Antireciprocal Two-ports based on Equivalent Circuits

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Abstract: - The paper analyzes the process of establishing the equivalent schematic of an antireciprocal quadripol (gyrator) from the results generated based on quadripol theory. To avoid confusion, the paper emphasizes the importance of association rule to specify the reference sense adopted, depending on which is expressed differently the antireciprocity condition. In this regard, some final results, the obtained expressions are written under the form which shows explicitly the sign. Analysis refers to one type of equivalent diagram, presenting the use in a problem of powers balance with particular emphasis on determining reactive power balance. The case study evaluates the behaviour of a lossy gyrator with operational amplifiers dimensional presenting the main results obtained, both analytically and by SPICE simulation.

Key-Words: - quadripol, equivalent schematic, antireciprocal, association rule, lossy gyrator.

1 Introduction

Setting up equivalent circuits for antireciprocal two-port, or gyrators, as these two-ports are usually named, can be done starting from known results from two-port theory and inserting there the specific antireciprocity condition. The insertion can be done from the very beginning, or it can be done at a specific point as the analysis proceeds. As the antireciprocity conditions also depends on the association rule adopted for the positive sign reference for currents and voltages the ports, if the adopted rule is not explicitly mentioned, some possible confusions may occur. Therefore, it is useful to represent the results explicitly in terms of the adopted association rule.

In this paper only resistive two-ports structures will be analyzed. Such structures are well suited for the gyrators we are referring to. Indeed, for the Hall gyrator we may count for a resistive behaviour up to the highest frequencies encountered in techniques [5]. As far as electronically gyrators are concerned, their behaviour may count as resistive in the working range of tens of kHz [3]. Therefore, instead of impedance or admittance two-port parameters, in analyzing gyrators we may base the computations on the simpler set of resistance or conductance two-port parameters.

As for any nonreciprocal two-port, for gyrators the antireciprocity condition will be implemented into the equivalent circuit via one or more voltage or current controlled sources. Due to their relatively simple structure, equivalent circuits can be conveniently used to make some check-up computations and power balances. As far as gyrators are concerned, it is of a peculiar interest the possibility of working-out the reactive power balance based on the equivalent circuits.

The paper focuses only on a single type of equivalent circuit, namely the Π-type circuit. Other types of equivalent circuit can be treated in a similar
2 Π-type equivalent circuit for antireciprocal two-port networks

The short-circuit conductance matrix of an arbitrarily resistive two-port may be decomposed as [2]:

\[
\begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix}
= \begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix} + 0 \begin{pmatrix}
0 & 0 \\
0 & G_4
\end{pmatrix}
\]

(1)

where \(G_4 = G_{21} - G_{12}\). Adopting the passive ("receptor") association rule at both access ports, the physical significance of two-port parameters is as follows: \(G_{11} = G_{1k}\) and \(G_{22} = G_{2k}\) are the driving-point short-circuit conductances at port one and respectively port two; \(G_{21} = (G_{k})_1\) and \(G_{12} = (G_{k})_2\) are the transfer short-circuit conductances, the two-port being forward, respectively backwards supplied.

The antireciprocity is expressed by the more general condition \(G_{12} \cdot G_{21} < 0\). If the two-port’s transfer parameters are of same magnitude, but of opposite signs, then the antireciprocity condition becomes \(G_{12} = -G_{21}\), [4].

The basic decomposition expressed by eq.(1) leads to an equivalent circuit consisting from a reciprocal two-port \((G_{12} = G_{21})\) of a Π-type, and a voltage-controlled current source \((G_4 U_1)\) connected at port \(22'\), which accounts for the non-reciprocity (antireciprocity) of the two-port (fig.1).

![Fig. 1 Equivalent circuit of a gyrator](image)

Writing the circuit equations for the equivalent circuit in fig. 1 we can determine the relations between the two-port parameters \(G_{11}, G_{12}, G_{21}, G_{22}\) and the parameters \(G_1, G_2, G_3, G_4\) of the equivalent circuit:

\[
\begin{align*}
G_{11} &= G_1 + G_2 \\
G_{12} &= G_2 \\
G_{21} &= G_2 - G_4 \\
G_{22} &= G_2 + G_3 \\
G_1 &= G_{11} - G_{12} \\
G_2 &= G_{12} \\
G_3 &= G_{22} - G_{12} \\
G_4 &= G_{12} - G_{21}
\end{align*}
\]

(2)

(3)

With these relations who, in fact, are of general nature, the schematics in figure 1 can be transformed into that in fig.2a. To take account of the two-ports antireciprocity, we may consider in eqs. (2, 3), for e.g., \(G_{12} > 0\) and \(G_{21} < 0\), or vice versa (according to the general antireciprocity condition). If we consider, for example, \(G_{12} > 0\), then the expressions for \(G_1, G_2, G_3\) will not change, but for \(G_4\) we will get

\[
G_4 = G_{12} - G_{21} = G_{12} + |G_{21}|
\]

(4)

If on the contrary, we take \(G_{12} < 0\), and consequently \(G_{21} > 0\), then

\[
G_4 = G_{12} - G_{21} = -(G_{12} + |G_{21}|)
\]

(5)

We should note that the relations for the parameters \(G_1, G_2, G_3\) and \(G_4\) are valid whether the transfer conductances are of same magnitudes or not. If, and only if, the magnitudes of the transfer conductances are equal, then, if needed, instead of eq.(3) we may write: \(G_1 = G_{11} + G_{21}, G_2 = G_{21}, G_1 = G_{22} + G_{21}, G_4 = -2G_{21} = 2G_{12}\). In these relations the transfer conductances are to be considered with their corresponding intrinsic signs.

Informatively, if we had adopted the passive association rule at port 1 1' and the active rule at port 2 2', then the relations between the two-port parameters and the equivalent circuit parameters had...
been:
\[ G_{11} = G_1 + G_2 \]
\[ G_{12} = -G_2 \]
\[ G_{21} = G_2 - G_4 \]
\[ G_{22} = -(G_2 + G_3) \] (6)

\[ G_1 = G_{11} + G_{12} \]
\[ G_2 = -G_{12} \]
\[ G_3 = -G_{22} + G_{12} \]
\[ G_4 = -G_{12} - G_{21} \] (7)

We should note that certain parameters in eqs.(6,7) have different signs as compared with those corresponding from eqs.(2, 3). Therefore, to avoid confusion, it is also necessarily to specify the corresponding association rule we based on when derived these relations.

It is important to point out that if we would make use of the physical significance of the two-port parameters we would obtain identical expressions, whether we consider the same association rule at both ports or not:
\[ G_1 = G_{1k} - (G_{k1}) \]
\[ G_2 = (G_{k2}) \]
\[ G_3 = (G_{k3}) \]
\[ G_4 = (G_{k4}) \] (8)

If we consider the passive association rule at port 22' in the more usual form as shown in fig.2b, then the short-circuit transfer conductances in eq.(2, 3) will change signs:
\[ G_{11} = G_1 + G_2 \]
\[ G_{12} = -G_2 \]
\[ G_{21} = G_2 - G_4 \]
\[ G_{22} = G_2 + G_3 \] (9)

Sometimes it is useful to show explicitly the signs of the transfer parameters in the equivalent circuit. If, for example, we consider the case \( G_{12} > 0 \), and consequently \( G_{21} = -|G_{21}| < 0 \), then the equivalent circuit that corresponds to eq.(3) is shown in fig.3a, whereas that corresponding to eq.(9) is shown in fig.3b.

### 3 Analysis of antireciprocal two-ports based on equivalent circuits

We begin the analysis with the simple case of an ideal antireciprocal two-port. In practice such gyrators can be building using operational amplifiers and resistors properly connected an valued. Lets suppose that the gyrator’s load is a capacitor \( C \) (fig.4). It is known that a gyrator acts as an impedance inverter, and therefore a capacitively-loaded gyror will simulate an inductor at the input port [10]. Capacitively-loaded gyror simulation of inductor; is a common practice in modern integrated circuits technologies, [I].

The general equations for an ideal antireciprocal two-port are:
\[ L_1 = G_{12} U_2 \]
\[ L_2 = G_{21} U_1 \] (10)

where \( G_{12} > 0 \). To eq.(10) we must add the load equation:
\[ U_2 = -L_2 Z_C = jX_C L_2 \]. By combining this equations we get the port-variables equations in terms of \( U_1 \):
\[ L_2 = G_{21} U_1 \]
\[ U_2 = jX_C G_{21} U_1 \]
\[ L_1 = jX_C G_{12} G_{21} U_1 \] (11)

![Fig.3 Equivalent circuits showing explicitly the signs of transfer conductances for the case \( G_{12} > 0, \ G_{21} < 0 \)](image)

![Fig.4 Capacitively-loaded gyror](image)
Fig. 5 Corresponding equivalent circuit

The relative phase angles for the port variables \((U_1', U_2', I_1', I_2')\) can be followed in the complex phasor diagram shown in fig.6, for the case \(G_{12}>0\). It is easy to check-up from this diagram that at output port the current \(-I_2'\) leads the voltage \(U_2'\) by 90° (capacitive phase angle) and that at the input port the current \(I_1'\) lags the voltage \(U_1'\) by 90° (inductive phase angle).

Fig.6 Complex phasor diagram for an capacitively loaded ideal gyrator

Based on relations (11) we may now determine the complex powers at the two-port terminals:

\[
S_1 = U_1' I_1' = P_1 - jQ_1 = jX_C G_{12} G_{21} U_1'^2
\]

\[
S_2 = U_2' (-I_2') = P_2 - jQ_2' = jX_C G_{21}^2 U_2'^2
\]

The analyzed gyrator being assumed lossless, it is obvious that the active powers at the ports should be zero. From eq.(12) it can be seen that the reactive power absorbed at port 1' is of positive sign, whereas the reactive power supplied to the capacitor at port 22' is of negative sign:

\[
Q_1 = -X_C |G_{12} G_{21}| U_1'^2 > 0
\]

\[
Q_2' = -X_C G_{21}^2 U_2'^2 < 0
\]  

(We should remember that \(G_{12} \cdot G_{21} < 0\)). To show explicitly the signs, we may write eq.(13) as:

\[
Q_1 = X_C |G_{12} G_{21}| U_1'^2 > 0
\]

\[
Q_2' = -X_C G_{21}^2 U_2'^2 < 0
\]

Instead of the “delivered” reactive power \(Q_2'\) at that port, which is of opposed sign to the “delivered” power \(Q_2\).

Therefore, if we consider at both ports the reactive powers as being “absorbed” by the two-port, then:

\[
Q_1 = X_C |G_{12} G_{21}| U_1'^2 > 0
\]

\[
Q_2 = X_C G_{21}^2 U_2'^2 > 0
\]

Let us note that if we consider just the reactive power at the ports, then it seems that there is no balance of those powers. Either there is no sign concordance between the reactive power absorbed at port 1’ \((Q_1>0)\) and the “delivered” power at port 22’ \((Q_2'<0)\), or there are “absorbed” reactive powers at both ports. Of course, this unusual situation is due to the fact that the gyrator acts as an impedance inverter. Unfortunately, based on the two-port equations (eq.10), only the powers at the two-port terminals can be calculated.

A formal explanation of this apparently unbalance of reactive powers can be given only with the help of an equivalent circuit. Considering the equivalent circuit in fig.5, with \(I_2' = -j \omega C U_2\) and \(U_2 = jX_C G_{21} U_1\), we determine the complex power absorbed by the controlled source:

\[
S_S = U_2'^* I_S = P_S - jQ_S = U_2'^* (G_{21} - G_{12}) U_1 = -jX_C (G_{21}^2 + |G_{12} G_{21}|) U_1^2
\]

Consequently, the active power is zero as expected, and the reactive power has the expression:

\[
Q_S = X_C |G_{12} G_{21}| U_1'^2 + X_C G_{21}^2 U_1'^2 > 0
\]

Comparing equations (15) and (17), we can state that the sum of the reactive powers “supplied” to the two-port at each port equals the reactive power absorbed by the controlled source of the equivalent circuit:

\[
Q_1 + Q_2 = Q_S
\]

We may also note that the ratio of the reactive powers at the ports, \(Q_1/Q_2\), equals the ratio of absolute values of short-circuit transfer conductances, \(|G_{12}|/|G_{21}|\).

Now we will analyze a lossy antireciprocal two-port \((G_{11} \neq 0 \text{ and } G_{22} \neq 0)\). The well known two-port equations are:

\[
L_1 = G_{11} U_1 + G_{12} U_2
\]

\[
L_2 = G_{21} U_1 + G_{22} U_2
\]

where again \(G_{12} \cdot G_{21} < 0\). Stalling from these equations and considering the passive association rule at both ports, we can derive the equivalent input admittance \(Y_{el} \). [4]:

\[
\text{“absorbed” reactive power } Q_2 \text{ at that port, which is of opposed sign to the “delivered” power } Q_2'.
\]

Therefore, if we consider at both ports the reactive powers as being “absorbed” by the two-port, then:

\[
Q_1 = X_C |G_{12} G_{21}| U_1'^2 > 0
\]

\[
Q_2 = X_C G_{21}^2 U_2'^2 > 0
\]
\( Y_{el} = G_{11} - \frac{G_{12}G_{21}}{G_{22} + Y_S} = G_{11} + \frac{|G_{12}G_{21}|}{G_{22} + Y_S} \) \hspace{1cm} (20)

Therefore, for a capacitively-loaded gyrator \((Y_S = j\omega C)\), we have:

\[
Y_{el} = G_{el} - jB_{el} = G_{11} + \frac{|G_{12}G_{21}|}{G_{22} + (\omega C)^2} \frac{G_{22} - j\omega C}{G_{22} + (\omega C)^2} \frac{\omega C}{G_{22} + (\omega C)^2}
\]

\(\omega = \sqrt{1 + \frac{G_{22}}{G_{22} + (\omega C)^2}} \)

which clearly reveals the inductive behaviour at port 11’ of the capacitively-loaded gyrator.

Now we determine the complex power at port 11’:

\[
S_1 = U_1^*I_1 = Y_{el}U_1^2 = P_1 - jQ_1 = \left[ G_{11} + \frac{|G_{12}G_{21}|}{G_{22} + (\omega C)^2} \frac{G_{22} - j\omega C}{G_{22} + (\omega C)^2} \right] U_1^2 - j\left[ \frac{|G_{12}G_{21}|}{G_{22} + (\omega C)^2} \frac{\omega C}{G_{22} + (\omega C)^2} \right] U_1^2
\]

\hspace{1cm} (22)

and consequently, the active and reactive power at that port:

\[
P_1 = \left[ G_{11} + \frac{|G_{12}G_{21}|}{G_{22} + (\omega C)^2} \right] U_1^2
\]

\[
Q_1 = \left[ \frac{|G_{12}G_{21}|}{G_{22} + (\omega C)^2} \right] \frac{\omega C}{G_{22} + (\omega C)^2} U_1^2
\]

\hspace{1cm} (23)

With the help of relation \( \left( \frac{U_2}{U_1} \right)^2 = \frac{G_{21}^2}{G_{22}^2 + (\omega C)^2} \),
we determine the complex power at output port 22’ in terms of \( U_1 \):

\[
S_2 = U_2^*(-I_2) = P_2 - jQ_2 = -j\omega \left[ \frac{G_{21}^2}{G_{22}^2 + (\omega C)^2} \right] U_1^2
\]

\hspace{1cm} (24)

We may note that the active power \( P_2 \) is indeed zero (purely reactive load) whereas the reactive power supplied to the two-port is:

\[
Q_2 = \omega C \left[ \frac{G_{21}^2}{G_{22}^2 + (\omega C)^2} \right] U_1^2
\]

\hspace{1cm} (25)

In order to check-up the balance of reactive powers we should again refer to an equivalent circuit. At the same time, we will also check-up the balance of active powers. Referring to the schematics in fig.2b, at the capacitively-loaded output port 22’ the Kirchhoff’s current law gives us:

\[
-j\omega U_2 = (G_{21} - G_{12})U_1 + (G_{22} + G_{12})U_2 - (U_2 - U_1)G_{12}, \text{ and hence:}
\]

\[
\frac{U_2}{U_1} = -\frac{G_{21}}{G_{22} + j\omega C} = -\frac{G_{21}(G_{22} - j\omega C)}{G_{22} + (\omega C)^2}
\]

\hspace{1cm} (26)

Now we can determine the complex power absorbed by the controlled source in terms of \( U_1 \):

\[
S_S = U_1^*I_S = U_1^2(G_{21} - G_{12})U_1 = \frac{G_{21}(G_{12} - G_{21})G_{22}U_1^2}{G_{22}^2 + (\omega C)^2} = P_S - jQ_S
\]

\hspace{1cm} (27)

The corresponding active and respectively reactive power are:

\[
P_S = \frac{G_{21}(G_{12} - G_{21})G_{22}U_1^2}{G_{22}^2 + (\omega C)^2}
\]

\hspace{1cm} (28)

\[
Q_S = \frac{G_{21}(G_{12} - G_{21})\omega C U_1^2}{G_{22}^2 + (\omega C)^2}
\]

\hspace{1cm} (29)

We note that the active power, being of negative sign, is effectively generated, whereas the reactive power is absorbed by the controlled source. From eqs.(23), (25) and (29) we see that the balance of reactive powers for an lossy antireciprocal two-port is again described by the relation \( Q_1 + Q_2 = Q_S \), as for the lossless two-port.

Now, based again on the equivalent circuit, we will check-up the balance of active power. After some mathematical transformations, we get for the power dissipated in the circuit’s resistors, \( P_R \), the following relation:

\[
P_R = \left[ G_{11} + \frac{|G_{12}G_{21}|}{G_{22}^2 + (\omega C)^2} \right] U_1^2 + \left[ \frac{|G_{12}G_{21}|}{G_{22}^2 + (\omega C)^2} \right] \frac{G_{21}^2}{G_{22}^2 + (\omega C)^2} U_1^2
\]

\hspace{1cm} (30)

The first term in eq.(30) represents precisely the active power absorbed by the two-port at port 11’ (eq.23), whereas the second represents the active power delivered by the controlled source (eq.28). Therefore, the active powers balance can be written as \( P_1 + P_S = P_R \).

Some of the main results obtained for an lossy antireciprocal two-port can be checked-up analytical as well as by computer simulation if we consider...
an OA gyrator designed such that it should have
controlled losses [6, 7]. Of course, this is not an
usual approach because in practice we want the
antireciprocal two-port to behave like an ideal one,
which means that the driving-point admittances have
to be negligible as compared with the transfer
admittances. We will admit such a design just to do
some checking-ups, as mentioned before.
Specifically, we have considered two situations:
a), the two short-circuit transfer conductances have
the same values;
b), the short-circuit transfer conductances have
different values.
For the first case the gyrator's quality factor
\(k = \frac{|G_{12}G_{21}|}{(G_{11}G_{22})} = 8.79\), whereas for the second,
k=3.05. Considering C=10nF, \(f=1kHz\) and \(U_1=1V\),
the most important results are shown in table. As can
be noted from this table, there is a good agreement
between analytical and simulated values.

<table>
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<th>(G_{12}) = (G_{21})</th>
<th>(G_{11}) [mS]</th>
<th>(G_{12}) [mS]</th>
<th>(G_{21}) [mS]</th>
<th>(G_{22}) [mS]</th>
<th>(G_{e1}) [mS]</th>
<th>(B_{e1}) [mS]</th>
<th>(L_e) [mH]</th>
<th>(P_1) [mW]</th>
<th>(Q_1) [mVAr]</th>
<th>(Q_s) [mVAr]</th>
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<tbody>
<tr>
<td>analytical</td>
<td>0.197</td>
<td>0.303</td>
<td>-0.303</td>
<td>0.047</td>
<td>0.917</td>
<td>0.853</td>
<td>86.6</td>
<td>0.917</td>
<td>0.853</td>
<td>1.706</td>
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<tr>
<td>PSpice</td>
<td>0.195</td>
<td>0.302</td>
<td>-0.303</td>
<td>0.053</td>
<td>0.912</td>
<td>0.852</td>
<td>87.05</td>
<td>0.914</td>
<td>0.844</td>
<td>-</td>
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<th>(G_{12}) ≠ (G_{21})</th>
<th>(G_{11}) [mS]</th>
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<th>(G_{22}) [mS]</th>
<th>(G_{e1}) [mS]</th>
<th>(B_{e1}) [mS]</th>
<th>(L_e) [mH]</th>
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<th>(Q_1) [mVAr]</th>
<th>(Q_s) [mVAr]</th>
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<td>-1</td>
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<td>0.5</td>
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<td>5.27</td>
<td>0.498</td>
<td>0.248</td>
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