Symbolic equation using Modified Nodal Analysis for linear electrical circuit using Matlab

Zoltan ERDEI, Luiza Alexandra DICS0, Liviu NEAMT, Oliver CHIVER
Department Electrical Engineering
North University of Baia Mare
Str. Victor Babes Nr. 62/A, 430083
ROMANIA
erdeiz@ubm.ro

Abstract: - In this paper it is presented a program which generates the modified nodal equation for electric analog circuits in a symbolic, partial symbolic and numerical mode. The program is an application, made in the environment of the program MATLAB version 7.1, which has a powerful symbolic math toolbox.

Key-Words: - analog circuits, circuit functions, symbolic equations, Matlab, netlist, modified nodal analysis.

1 Introduction
The modified nodal methods have been implemented in a program on a Pentium Dual Core computer compatible to obtain symbolic forms, partly symbolic or numerical equations for electronic linear analog circuits. Between the input gate and the output one specified by the user, any of the four types of circuit functions (transfer impedance - \( Z_{ei} \), admittance transfer function - \( Y_{ei} \), voltage transfer factor (amplifier) - \( A_{ei} \) and current transfer factor (amplifier) – \( B_{ei} \) for linear analog electric circuits around a functionary point. Starting from the description of the circuit through to a file-type input netlist, according to the window shown in figure 1, the following capabilities:
- generates symbolic, partly symbolic or numerical equations using modified nodal analysis for linear electrical circuits;
- generates symbolic form of the extensive matrix of the system equations and determines the symbolic solution, partly symbolic or numerical to the circuit system;

Data entry is a set of lines describing nl circuit branch. Each branch of the circuit is described by a proper letter which indicates the type of circuit elements, the first node, final node, the nominal value and the value of the tolerance. In the case of controlled sources we should also introduce nodes of command branch. The netlist is similar to those described in the program SPICE.

Fig.1 Application window
2 Modified Nodal Analysis (MNA)
To solve the circuit and obtain the desired circuit function we use the modified nodal analysis.
In the modified nodal analysis the circuit matrix is obtained by the augmentation of the nodal conductance (admittance) matrix corresponding to the NA-compatible circuit elements with additional rows and columns for non-NA-compatible circuit elements.
The circuit equations in operational corresponding to the modified nodal analysis method, have the following form:

\[
\begin{bmatrix}
Y_{n-1,n-1}(s) & B_{n-1,m}(s) \\
A_{m,n-1}(s) & Z_{m,m}(s)
\end{bmatrix}
\begin{bmatrix}
V_{n-1}(s) \\
I_{m}(s)
\end{bmatrix}
= \begin{bmatrix}
I_{s,n-1}(s) \\
E_{m}(s)
\end{bmatrix}
\]

where: - is the admittance matrix corresponding to the n-1 independent nodes; is an (n-1)xm matrix and it contains the elements -1, 0, +1 and the current gains of the CCCS’s; is an mx(n-1) matrix containing the elements -1, 0, +1 and the voltage gains of the VCVS’s; is a mxm matrix having the transfer impedances of the CCVS’s; is node voltage vector corresponding to the n-1 independent nodes. The vector represents the current vector corresponding to the non-NA compatible circuit branches and it has the following structure:

\[
I_{m}(s) = \left[ (I_{c}(s))^T, (I_{c}(s))^T, (I_{c}(s))^T, (I_{c}(s))^T \right]^T
\]

where : (I_{c}(s))^T is the ideal voltage source current vector; (I_{c}(s))^T the controlled (output) branch current vector of all controlled voltage sources; (I_{c}(s))^T the controlling (input) branch current vector of the CCVS’s; (I_{c}(s))^T the controlling branch current vector of the CCVS’s. The vectors I_{s,n-1}(s) and E_{m}(s) represent the contributions of the excitation sources (independent current and voltage sources).

3 Description of the application
To explain and understand this application we consider the following circuits (figure 2). This application describes how to use a lowpass Sallen-Key filter with a dual supply voltage. The circuit provides a Butterworth response with a 30MHz bandwidth, and is ideal for video-reconstruction filtering in HDTV applications. In HDTV applications, lowpass filters are used for reconstruction of RGB and component video (Y, Pb, and Pr) signals. They are placed following the video DAC to remove the higher frequency replicas of the signals, as well as before the ADC for anti-aliasing. Figure 2 shows a one-channel, dual-supply configuration incorporating the MAX4382. It is a three-pole, Sallen-Key lowpass filter, in which the current DAC generates the video signal, and the resistor (RL) sets the amplitude. With the MAX4382, the RL, R1, R2, C1, and C2 form a two-pole, Sallen-Key lowpass filter having a gain of 2. The driving load (75Ω) at the output, plus RT and Cp, sets the real pole.

In the Figure 2 circuit, the -3dB bandwidth is about 30MHz. The attenuation is approximately 14dB at 44.25MHz, and 28dB at 74.25MHz. The group delay is roughly 6.5ns. If the current DAC load is different than 75Ω, just use the following relationship to set the value of R1: R1 + RL = 150Ω. For RL greater than 150Ω, C1, C2, R1, and R2 will need to be adjusted.

![Fig.2 Lowpass Sallen-Key filter.](image-url)
The coefficient matrix obtained with the program for this example will be as belows:
The admittance transfer function for this example in a symbolic form:

\[ Y_{ei} = GA*(R5+R4)/(1+s*C3*R6)/(R4*R5+R5*Rf+R4*Rf+R5*R3+R5*s*C2*R2*R3-R2*R5*GA*Rf*s*C2+R2*R5*Rf*s*C2+R5*Rf*s*C1*R3+R2*R5*Rf*s*C1+R2*R5*Rf*s*C1*R3*R2+R2*R5*Rf*s*C1*R3*R2+R2*R5*Rf*s*C1*R3*R2+R2*GA*Rf*R4+R2*R5*Rf*s*C1*R3*R2+R2*R5*Rf*s*C1*R3*R2)*Rf*E1 \]

\[ 4 \text{ Conclusion} \]

The main advantage of the MNA method is that the contributions of all individual circuit elements can be inserted directly in appropriate places in (1), independently of whether the primary variable of a particular element is a voltage or a current.

MNA method associated with an efficient nod tearing leads to an important reduction in computing time and to an increase of the accuracy.

This is a new software environment that allows the schematic representation, analysis and design for linear analog circuits.

References:
[10] Zoltan ERDEI Analiza toleran\c{t}elor circuitelor electrice – proiect de diplom\a, Facultatea de electrotehnici\c{t}, U:P:B. 2002.