EVALUATION OF FORM DEVIATIONS USING GENETIC ALGORITHMS:
CIRCULARITY PROBLEM
DANIEL-SILVIU MANOLACHE
Department of Industrial and Manufacturing Engineering, I.M.S.T. Faculty
University POLITEHNICA of Bucharest
Splaiul Independentei 313, București, 060042
ROMANIA
daniel.manolache@upb.ro  http://www.imst.pub.ro/tcm

Abstract: - This paper presents a method for obtaining circularity deviation when real circular geometric features are inspected using coordinate metrology. The method is based on genetic algorithms for computing the geometrical ideal feature which the best approximate the measured set of points according with standardized association criteria. Then could be calculated the form deviation of real profile from computed substitution element. The performance of the algorithm was tested on developed computer application using LabVIEW programming language.

Key-Words: - Form deviation, Circularity, Coordinate metrology, Genetic algorithms, Optimization

1 Introduction
Dimensional and geometric inspection has a key role in process control and product quality evaluation. Coordinate measuring technology is used actually on large scale in industrial environment for unitary determination of part precision, being applicable on any kind of product typology.

Coordinate metrology is using a representative finite set of probed points on part features, each point being recorded thru its coordinates into coordinate system. Thus is not possible to evaluate directly the dimensional and geometric precision without to use an analytical model of part obtained mathematically. This model is formed by imaginary geometrical ideal features called substitute elements [2, 3, 7] or associate elements [1].

The association process between the set of points and ideal feature represents a mathematical optimization problem. Based on optimization criteria could be obtained different solution which fulfils international or local standards requirements depending by drawing requirements. To obtain the solution of the problem mainly are used numerical methods or geometric computational methodologies.

In case of circular features, which represent one of the most common feature on any actual products, are reported in the literature different methods and algorithms for resolving the association process between the set of points and ideal feature. For all optimization criteria [2] the mathematical models are nonlinear and raise problems in obtaining the solution. One direction of resolving the problem is based on Small Displacement Torsor method [1] which allows transforming the system nonlinear equations into one of linear equations which can be resolved with classical numerical methods.

Several other approaches for circularity evaluation are using geometric computational methodologies based on Voronoi diagrams [4, 9, 11, 12]. One set of points allows defining nearest diagram as locus of points with minimum distance from a given point and furthest diagram as locus of points with maximum distance from a given point, the solution being at geometrical intersection of those.

Another type of solutions for evaluation of circularity deviation is based on heuristic algorithms like selective analysis of the point data set [5, 10].

Genetic algorithms are stochastic optimization methods using models which mimic biological evolution process, algorithms known also as evolutionary or evolutive algorithms - EA [8]. Comparative with random solution search methods, those heuristic optimisation methods do not operate with a single numerical solution but with set of possible solutions spread into search space of variables boundaries, solutions which evolve to optimum problem solution.

2 Problem Formulation
Association of geometrical ideal feature to a set of points is related to a mathematical optimization problem. Optimization, which represents a mathematical regression process against different approximation criteria, is applied on mathematical model used to define the objective function for genetic optimization algorithm, known as fitness function. The optimization process is based on Euclidian norm concept, the distance from probed points on real surface to virtual ideal feature to be computed.

For each measured point $M_i(x_i, y_i, z_i)$ can be defined the deviation $e_i$ between the point and the substitution
feature \( S(Z(x, y); (a_1, a_2, \ldots, a_m) \in \mathbb{R}) \) as distance function \( d(M, S) \): \[ e_i = d(M, S) \]

(1)

2.1 General model

On real situation the deviations defined as distance function will not be equal with zero and to obtain the feature which interpolate the measurements points then should be applied an optimization operation on deviation set. The general form of norm concept \( L_p \) is [2]:

\[
\|e\|_p = \left( \sum_{i=1}^{N} |e_i|^p \right)^{\frac{1}{p}}
\]

(2)

where \( p \in (1, \infty) \), \( N \) represents number of measurements points and \( e_i \) is the deviation between measurement point and substitute element.

Depending by value of parameter \( p \) the optimization problem can take different forms which fulfils one of form or functional criteria. Are common the following developments:

\[ p=1 \Rightarrow \|e\|_1 = \sum_{i=1}^{N} |e_i| = \min \]

(3)

\[ p=2 \text{ (Gauss case)} \Rightarrow \|e\|_2 = \sum_{i=1}^{N} e_i^2 = \min \]

(4)

\[ p=\infty \text{ (Cebyshev case)} \Rightarrow \lim_{p \to \infty} \|e\|_p = \max |e_i| = \min \]

(5)

Relation (5) could be transformed more into following model

\[
\min \left( \max_{i \leq N} |e_i| \right)
\]

(6)

in concordance with applied form criteria and with

\[
\max_{i \leq N} |e_i|
\]

(7)

or

\[
\min \left( \max_{i \leq N} |e_i| \right)
\]

(8)

where \( r_i \) is radius of substitute feature, in concordance with applied functional criteria (tangency criteria).

2.2 The models for substitute circle

In case of circular features are possible two association criteria: form criteria - when objective function is to minimize form deviations, and functional criteria (tangency criteria) – when objective function is to minimize or maximize feature size. Based on this are resulting four objective functions which resolved are leading to four different substitute circles used as reference to evaluate form deviation of real feature probed using coordinate technology or as datum for evaluating positional deviations of other features. Those four situations are in concordance with ISO 1101 standards or standard SR ISO 4291-97 for evaluating form deviations.

The equation of circle defined into XY plane and having the centre point of coordinate \( x_c, y_c \) and radius \( r \) is as follow:

\[
(x - x_c)^2 + (y - y_c)^2 = r
\]

(9)

Thus for any point probed on real circular profile where number of measurements points \( i \geq n=4 \), in order to analyze a real situation, the distance from circle centre to any probed point is:

\[
r_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}
\]

(10)

and deviation \( e_i \) will be \( e_i = r_i - r \). The problem of determination substitution circle is done according with following forms of possible objective functions.

I. Minimum zone condition (MZC) or Minmax condition.

The objective function is minimizing the distance between two concentric circle tangent exterior and interior to real profile. The function has following form:

\[
f_z(x; x_c, y_c) = \operatorname{Min} \left[ \operatorname{Max}(r_i) - \operatorname{Min}(r_i) \right]
\]

(11)

where \( \operatorname{Max}(r_i) \) represents the radius of outer tangent circle to measured points set and \( \operatorname{Min}(r_i) \) represents the radius of inner tangent circle to measured points set (see Figure 1). Developing the relation (5) using relation (4) is resulting:

\[
f_z(x; x_c, y_c) = \operatorname{Min} \left[ \operatorname{Max} \left( \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \right) - \operatorname{Min} \left( \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \right) \right]
\]

(12)

Using this objective function is allowing to search into solution space for coordinates of one single point, point which represents the centre of both concentric circles.

II. Minimum circumscribed condition (MCC) – functional criteria for shaft type features.

The objective function is minimizing the radius of circle tangent exterior to real profile (see Figure 2). The relation of objective function is as follow:
\[ f_i(x; x_c, y_c) = \text{Min} [\text{Max} \left( \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \right) \] (13)

Solution search space for this model is two-dimensional, related also to coordinates of centre of circumscribed circle.

\[ f_q(x; x_c, y_c, r) = \text{Min} \left[ \sum_{i=1}^{n} \left( \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} - r \right)^2 \right] \] (15)

Using this objective function is necessary to search into solution space for coordinates of one single point, point which represents the centre of both concentric circles, and radius of circle with determined centre. Searching for three variables the search space in this case is three-dimensional.

III. Maximum inscribed condition (MIC) – functional criteria for hole type feature.

The objective function is maximizing the radius of circle tangent interior to real profile (see Figure 3). The function is as follow:

\[ f_d(x; x_c, y_c) = \text{Max} \left( \text{Min} \left( \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \right) \right) \] (14)

Solution search space for this model is the same as for previous two conditions.

IV. Least square (Gauss) condition (LSC) – regression reference circle.

The objective function is minimizing the sum of square deviations from measurement points to radius of substitute circle (see Figure 4). Therefore the regression circle is preferred in coordinate metrology because generate sufficiently stable substitute feature on fewer measurements points that other substitute conditions. The objective function has following form:

\[ f_q(x; x_c, y_c, r) = \text{Min} \left[ \sum_{i=1}^{n} \left( \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} - r \right)^2 \right] \] (15)

Using this objective function is necessary to search into solution space for coordinates of one single point, point which represents the centre of both concentric circles, and radius of circle with determined centre. Searching for three variables the search space in this case is three-dimensional.

3 Problem Solution

Developed evolutionary algorithm requires completing the following steps for reaching a solution into solutions space [6]:

1. Generate the initial population POP(t=0);
2. Compute the fitness (objective function) for each individual of initial population;
3. Sort the initial population based on their fitness;
4. Repeat
   4.1. Select the genetic operator;
   4.2. Select the individuals for reproduction;
   4.3. Perform reproduction;
   4.4. Compute the fitness for offspring;
   4.5. Re-insert into population the offspring;
   4.6. Sort the new population based on their fitness;
   4.7. Select the new population POP(t=t+1);
   4.8. Verify if the individual with best fitness kept into population otherwise introduce it;
Until (t > imposed number of generations)
3.1 Implementation aspects

The implementation of the algorithm require to resolve the following aspects: genetic representation of optimization problem, procedure to generate initial population, fitness function, genetic operators, parameters related to variation domain of individual genes, number of individuals, number of generations, probability of selecting the genetic operators.

The genetic representation of the solution is an array of real numbers called genes [8]. Depending by model of substitution circle the structure of chromosome will be formed by \( x_p \) and \( y_p \) coordinates of circle centre for all models plus radius \( R \) in case of determining the substitute circle according least square (Gauss) criteria.

The dimension of search space for solutions will be defined by domain variation for each gene. For all genes, representing the circle centre or radius, the variation domain will be around the calculated average value of data set parameter plus and minus an imposed deviation \( \Delta \) as Table 1.

The procedure of creating initial population of individuals is generating randomly individuals with gene values in variation domain, the number of individuals being a parameter of simulation program.

<table>
<thead>
<tr>
<th>Association condition</th>
<th>Genes variation domain</th>
<th>Location</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>MZC: minimum zone (MinMax)</td>
<td>( x_p = \sum_{i=1}^{n} x_i / n ) ( \pm \Delta x_p )</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>MIC: maximum inscribed</td>
<td>( y_p = \sum_{i=1}^{n} y_i / n ) ( \pm \Delta y_p )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCC: minimum circumscribed</td>
<td>( R = \sum_{i=1}^{n} R_i^{(CG)} / n ) ( \pm \Delta R )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LSC: least square (Gauss)</td>
<td>( R = \sum_{i=1}^{n} R_i^{(CG)} / n ) ( \pm \Delta R )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 – Formulas for defining the solution space domain

Fitness function used is according with models for substitute circles objective functions presented in paragraph 2.2, representing the argument of minimization or maximization function, depending by problem case.

Generation of new solutions is performed from those selected through genetic operators. The developed algorithm is using one of 5 types of crossover (recombination) genetic operators and 8 types of mutation genetic operators. The selection of one of 13 genetic operators is done randomly based on uniform probability distribution or Gaussian probability distribution from a list ordered as follow: boundary mutation, complete uniform mutation, arithmetic crossover, heuristic crossover, mixed type 1 crossover, complete non-uniform mutation, Gaussian mutation, non-uniform mutation, complete Gaussian mutation, mixed type 2 crossover, central heuristic crossover, complete arithmetic crossover, non-linear elitist mutation, uniform mutation. The resulted list was done through simulation tests performed during algorithm development with the objective to get the solution convergence with higher precision imposed by application type - coordinate metrology.

3.2 Simulation

The algorithm was implemented as computer application using LabVIEW platform from National Instruments (USA) as programming language. The application developed, called TolEvAl [6], was used to simulate on randomly generated set of points or using data set from literature. The program allows also selecting the type of geometrical element, association criteria and parameters of genetic algorithm: number of individuals from population, number of generation, parameters associated to genetic operators.

The evolution of solution into search space is visible graphically. Figure 5 shows the initial population of candidate solution for determining the maximum inscribed circle associated to a hole feature and evaluation of circularity error according Romanian standard STAS 7384-85 using a generated data set.
solutions. The evolution of fitness function during generations (see Figure 7) presents the evolution of radius value from a starting value done by best individuals in initial population to final one showing a good convergence of the developed algorithm. Also during simulation is possible to visualize the histogram of genetic operator usage as presented in Figure 8.

The analysis of performance for developed algorithm was performed using reported data set in literature. Using as parameters of evolutionary algorithms the values from Table 2, the run of developed algorithm on known data sets from literature demonstrated similar results. Table 3 presents the results of simulations done using five data set from literature, the problem resolved being to determinate the circle associated to minimum zone criteria, criteria which define the circularity deviation evaluation according ISO1101.

<table>
<thead>
<tr>
<th>Population dimension</th>
<th>Number of generations</th>
<th>Type of selection for genetic operators</th>
<th>Nonlinearity degree for selection</th>
<th>Non-uniformity degree of mutation</th>
<th>Variation domain for non-linear mutation</th>
<th>Dispersion domain for Gaussian mutation</th>
<th>Number of parents for arithmetic crossover</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>10000</td>
<td>gaussian</td>
<td>0,05</td>
<td>5</td>
<td>0,05</td>
<td>0,1</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2 – Parameters used into evolutionary algorithm during simulation
4 Conclusion

The numerical simulations prove that the proposed algorithm is leading to results for circularity deviation similar with those reported in literature and obtained with different other resolving algorithms, simulations done on same data set and for all association criteria: minimum zone (MiniMax or Cebyshev), minimum circumscribed, maximum inscribed, least square (Gauss).

Obtained parameters for substitute geometry, circle for this case, parameters which define location and size of substitute circles are similar also.

Simulation shows also that for each association criteria are obtained different values for circularity deviation of real feature calculated against each type of reference circle, the user being aware of correlating program association criteria with evaluation standard imposed by real situation.

References:

<table>
<thead>
<tr>
<th>Data set</th>
<th>Circle centre coordinates obtained using developed EA algorithm</th>
<th>Radius of substitute circle obtained using developed EA algorithm</th>
<th>Circularity deviation obtained with developed EA algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$ [mm]</td>
<td>$y$ [mm]</td>
<td>$R$ [mm]</td>
</tr>
<tr>
<td>[5]</td>
<td>0.0335614</td>
<td>-0.052930</td>
<td>1.000225</td>
</tr>
<tr>
<td>[10]#1</td>
<td>0.034428</td>
<td>-0.016553</td>
<td>9.992003</td>
</tr>
<tr>
<td>[10]#2</td>
<td>0.042061</td>
<td>-0.098052</td>
<td>8.983260</td>
</tr>
<tr>
<td>[12]#1</td>
<td>39.999682</td>
<td>30.002219</td>
<td>25.003004</td>
</tr>
<tr>
<td>[12]#2</td>
<td>40.000742</td>
<td>50.001530</td>
<td>30.000065</td>
</tr>
</tbody>
</table>

Table 3 – Results associated to calculated substitute circle according with minimum zone association criteria