linearities, a method also known as exact linearization;
- using other command strategies: robust controls, multi-model controls, auto-adaptive controls.

Methods relying on exact linearization of non-linear systems and input-output decoupling are presented here. Discussing the issue of systems input-output decoupling is also motivated by practical reasons: it allows easier determination of the global command system and answers to systems security imperatives by isolating command paths.

Next we will show how a linearly behaving input-output system can be obtained by using a non-linear command and a change of variable (the entry is modified).

Let us consider the system:

\[
\begin{align*}
\dot{x} &= f(x) + g(x) \cdot u \\
y &= h(x)
\end{align*}
\]  

The issue is: given a point \(x^0\), find a reaction (if possible) in a vicinity \(V\) of \(x^0\):

\[
u = \alpha(x) + \beta(x) \cdot v
\]  

defined on \(V\) and a coordinates transformation \(z = \Phi(x)\) also defined on \(V\) so that the corresponding closed loop system

\[
\dot{x} = f(x) + g(x) \cdot \alpha(x) + g(x) \beta(x) \cdot v
\]  
in coordinates \(z = \Phi(x)\) should be linear and controllable.

In other words, let us consider the non-linear system:

\[
\begin{align*}
\dot{x} &= f(x) + g(x) \cdot u \\
y &= h(x)
\end{align*}
\]  

having relative order \(r\) in a point \(x^0\).

The state reaction:

\[
u = \alpha(x) + \beta(x) \cdot v
\]  

transforms this system into one whose input-output behavior is identical to the one of a linear system having a transfer function:

\[
H(s) = \frac{1}{s}
\]  

5 Applications of Exact Linearization by Reactions in Continuous Current Drives

The mono-variable case. The state equations attached to the regulation system are:

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{R}{L} i - \frac{k_r \cdot \varphi_{es}}{L} - \omega + \frac{U}{L} \\
\frac{dx_2}{dt} &= \frac{U}{L} - \frac{k_r \cdot \varphi_{es}}{L} i_s \\
\frac{dx_3}{dt} &= \frac{k_m \cdot \varphi_{es}}{J} i_s - \frac{M}{J}
\end{align*}
\]  

But, in this way, the equation system becomes:

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{R}{L} i - \frac{k_r \cdot \varphi_{es}}{L} \cdot x_1 - \frac{k_e}{L} x_1 \cdot x_2 \cdot x_3 + \frac{U}{L} \\
\frac{dx_2}{dt} &= \frac{U}{L} - \frac{k_r \cdot \varphi_{es}}{L} x_2 \\
\frac{dx_3}{dt} &= \frac{k_m \cdot \varphi_{es}}{J} x_1 \cdot x_2 \cdot x_3 - \frac{M}{J}
\end{align*}
\]  

If we note:

\[
\begin{align*}
x_1 &= i \\
x_2 &= i_{es} \\
x_3 &= \omega
\end{align*}
\]  

The previous equation system can be written in the following form:

\[
\dot{x} = f(x) + g(x) \cdot u
\]  

where:

\[
f (x) = \begin{bmatrix}
- \frac{R}{L} \cdot x_1 & - \frac{k_e}{L} \cdot x_1 \cdot x_2 \cdot x_3 \\
\frac{U}{L} - \frac{R}{L} \cdot x_2 & - \frac{k_r \cdot \varphi_{es}}{L} \cdot x_2 \\
\frac{k_m \cdot \varphi_{es}}{J} \cdot x_1 \cdot x_2 \cdot x_3 & - \frac{M}{J}
\end{bmatrix}
\]  

\[
g (x) = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]  

The non-linear command is thus calculated:

\[
u = \alpha(x) + \beta(x) \cdot v
\]  

\[
\alpha(x) = - \Delta^{-1}(x) \cdot \Delta(x) =
\]  

\[
= \frac{u_m \cdot x_1 \cdot L}{L_{es}} + \frac{R_e \cdot x_1}{L_{es}} + \frac{R \cdot x_1 + k_e \cdot \varphi_{es} \cdot x_2 \cdot x_3 + M \cdot L}{k_m \cdot L_{es} \cdot x_2}
\]
Next, the vector fields are determined:

\[
\tilde{f}(x) = f(x) + g(x) \cdot \alpha(x) \\
\tilde{g}(x) = g(x) \cdot \beta(x)
\]

(19)

In the new coordinates the system becomes:

\[
\dot{x} = \tilde{f}(x) + \tilde{g}(x) \cdot v \\
y = \omega
\]

(20)

where \(v\) is the new entry of the system.

The equation system can also be written as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix}
= \begin{bmatrix}
R_{\text{es}} \cdot x_1 \\
R_{\text{es}} \cdot x_2 \\
\end{bmatrix} + \begin{bmatrix}
\frac{M_r}{J} \\
\frac{M_r}{J} \\
\end{bmatrix} v
\]

\[
+ \begin{bmatrix}
R_{\text{es}} \cdot x_1 \\
R_{\text{es}} \cdot x_2 \\
\end{bmatrix} + \begin{bmatrix}
\frac{M_r}{J} \\
\frac{M_r}{J} \\
\end{bmatrix} v
\]

\[
= \begin{bmatrix}
R_{\text{es}} \cdot x_1 \\
R_{\text{es}} \cdot x_2 \\
\end{bmatrix} + \begin{bmatrix}
\frac{M_r}{J} \\
\frac{M_r}{J} \\
\end{bmatrix} v
\]

(21)

Next, the newly obtained system is checked for linearity.

\[
y = \omega = x_3
\]

\[
\dot{y} = \dot{x}_3 = \frac{k_{\text{m}} \cdot L_{\text{es}}}{J} \cdot x_1 \cdot x_2 + \frac{M_r}{J},
\]

(22)

\[
\ddot{y} = x_3 = \frac{k_{\text{m}} \cdot L_{\text{es}}}{J} \cdot x_1 \cdot x_2 + \frac{k_{\text{m}} \cdot L_{\text{es}}}{J} \cdot x_1 \cdot x_2 - \frac{M_r}{J}.
\]

Therefore,

\[
\ddot{y} = v
\]

(23)

which means that the system with the new coordinates (\(v\)-input and \(\omega\)-output) is linear.

In order to obtain high performance, new \(v\) must be:

\[
v = k_1 \cdot (\omega_{\text{ref}} - \omega) - k_2 \cdot \dot{\omega}
\]

(24)

where \(k_1, k_2\), are coefficients obtained by poles allotting technique, and they can have the form:

\[
k_1 = \omega_0^3
\]

\[
k_2 = 2 \cdot 466 \cdot \omega_0^2
\]

\(\omega_0\) being pulsation of break in an open circuit.

An example of scheme for such a system is presented in Fig. 7.

![Fig. 7: Mechanical characteristics of the asynchronous motor.](image)

Fig 8 [8] illustrates the results of simulating drives functioning system in the case of using the non-linear command.

![Results of simulating drives functioning system.](image)
6 Conclusions

The relative order $r$ of a linear system can be interpreted as being the excess poles-zeros of the transfer function.

A linear system was shown to be obtained from a non-linear system if a non-linear command and a change of coordinates are used.

In practice, for implementing this command algorithm, it is absolutely necessary that we should have a data acquisition system enabling: acquisition of the values in the state vector as well as the input and output in the regulation system, together with non-linear command computation and command elaboration by the execution element.

References


