Simulation Software for Transient, Multidimensional Heat Conduction

SORIN RAȚIU\textsuperscript{1}  GABRIEL NICOLAE POPA\textsuperscript{2}  ANA JOSAN\textsuperscript{1}  VASILE ALEXA\textsuperscript{1}
\textsuperscript{1}Department of Engineering and Management
\textsuperscript{2}Department of Electrical Engineering and Industrial Informatics
Politechnica University Timișoara
Str. Revoluției, no.5, Hunedoara
ROMANIA
sorin.ratiu@fih.upt.ro  www.fih.upt.ro/np/depingman/dep_ingman.html

Abstract: The purpose of this work consists in determining the three-dimensional distribution of the non-stationary temperature field in a steel half-finished material, S235 JRG2 (OL 37-2k) mark, heated in an experimental flame oven, by building a numerical model with finite differences in the C++ programming software. The results obtained will be compared to the ones determined through experimental measurements.

Key-Words: temperature, finite differences, C++, experimental measurements, compared analysis

1 Introduction
The first problem that arises when it comes to finding solutions to the equations that describe phenomena of heat transfer is that of finding a form that should allow the numeric solving, i.e. a form that can be computer processed. In order to achieve this, the field in which the phenomenon is analyzed is not considered as a continuous medium, but as a finite multitude of points, making up the knots of a network.
In other words, the system of differential equations with partial derivatives, valid for any point in the field of analysis becomes a system of algebraic equations, valid for only certain points of the field, points that define the discretizing network of the model. Figure 1 gives a network that covers a three-dimensional field [1].

![Fig.1. The three-dimensional network](image)

The network pitch in a certain direction can be constant or variable. Also, with respect to the three directions, the network can have the same pitch or different ones. Generally, for the sake of simplicity, we consider the network pitches to be equal in all directions. When we have high gradients of parameter fields (high temperature gradients) the network pitch should be denser than in the rest of the field. The smaller a network pitch, the higher the precision of the calculation, but at the same time the memory requirements increase and this is why we have to look for an optimal solution between the expenses related to computer use and the precision of the results.
The purpose of this study is the development of a numerical model with finite differences for the study of the thermal field inside a steel half-finished material, heated with a view to the subsequent plastic strain and its validation based on the experimental data obtained on a laboratory model that simulates the industrial heating conditions [2], [3].

2 Experimental Installation
The experimental installation, a chamber that is similar to the work space of a chamber furnace, allows direct temperature measurements in certain points inside the work area, on the inner hearth surface, on the lateral walls and particularly in different points inside the material to be heated up (figure 2). The thermal effect is obtained by burning methane gas from the local network, using a burner with a maximum outflow of 9 m\textsuperscript{3}/h. The exhaust of burned gases is made at the upper part of the chamber.
In order to increase the maximum heating temperature, the combustion air, flown in by means of a fan, is preheated in a coil copper placed on top of the heating chamber. Burned gases are exhausted by means of an ejector blowing air in the exhaust pipe, thereby creating a certain depression inside the work chamber.

The bloom to be heated up (figure 3.a.) is a semi finished part obtained by continuous casting at S.C. Arcelor Mittal Steel Hunedoara. The steel grade is OL 37 - 2k. Inside the bloom we bore five holes (numbered 1, 2, 3, 4 respectively 5) for the introduction of the thermocouples, at different levels and depths, as it can be noticed in figure 3.b. The figure next to each hole indicates the respective thermocouple.

The aim of our experiment was to monitor the dynamics of the thermal field under conditions as close as possible to the real ones. In order to achieve this target we collected data referring to the temperatures in eight points (five inside the bloom, one on the inner surface of the masonry, one in the hearth and one inside the work chamber), points that we considered to be significant in solving the problem under consideration.

The branch box, comprising the data acquisition system, made of the two modules (the acquisition module and the signal converter) together with the feeding module, connected to the supply network, is given in figure 4.

The electric connection diagram of the measuring devices is given in figure 5.

The voltage signals generated by the thermocouples were received by an 8-channel analogical acquisition module of the type ADAM- 4018. The data obtained were input to a PC by means of a signal converter of the type ADAM-4520 [4].
3 The Numeric Model with Finite Differences

We tried to determine the three-dimensional distribution of the temperature field in the bloom made of steel S235 JRG2 (OL 37-2k), heated up in the experimental pilot installation.

The numeric model requests certain initial data, namely:
- the value of the initial temperature of the bloom, considered to even all over its mass: \( t_0 = 20 \, ^\circ\text{C} \);
- the value of the room temperature: \( t_a = 20 \, ^\circ\text{C} \);
- the geometrical dimensions of the semi-finished part: \( 0.24\times0.30\times0.27 \, \text{m}^3 \);
- the number of knots within the field of analysis, according to the three axes: 20 for each;
- the values of the thermal conductivity of the material the bloom is made of, according to temperature;
- the values of the enthalpy of the material the bloom is made of, according to temperature;
- the convection coefficient between the surface of the bloom and the burnt gases;
- the total duration of the heating up: \( \tau_{\text{total}} = 36000 \, \text{s} \) (10 h);
- the maximum of the enthalpy for one iteration.

For the particular case of the three-dimensional non-stationary conduction, without internal sources of heat, the equation describing this phenomenon has the following form [5]:

\[
\frac{\partial \Phi}{\partial t} = a \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \tag{1}
\]

Considering the dependency of thermal conductivity \( \lambda \) on temperature, we introduced the notion of low temperature, which includes this variation:

\[
\Phi = \int_{t_0}^{t} \frac{\lambda}{\lambda_0} \, dt \tag{2}
\]

where \( \lambda \) and \( \lambda_0 \) are the thermal conductivities at temperature \( t \) and at an arbitrary temperature \( t_0 \).

For the specific heat of the material we used the expression:

\[
c = \frac{\partial H}{\partial \tau} \tag{3}
\]

After discretizing the field of analysis according to figure 1, we will express the temperature of one of its points \((i,j,k)\) according to the temperature of the neighboring points. The enthalpy variation expressed as finite differences has the following form:

\[
\frac{\partial H}{\partial \tau} \approx \frac{H_{i,j,k,n+1} - H_{i,j,k,n}}{\tau_{n+1} - \tau_n} = \Delta H_{i,j,k} \tag{4}
\]

Index \( n \) from (4) expresses the time moment.

\[
\Delta H_{i,j,k} = \frac{\lambda_0}{\rho A \Delta \tau} \left[ \Phi_{i-1,j,k} + \Phi_{i+1,j,k} - 2\Phi_{i,j,k} \right] + \frac{\lambda_0}{\rho A_2 \Delta \tau} \left[ \Phi_{i,j-1,k} + \Phi_{i,j+1,k} - 2\Phi_{i,j,k} \right] + \frac{\lambda_0}{\rho A_3 \Delta \tau} \left[ \Phi_{i+1,j+1,k} + \Phi_{i-1,j-1,k} - 2\Phi_{i,j,k} \right] \tag{5}
\]

The equation with finite differences makes it possible to determine the enthalpy variation in a time interval \( \tau_{n+1} - \tau_n \) according to the temperature of the neighboring points. If we know the initial temperature distribution \( \Phi_{i,j,k,0} \) (or enthalpies \( H_{i,j,k,0} \)) we can determine by means of relation (5), the temperature variation after a given time interval, the result being \( \Phi_{i,j,k,1} \). Starting from this result we can determine the temperature distribution \( \Phi_{i,j,k,2} \) etc. So, by an iterative method, one can obtain the evolution of temperature distribution during the heating up of the semi-finished part.

Equation (5) is valid for a point inside the material. For a point placed on the limit surface (frontier), the respective equation will be modified.

Simplifying hypotheses:

Achieving the numeric model with finite difference implies taking into consideration certain simplifying hypotheses referring particularly to establishing the means of heat transfer from the heating chamber to the surface of the material. In this sense:
- we neglected the radiation effect of the burned gases and of the inside surface of the furnace brickwork, the virtual heat exchange being obtained by convection alone, with a coefficient \( \alpha_C \) that is time constant and equal for each of the free surfaces of the bloom.
- its high value (1000 W/(m²K)) suggests the fact that the temperatures of the free surfaces of the bloom closely emulate the temperature of the gases burned in their immediate vicinity.
- for the surface lying on the furnace hearth the value of coefficient \( \alpha_C \) was given an exaggerated value (10⁷ W/(m²K)) while the temperature that is exterior to the respective surface actually becomes the hearth temperature. In this way we simulate the thermal transfer by contact between the hearth and the base of the semi-finished part [6].
4 Description and Functioning of the Source Program Used to Simulate the Thermal Field

The computer program is written in C++ and works under Win32 (i.e. Windows 95, 98, Me, NT4, 2000, XP – with Intel processor).

For the graphic interface, the program uses MFC (Microsoft Foundation Classes), a class library that encloses the functional character of the standard programming interface Windows API – Application Program Interface.

The source program has a modular, object-oriented architecture. One C++ module consists in general in a pair of files: one with the extension .H (form header) that contains function and/or class declarations, used as interface with the other modules, and one with the extension .CPP (from C++) that contains definitions (implementations of the functions and classes declared in the header). The program in question contains 28 such modules, but only 4 are directly related to calculations. The other ones are auxiliaries of these or are meant to implement graphs, windows, dialogue cases, etc.

The most important C++ involved in the simulation process are:
- Material – containing the description of a material;
- HeatFlowSequence – making the calculations related to one iteration.
- SequenceInterpolator – interpolates the values obtained for the graphs;
- CProgressDialog – is a derivative of class MFC CDialog and implements a modal dialogue (which does not allow the access to the main window). This will open when the calculation process starts – it is indirectly controlled by it – and closes when calculation is over, either by manual command, or by reaching the halt condition [7].

Figure 6 gives the dialogue window opened at the moment of program initializing, and figure 7 shows the one through which the data needed by program running are input.

5 Presentation of the Results

The program we wrote allows the qualitative visualization of the thermal field for different time moments, in plane cross-sections of the bloom, perpendicular to one of the coordinate axes and situated at different heights with respect to the base plan to which they are parallel. For example, in order to visualize temperature distribution in a cross section that is parallel to plane xOy placed at distance z = 0 from it (in this case the cross section is the very plane xOy), after one hour from the beginning of the heating process we will activate the graphical window by pressing the “3DZ” button from the main window, given in figure 6, after having opened the “Dialog” window given in figure 8 and we will move the cursor according to the percent distance z, in position 0, and the time cursor to position 60. The thermal field for this case is given in figure 9.

Similarly, we can visualize the temperature distributions in different cross sections and at different time moments. A few examples are given hereinafter. The dimension along axis Ox of the semi-finished part is labeled B, along Oy it is L, and along axis Oz it is labeled H.
Fig. 9. The thermal field in the base surface of the bloom after one hour from the beginning of the heating process.

Fig. 10. The thermal field in a plane that is parallel to xOy, placed at z = ½ H, after 5 h of heating.

Fig. 11. The thermal field in a plane that is parallel to yOz, placed at x = ¾ B, after 9 h of heating.

The data obtained can be exported so as to compare them with the values obtained through experimental measurements. As we have already mentioned, the aim of this study is the validation of the numerical methodology used and, consequently, of the numerical results, through experimental results. This validation, in the case of the finite differences method, involves a comparison between the temperature values obtained by solving the numerical model built with the C++ programming software and the values determined experimentally. Having available only the experimental curves of temperature variation in the measurement points (figure 3.b), the comparison of results will be done only for the values corresponding to these points. In figure 13 the time variations of the temperature obtained through experimental measurements and numerically computed with C++ were represented comparatively for no. 1 characteristic point. The signification of the graphs in figures 14, 15, 16 and 17 is the same, only that it is for the characteristic points 2, 3, 4, and respectively 5.

Fig. 12. The graphic evolution of characteristic points temperature.

The characteristic points temperature evolution:

By characteristic points we defined those points of the discretizing network of the numerical model field with finite differences, which correspond to the points of the real field (the bloom) in which the thermal-couples were introduced for temperature measurements in the pilot experimental installation. The time evolution of the characteristic joints temperatures can be supervised with the help of the MetalGraphFive dialogue window, shown in the figure 12.

Fig. 13. The comparative results corresponding to no. 1 measurement point.
In absolute value, the differences between the experimental values and the ones provided by C++ are bigger for the characteristic points corresponding to 3 and 5. The best precision was obtained for no. 4 characteristic point.

At the beginning of the heating period, the values computed with C++ are bigger than the ones measured experimentally, and after this period the situation reverses, with the exception of point 1, for which the computed values are bigger during the entire heating period. The explanation is that the numerical model does not take into consideration the thermal radiation effect when the bloom is heated virtually. On the other hand, the convection coefficients under discussion show values which are not conform to reality, due to the simplified numerical model.

We can say that, during the first heating hour, the relative errors of the numerical model are big enough, reaching even percentages of 80%, and in the next heating stages they decrease significantly, fitting into a tolerable range of 10%.

As a general conclusion relatively to the temperature values obtained with the numerical model implemented in C++, we can say that it provides accurate data, comparatively to the experimental ones, which are considered to be our reference. During the starting heating period on the other hand, the model falls short, the values computed during this interval being wrong. In order to remedy this shortcoming we should take into consideration a global coefficient of heat transfer which varies with temperature. For the most part of the temperature range under discussion the numerical simulation proves to be an efficient tool in the study of the thermal transfer phenomenon.

References:

6 Conclusions
Relatively to the dynamics of the temperature variation measured experimentally in the five points, we can say that the values computed with the C++ software in all the five characteristic joints, corresponding to the measurement points, follow this variation closely.