The Implementation of a Unidirectional Topology Using Hall Gyrator

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Abstract: - The topology enclosing a Hall gyrator with negative transfer conductance and an elementary resistive two port is approached from the standpoint of the unidirectionallity condition. A Hall gyrator is a non-reciprocal device in which the Hall Effect reverses the polarity of a signal when input and output are interchanged. The Hall generator is the basic antireciprocal device; the negative resistances are needed to cancel out the effect of the Hall-plate input and output resistances. A gyrator realisation consisting of a Hall generator and two negative-resistance circuits is described. The circuit model, experimentally validated through electrical measurements, at unidirectionallity has with a relatively high attenuation value, due to the behavior of the Hall generator as a real gyrator. The attenuation constant may be improved using an ideal gyrator. The good concordance between values obtained experimentally and analytically confirms the studied model.

Key-Words: - Hall gyrator, negative transfer conductance, unidirectionallity.

1 Introduction

It is known that a two-port circuit is unidirectional if the transfer of electro-magnetic energy has a single sense. The unidirectional condition from port 1¹ to 2² it is that the no-load impedance and short-circuit impedance at driving-point must be equal, \( Z_{10} = Z_{1k} \). Taking account of this condition depending on \( Z \) and \( Y \) the no-load transfer impedance and the short-circuit transfer admittance are zero, \( Z_{e12} = Y_{e12} = 0 \) thus, the voltage \( U_1 \) and current \( I_1 \) at port 1¹ are not depending on \( U_2 \) and \( I_2 \) at port 2². The behavior of this two-port may be symbolized as 1* → 2, respectively 2 → 1*, where the asterisk show that the circuit 1¹ is supplied.

The unidirectional schematics contain a gyrator serial or parallel connected with a elementary quadripol resistive, depending on transfer equivalent impedance or equivalent transfer admittance in short circuit are null.

In this paper only resistive two-port structures will be analyzed. Such structures are well suited for the gyrators we referring to. Indeed, for the Hall gyrator we may count for a resistive behavior up to the highest frequencies encountered in techniques [4]. There for instead of impedance or admittance two-port parameters, in analyzing gyrators we may base the computations on the simpler set of resistance or conductance two-port parameters. The unidirectional condition become: \( R_{e12} = G_{e12} = 0 \), where \( e \), equivalent index for the equivalent two-port.

We use the association rule for receptors both to input and output ports.

2 Formal development of a unidirectional topology

The equations of Hall generator, we may write as:
\[
U_1 = R_{11}I_1 + R_{12}I_2
\]
\[
U_2 = -[R_{21}]I_1 + R_{22}I_2
\]
(1)

Regarding the expression of \( U_1 \) tension, the term \( R_{12}I_2 \) is corresponding secondary Hall tension, with
actions in command circuit when Hall generator is in run regime. These voltage actions in reverse sense of command voltage circuit and appear in relation with “+” sign. This result is characteristic for gyrator like behavior of Hall circuit.

The expression of matrix conductance \([G']\) of Hall gyrator is computed as follows:

\[
[G'] = \begin{bmatrix} G'_{11} & G'_{12} \\ G'_{21} & G'_{22} \end{bmatrix} = \frac{1}{\det[R']} \begin{bmatrix} R_{22} & -R_{21} \\ R_{12} & R_{11} \end{bmatrix}
\]  

(2)

where: \(\det[R'] = R_{11}R_{22} - R_{12}R_{21}\). Matrix \([R']\) is resistance matrix of Hall generator, resulting from rel. (1). We note that \(G'_{12}\) is negative. For realize a unidirectional circuit in parallel with Hall gyrator is connected a quadripol build from two longitudinal resistances equal in parallel (fig. 1).

The equivalent conductance \([G_e]\) of elementary resistive quadripol can be written under the form:

\[
[G_e] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
\]

(3)

Fig.1 Unidirectional quadripol with Hall gyrator

\[
[G_e] = \begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} = \begin{bmatrix} G_{e11} & G_{e12} \\ G_{e21} & G_{e22} \end{bmatrix} = \begin{bmatrix} G'_{11} + \frac{1}{2R_L} & G'_{12} + \frac{1}{2R_L} \\ G'_{21} + \frac{1}{2R_L} & G'_{22} + \frac{1}{2R_L} \end{bmatrix}
\]

(4)

The equivalent conductance in short circuit of unidirectional quadripol supplied at port 11’ namely:

\[
G_{e10} = G'_{11} + \frac{1}{2R_L} = \frac{R_{22}}{R_{11}R_{22} + R_{12}R_{21}} + \frac{1}{2R_L}
\]

(5)

The equivalent resistance matrix corresponding to equivalent quadripol becomes:

\[
[R_e] = \begin{bmatrix} R_{e11} & R_{e12} \\ R_{e21} & R_{e22} \end{bmatrix} = \frac{1}{\det[G_e]} \begin{bmatrix} G_{e22} & G_{e21} \\ G_{e12} & G_{e11} \end{bmatrix}
\]

(6)

We note with \(\det[G_e]\) the determinate matrix conductance equivalent may be written as follows:

\[
\det[G_e] = \left( \frac{1}{2R_L} \right) \left( G'_{11} + \frac{1}{2R_L} \right) \left( G'_{22} + \frac{1}{2R_L} \right)
\]

(7)

In no load equivalent resistance of equivalent quadripol supplied at port 11’ is:

\[
R_{e10} = \frac{G_{e22} + \frac{1}{2R_L}}{\det[G_e]} = \frac{G_{e22} + \frac{1}{2R_L}}{\det[G_e]}
\]

(8)

The no-load equivalent conductance of equivalent quadripol supplied at ports 11’ is:

\[
G_{e10} = \frac{1}{R_{e10}} = \frac{\det[G_e]}{G_{e22}} = \left( G'_{11} + \frac{1}{2R_L} \right) - \left( \frac{1}{2R_L} \right) G_{e22} + \frac{1}{2R_L} G_{e21}
\]

(9)

For \(G'_{12} = \frac{1}{2R_L}\), it results \(G_{e12}=0\), thus is obtain the unidirectionality as the relation (10):

\[
G_{e10} = G'_{11} + \frac{1}{2R_L} = G_{e1k}
\]

(10)

In this case, we can determine the transfer constant \(g_{1C}\) in direct sense [3]:

\[
g_{1C} = \ln \left( \frac{2\sqrt{G_{e11}G_{e22}^2}}{G_{e21}} \right) = \frac{2\sqrt{2R_LR_{22} + \det[R']}}{2R_LR_{12} + \det[R']} - \sqrt{\left( 2R_LR_{22} + \det[R'] \right)}
\]

(11)

The reverse transfer constant is infinitely:

\[
g_{2C} = \ln \left( \frac{2\sqrt{G_{e11}G_{e22}^2}}{G_{e21}} \right) = \frac{2\sqrt{2R_LR_{22} + \det[R']}}{2R_LR_{12} + \det[R']} - \sqrt{\left( 2R_LR_{22} + \det[R'] \right)}
\]

(12)

The unidirectionality is verified by connecting Hall gyrator in serial connection with transversal resistance \(R_T\) as is shown in fig. 2.
3 The Applicable circuit synthesis techniques

Taking account that the Hall transducer parameters have a resistive behaviour up to highest frequencies we consider the parameter conductance (resistances) instead of admittance (impedance). The Hall gyrator are general two-port, (fig.3) have the equations:

\[
\begin{align*}
[U] &= [R] [I] \\
[I] &= [G] [U]
\end{align*}
\]

The matrix \([R]\) elements from equation (13) are experiment determined from peculiar no-load regime.

\[
\begin{align*}
R_{11} &= (U_1/I_1)_{t=0} \\
R_{21} &= (U_2/I_1)_{t=0} \\
R_{12} &= (U_1/I_2)_{t=0} \\
R_{22} &= (U_2/I_2)_{t=0} \\
R_{13} &= (U_1/I_3)_{t=0} \\
R_{23} &= (U_2/I_3)_{t=0} \\
R_{31} &= (U_3/I_1)_{t=0} \\
R_{32} &= (U_3/I_2)_{t=0} \\
R_{33} &= (U_3/I_3)_{t=0}
\end{align*}
\]

The G parameters of general two-port may be calculated on the basis of link relation between different general two-port [3]. For the equivalent schematic valid in the general two-port theory, the conductance matrix in decomposed in the following way: The first three matrix from matrix relation (15) corresponds, in the order of decomposition to the three two port are shown in fig.4. The fourth matrix from relation (15) corresponds to current sources commanding voltage connected adequate to the terminals.

\[
\begin{align*}
[G] &= \begin{bmatrix}
G_{11} & G_{12} & G_{13} \\
G_{21} & G_{22} & G_{23} \\
G_{31} & G_{32} & G_{33}
\end{bmatrix} \\
&= \begin{bmatrix}
G_1 & -G_1 & -G_1 \\
-G_1 & G_1 & G_1 \\
-G_1 & G_1 + G_1 & -G_3
\end{bmatrix} \\
&+ \begin{bmatrix}
G_4 & 0 & 0 \\
0 & G_5 & 0 \\
0 & 0 & G_8
\end{bmatrix}
\end{align*}
\]

The equivalent schematic of Hall generators as general two-port is shown in fig.5; we assign the following relation:

\[
\begin{align*}
G_1 &= 1/2(2G_{23} - G_{13} - G_{33}) \\
G_2 &= 1/2(2G_{13} + G_{33}) \\
G_5 &= 1/2(G_{23} - 2G_{33} - G_{13}) \\
G_4 &= G_{11} + G_{13} \\
G_5 &= G_{22} + G_{23} \\
G_6 &= G_{12} + 1/2(2G_{23} - G_{13} - G_{33}) \\
G_7 &= G_{21} + 1/2(2G_{23} - G_{13} - G_{33}) \\
G_8 &= G_{31} + G_{13} \\
G_9 &= G_{32} + G_{23}
\end{align*}
\]

In order to check up obtain results we
determining the G parameters using the equivalent schematic in fig.5 and taking account of their physical significations. The nine G parameters defined in short circuit are:

\[ \begin{align*}
G_{11} &= (I_1/U_1)U_2 = U_3 = 0 = G_1 + G_3 + G_4 \\
G_{21} &= (I_2/U_1)U_2 = U_3 = 0 = -G_1 + G_7 \\
G_{31} &= (I_3/U_1)U_2 = U_3 = 0 = -G_1 - G_3 + G_8 \\
G_{12} &= (I_1/U_2)U_1 = U_3 = 0 = -G_1 + G_6 \\
G_{22} &= (I_2/U_2)U_1 = U_3 = 0 = G_1 + G_2 + G_5 \\
G_{32} &= (I_3/U_2)U_1 = U_3 = 0 = G_1 + G_2 + G_9 \\
G_{13} &= (I_1/U_3)U_1 = U_2 = 0 = -G_1 - G_3 \\
G_{23} &= (I_2/U_3)U_1 = U_2 = 0 = G_1 + G_2 \\
G_{33} &= (I_3/U_1)U_1 = U_2 = 0 = G_1 + 2G_2 + G_3 
\end{align*} \]  

(17)

The expressions are concordant with relation (16).

4 The unidirectionality condition in the theory of general two-port

For a unidirectional two-port in which the transfer of electromagnetic power is made only at terminals 11' and 22' the no-load impedances and short circuit at these terminals are equal. The input admittance must be the same whether the function regime at terminal 22' is. In the other hand, the current at port 33' must be zero. Those conditions are written as follows

\[ \begin{align*}
Y_{1k} &= Y_{10}, \quad I_3 = 0 \\
Y_{1k} - \text{the input admittance in short circuit at port 22'} \\
Y_{10} &\quad \text{the input admittance for no-load regime at port 22'}
\end{align*} \]  

(18)

Determining the two input admittances and taking account of relation (18) we obtain the relation that must be fulfill by the G parameters for a two-port circuit

\[ G_{22}(G_{13}G_{32} - G_{12}G_{33})(G_{23}G_{31} - G_{21}G_{33}) = 0 \]  

(19)

The relation (19) is fulfilled if one of two conditions are carried ant:

\[ G_{13}G_{32} - G_{12}G_{23} = 0, \quad G_{23}G_{31} - G_{21}G_{33} = 0 \]  

(20)

5 The unidirectional scheme obtain by connecting same unequal resistances in parallel with Hall gyrator

The Hall transducer as general two-port become a unidirectional schematic by connecting r' and r" in parallel with Hall plate longitudinal or in X. (fig.6.)

Thus, the Hall transducer considered general two-port shown in fig.5 is in parallel with one of general two-port shown in fig.7a. or fig.7b. For two-port from fig.7a and fig.7b in which we note G'=1/r' and G"=1/r" the conductance matrix are:

\[ [G] = \begin{bmatrix}
G' & -G' & -G' \\
-G' & G' & G' \\
-G' & G' & G' + G''
\end{bmatrix} \]  

(21)

\[ [G] = \begin{bmatrix}
G' & 0 & -G' \\
0 & G'' & G'' \\
-G' & G' & G' + G''
\end{bmatrix} \]  

(22)

The equivalent conductance matrix [G_e] of the corresponding schematics fig.6a and fig.4b are:
Adopting the unidirectionality condition (relation (20)) for matrix (23) we get relation (25) and for matrix (24) relation (26).

\[
G' G'' + (G_{13} - G_{32} - G_{12} + G_{33})G' - G_{12}G'' = 0
\]

(25)

\[
G' G'' + (G_{12} - G_{32})G' + (G_{12} + G_{13})G'' = 0
\]

(26)

6 Unidirectional schematic obtain by connecting some equal resistances in parallel with gyrator Hall

We consider for Hall gyrator on equivalent schematic in symmetrical bridge: By connecting with supplementary equal resistances is full field the interaction condition in parallel for diport two-port, which means, by applying a voltage at input terminal of the two-port connected in parallel at output terminal of the constitutive two-port the voltage is null if the terminals of each two-port are in short circuit.

Fig.8 Two diport tow-port interconnected in parallel

The equivalent conductance matrix \([G_e]\) for schematic (fig.8) is obtain by totalizing conductance matrix of gyrator Hall with conductance matrix (21) particularized for this case (in which we note \(G=I/r\)).

\[
[G_e] = \begin{bmatrix}
G_{11} + G' & G_{12} - G' & G_{13} - G' \\
G_{21} - G' & G_{22} + G' & G_{23} + G' \\
G_{31} - G' & G_{32} + G' & G_{33} + G' + G''
\end{bmatrix}
\]

(23)

\[
[G_e] = \begin{bmatrix}
G_{11} + G' & G_{12} & G_{13} - G' \\
G_{21} & G_{22} + G' & G_{23} + G' \\
G_{31} - G' & G_{32} + G' & G_{33} + G' + G''
\end{bmatrix}
\]

(24)

Applying the unidirectionality condition (20) on matrix (27) we get the expression of longitudinal resistance \(r\).

\[
r = \frac{R_{11}R_{22} + R_{12}R_{21}}{-2R_{21}}
\]

(28)

If transducer Hall with no zero voltage relation (28) becomes:

\[
r = \frac{R_{10}R_{20} + (S_0B)^2}{2S_0B}
\]

(29)

If the Hall plate is simetrical we get relation (30) know in literature [3].

\[
r = \frac{(R_{10})^2 + (S_0B)^2}{2S_0B}
\]

(30)

If the resistance are connected in X, in parallel at plate Hall, following the same algorithm by applying the unidirectionality condition we get:

\[
r = \frac{R_{11}R_{22} - \left(\frac{R_{12} + R_{21}}{2}\right)^2 - \left(\frac{R_{12} - R_{21}}{2}\right)^2}{2R_{21}}
\]

(31)

If the Hall plate with no zero voltage the resistance value \(r\) coincides with relation (29).

7 Experimental results

For the beginning, we determine the resistance parameters of Hall generator for a frequency \(f=950\text{Hz}\), in no load regime, using a plate Hall from InSb with excitation circuit with \(U_e=4.4\text{V}, I_e=1\text{A}\), with provide an induction \(B=1\text{T}\).

<table>
<thead>
<tr>
<th>(I_2 = 0)</th>
<th>(I'_2 = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_1) mV</td>
<td>(I_1) mA</td>
</tr>
<tr>
<td>246</td>
<td>80.1</td>
</tr>
</tbody>
</table>

Table 1

From which we determine the following parameters:

\[
\begin{align*}
R_{11} &= (U_1/I_1) = 3.07 \Omega \quad \text{for } I_1 = 0; \\
R_{21} &= (U_2/I_1) = 0.137 \Omega \quad \text{for } I_1 = 0; \\
R_{22} &= (U_2'/I_1) = 3.44 \Omega \quad \text{for } I'_1 = 0; \\
R_{12} &= (U_1'/I_2) = 0.138 \Omega \quad \text{for } I'_2 = 0; \\
\end{align*}
\]

The Hall equations become with respect to association rule receptor- receptor:

\[
U_1 = 3.07I_1 + 0.138I_2 \\
U_2 = -0.137I_1 + 3.44I_2
\]

(32)

The conductance matrix \([G']\), result from resistance matrix \([R']\) has the expression:
The equivalent conductance matrix \([G_e]\) become:

\[
\begin{bmatrix}
0.32 & -0.012 \\
-0.012 & 0.29
\end{bmatrix}
\]

The equivalent conductance matrix \([G_e]\) become:

\[
G_e = \begin{bmatrix}
0.32 + \frac{1}{2RL} & -0.012 + \frac{1}{2RL} \\
-0.012 + \frac{1}{2RL} & 0.29 + \frac{1}{2RL}
\end{bmatrix}
\]

For different values of longitudinal resistances (fig.2), at a frequency of 950Hz, are obtain the values presented in table 2.

<table>
<thead>
<tr>
<th>(R_L) [Ω]</th>
<th>(U_1) [V]</th>
<th>(I_{e10}) [mA]</th>
<th>(I_{e1k}) [mA]</th>
<th>(G_{e10}) [mS]</th>
<th>(G_{e1k}) [mS]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.1</td>
<td>32.5</td>
<td>52.4</td>
<td>325</td>
<td>524</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>35.0</td>
<td>44.2</td>
<td>350</td>
<td>442</td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
<td>37.0</td>
<td>40.0</td>
<td>370</td>
<td>400</td>
</tr>
<tr>
<td>30</td>
<td>0.1</td>
<td>38.4</td>
<td>38.4</td>
<td>384</td>
<td>384</td>
</tr>
<tr>
<td>50</td>
<td>0.1</td>
<td>40.0</td>
<td>37.5</td>
<td>400</td>
<td>375</td>
</tr>
</tbody>
</table>

Analyzing the values from table 2, we note that for \(R_L=40\)Ω, the short circuit and no load conductances are equal, thus the unidirectionality condition is fulfill.

Analytical unidirectionality is obtain for a value of a longitudinal resistance \(R_L=1/(2*0.012)=41.6\)Ω, situation in which \(G_{e1k}=0\).

For different values of longitudinal resistance \(R_L\) we trace characteristics \(G_{e1k}=f(R_L)\) and \(G_{e10}=f(R_L)\), (fig.9).

The transfer constants in direct sense and reverse of equivalent unidirectional schematic have expressions:

\[ g_{ic} = a = \ln 2.38 = 3.27Np = 28.37dB \]

\[ g_{2c} = \infty \] (34)

Different values of transversal resistance \(R_T\), fig.2, obtain experimentally are shown in table 3.

<table>
<thead>
<tr>
<th>(R_T) [Ω]</th>
<th>(U_1) [V]</th>
<th>(I_{e10}) [mA]</th>
<th>(I_{e1k}) [mA]</th>
<th>(R_{e10}) [kΩ]</th>
<th>(R_{e1k}) [kΩ](^*)</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.165</td>
<td>3.17</td>
<td>59.2</td>
<td>5.20 (10^2)</td>
<td>2.78</td>
<td>(f=950) Hz</td>
</tr>
<tr>
<td>100</td>
<td>0.22</td>
<td>2.17</td>
<td>81.6</td>
<td>1.01 (10^3)</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>0.24</td>
<td>1.64</td>
<td>84.6</td>
<td>0.146</td>
<td>2.83</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.26</td>
<td>1.32</td>
<td>85.6</td>
<td>0.196</td>
<td>3.03</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0.28</td>
<td>0.95</td>
<td>90.4</td>
<td>0.294</td>
<td>3.09</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9 The characteristics \(G_{e1k}=f(R_L)\), \(G_{e10}=f(R_L)\)

8. Conclusions

Due to negative transfer conductance of Hall gyator, the unidirectional condition for an equivalent quadrupol is realized only by parallel connection with an elementary resistive two port. Both, experimentally and analytically, the concordance between values is rather good. At unidirectionality, the transfer constant in direct sense (attenuation) has a relatively high value because that the Hall generator is a real gyator. The attenuation constant may be improved using an ideal gyator.

References: