A Bayesian Framework for Parameters Estimation in Complex System

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Abstract: - The real-life complex development situations express that the methods applied to new product development process content reliability risks which require assessment and quantification at the earliest stage, extracting relevant information from the process. Reliability targets have to be realistic and systematically defined, in a meaningful way for marketing, engineering, testing, and production. Potential problems proactively identified and solved during design phase and products launched at or near planned reliability targets eliminate extensive and prolonged improvement efforts after start on. Once in the market, products standard procedures require monitoring of early signs of issues, allowing corrective action to be quickly taken. Reliability validation before a product goes to market by the means of Bayesian statistical method because the model has shorter confidence intervals than the classical statistical inference models, allowing a more accurate decision-making process. The paper proposes the estimation of the shape parameters in a complex data structures approached with exponential gamma distribution as model of life time, reliability and failure rate functions. The numerical simulation performed in the case study validates the correctness of the proposed methodology.

Key-Words: - failure, rate, Bayesian model, adequate function, distribution, simulation.

1 Introduction

The systematic reduction of product development time and cost without risking or sacrificing reliability includes procedures and standards regarding the choice of types of components of product, research and selection of manufacturers, suppliers, purchasing specifications, analysis of faults, the reliability of their introduction into manufacturing and thereafter etc. Based on experience with similar components, specifying target reliability prediction is based on laboratory tests and a large amount of data obtained in operational regime. Variables influencing the failure rate of components are:

1. criteria for failure: critical values of certain accessible parameters of the component as is considered defective (failure may be total, catalectic or derived)
2. electrical constraints such as: current, power, noise, etc..
3. thermal constraints which depend on the type of the studied components
   a. Passive components are characterized by ambient temperature and the capsule or layer temperature
   b. Active components are characterized by two types of temperature: a junction and normalized junction
4. constraints climate: humidity, pressure, dust, altitude
5. mechanical constraints: shock, vibration
6. other factors: technology, construction, etc. manufacturer. In reality because of the large number of variables that influence the intensity of failure, it is difficult to determine their influence. If the note by R (t) the reliability or the likelihood of good functioning at time t and dR crash probability between time t and t + dt, the intensity of failure can be considered as a function of time λ (t) or z (t) that vary by a curve in the "bathtub" result obtained by accumulating a large volume of data and is concordant with some general principles. The behavior of dynamical systems can be characterized using the so-called reliability bathtub curve, as shown in Figure 1, i.e. the initial decreasing failure rate period (infant-mortality, AB) is followed by a relatively long constant failure rate period (useful life), while the probability of failures sharply increases toward the end of the system’s design life (wear-out period).

2 Summarizing the modeling of failure rate

A particularly problem in determining the reliability of the elements is related to the complexity of environmental factors, acting simultaneously. The climatic factors, factors related to the chemical, thermal, mechanical (shock, vibration) and the variables that influence the rate of failure of components have to be taken into account. The component resistance and demands that characterize the spectrum of operation, determines the intensity of component failure in a given situation. It notes that over time, the component with quality level Q2 will fail several times assuming its replacement, unlike the component with quality level Q1 where the constraints have not exceeded the supported level. Even in the same batch, components don’t have exactly the same quality regarding not only the mechanical resistance to vibration, shock, pressure or acceleration but temperature resistance, to withstand the different voltage variations, resistance to moisture, corrosion, radiation, etc. Appears observable that the electronic component reliability can not be expressed by a single numerical value significantly, requesting a spectrum that cannot be expressed by a single numerical value but by an interval. Therefore, the usual models employed to describe the behavior over time of various applications based on physical phenomena, chemical and statistical findings are based on a large number of experiments, having a predictive character.

Among the best known laws of survival or mortality is the Gompertz law (1825). Gompertz hypothesis was that death may be caused by two distinct cases:
- By a hazard, accident, this could lead to the death of a healthy person, regardless of age;
- By the force of mortality which consists in the gradual weakening of the individual as the force of mortality ρ in dt is proportional to ρ dρ.

Neglecting first situation, Gompertz equation can be written under the form:

\[
\frac{1}{\rho} \frac{d\rho}{dt} = k \tag{1}
\]

where: \( \rho = a \cdot e^{kt} \)

If \( S_t \) is the number of survival to age at time t, and \( S_{t+dt} \) is the number of survival at time \( dt \), the mortality relative to the time interval \( dt \) is express as:

\[
\frac{S_t - S \cdot t + dt}{S_t} \tag{2}
\]

As a model of aging, taking into account chemical reactions in the dielectric is adopted the Arrhenius equation, which agree on the degradation rate:

\[
v = A \cdot e^{-\Delta E / KT} \tag{3}
\]

where A is a constant;

\( \Delta E \) - energy of activation (ie the corresponding energy level that any molecule reach to enter the reaction);

K - Boltzmann's constant

Considering the electrical solicitation (expressed by a function S), can be accepted Eyring relationship:

\[
v = A_1 \cdot AT^a \cdot \exp\left(-\frac{b}{t}\right) \cdot [S(e + \frac{d}{T})] \tag{4}
\]

where A1, a, b, c, and d are constant.

For the low power applications, Arrhenius relationship may be written as:

\[
\alpha = \tau \cdot e^{-b/T} \tag{5}
\]

\[
\frac{R}{R_0} = e^{-\tau \exp\left[-\frac{b}{T}\right]} \tag{6}
\]

the equation (6) may represent the general degradation low of thermal insulation. At T = constant, may be represented as:

\[
\ln\left(-\ln\frac{R}{R_0}\right) = \left(\ln \tau - \frac{b}{T}\right) + \ln t \tag{7}
\]
For common dielectrics, ratio \( R/R_0 \) is permitted 0.5 and \( t = t_{\text{lim}} \), considered under the forms (8), (9), (10):

\[
t_{\text{lim}} = \left\{ \ln[-\ln\left(\frac{R}{R_0}\right)_{\text{lim}}] - \ln \tau \right\} + \frac{b}{T} \tag{8}
\]

\[
\ln t_{\text{lim}} = a + \frac{b}{T} \tag{9}
\]

\[
t_{\text{lim}} = a + \frac{b}{T} + \frac{c}{T^2} + \frac{d}{T^3} + \ldots \tag{10}
\]

In the operating regime, for the electrical servomotors the temperature distribution in the winding is exponential, ie \( f(T) = \lambda \exp[-\lambda (T-T_0)] \). The distribution of failure times of winding can be express as:

\[
f(t) = \frac{\lambda \lambda}{t(tn-t-\alpha)} \exp\left[-\frac{\lambda b}{\ln t - A} - T_0 \right] \tag{11}
\]

Symmetrically, the stabilized form of the aging law of mechanical materials is reached if in the Arrhenius equation is taken into account the effects of mechanical demands on growth of aging rate:

\[
\ln t_{\text{lim}} = a + \frac{b - g \cdot \sigma}{T} \tag{12}
\]

where: \( g \) is the influence coefficient for mechanical load.

The time distribution function, damage due to fatigue, can be written under the form:

\[
f(t) = \frac{T}{t_\sigma \sqrt{2\pi}} \exp\left[\frac{-(T \ln R_{\text{lim}} - a - m)^2}{2s^2} \right] \tag{13}
\]

where: \( m \) is the mean and \( s \) is standard deviation referring to parameter \( b \).

The law for mechanical sub-assemblies, as rolling bearings, which failure is due to global and local warming, is formalized as stated in equation (14):

\[
\ln t = a + \frac{b}{T_0 + \frac{k_0}{(\delta - \delta_m)^{2/3}}} \tag{14}
\]

where: \( T_0 \) is the ambient temperature (in Kelvin degrees); \( k_0 \) - factor heat sizing of functional interfcer \( \delta \) and \( \delta_m \) - functional and minimal interfcer (acceptable).

Models widespread today are the following:

- Model of Bazovskz
- Tatár's model
- The type RADC TR - 67 - 108
- Model of MIL HDBK 217 B

Bazovskz's model, based on Arrhenius's law which assumes that chemical reaction rate in solution doubled for an increase in temperature to \( 1^\circ C \), concludes under the form:

\[
\lambda_2 = \lambda_1 \left(\frac{V_2}{V_1}\right)^n \cdot K^{(n-a)} \tag{15}
\]

where:

\( \lambda_1 \) is failure rate for the voltage \( V_1 \) and the temperature in Celsius;
\( \theta_1 \) and \( \theta_2 \) temperature in Celsius degrees;
\( \lambda_2 \), mechanical constraints

Exponent \( n \) and the value of \( K \), variation factor of the failure rate for a change in temperature with \( 1^\circ C \) should be determined in operational conditions for each type of component.

Tatár's model is an exponential model described through equation (16):

\[
\lambda = \lambda_b \cdot \exp\left[ h_1 T + b_2 S + b_3 S^2 + b_4 T^2 + b_5 S T \right] \tag{16}
\]

Model RADC TR 67-108 (Rome Air Development Center Reliability notebook) for condensers is presented under general form as eq. (17):

\[
\lambda = \lambda_b \left[ \frac{S}{NS} \right]^H \cdot \left[ \frac{T}{NT} \right]^G \tag{17}
\]

where: \( S \) is nominal voltage;
\( T \) is ambient temperature in °K.

\( \lambda_b, NS, H, NT, G \) are constant characteristics for each type of component; e.g., for the Tantalum capacitors with solid electrolyte: \( \lambda_b = 1.3 \times 10^{-8}; NS = 0.52, H = 3, NT = 358, G = 8 \)

A similar, but more comprehensive model for condensers is the MILHDB 217 B described by the equation:

\[
\lambda_b = K \left[ \left(\frac{P}{NS} \right)^H + 1 \right] \cdot \left[ \frac{T}{NT} \right]^G \tag{18}
\]

The doubts regarding dependency of the failure rate on temperature question the validity of the models used, in particular MILHDB 217. The Arrhenius formula, that relates physical and chemical process rates to temperature, has been used to describe the relationship between temperature and time to failure for electronic components. This formula is the basis of methods for predicting the reliability of electric systems. For the majority of modern electronic components, the failure mechanisms are not activated or accelerated by temperature increase. The materials and processes are stable up to temperatures higher of those recommended for use.
This inadvertence may be induced by the observation that a large proportion of components was observed to fail at higher temperatures. The current data do not show such a relationship [5]. For quantitative assessment of failure rate are employed two models:
- linear regression if model is linear or linear by logarithming;
- adjustment of gradient developed by Fletcher and Powell if RADC model is not linear.

These models consist of minimizing the sum of the squares differences between estimated values \( \lambda_i \) and adjusted values \( \hat{\lambda}_i, (\sum_{i=1}^{n} (\lambda_i - \hat{\lambda}_i)^2) \), apply allowed by above mentioned models.

Important issues are related to share data. Qualitative variables treat (technology, manufacturers, types of use, etc.) requiring a particular distinction between the variables made by constant term iteration.

The analysis made concludes that the charges which cumulate a small number of hours comparing to MTBF of the component lead to erroneous results and hence the accuracy of models is found depending on the quantity of available data. From this standpoint it is recognized that RADC 67-108 reflect better the average behavior of the components.

The lack of any precise formula linking specific environments to failure rates, even though empirical relationships have been established between certain failure rates and specific stresses, (voltage and temperature).

The omission of some factors that affect reliability, as:
- transient over-stress;
- temperature cycling;
- control of assembly;
- test and maintenance,
remain shortcomings reported in the dedicated literature.

A more profound research into the significance of the rate of failure for a component raised and other issues related to the validity of some multipliers used in the models. Adverse effects of increasing device complexity have generally been counteracted by process improvement.

Therefore, before presenting the model developed for the failure rate of components are considered necessary clarification and theoretical findings regarding the practical failure rate.

3. Dynamical model for failure rates via Bayesian theorem

Assuming that each component is characterized by a resistance degree to a constrain with determined nature (electricity, heating etc.), Bayes' theorem is operating by potential resilience for \( i \) component, \( i = 1, 2, ..., n \), \( \gamma_{ij} \), where \( n \) is the components number of the batch subject to spectrum \( j \) of solicitation. Generalizing, we can consider all the \( j \) constraints simultaneously, \( j=1, 2, ..., k, \).

Under these assumptions it can be assumed that the component failure phenomenon is actually the result of two factors independent:
- resistance \( r_{i,j} \) of the component to all the \( j \) constraints;
- the constraints spectrum of the component.

For \( i-I, i, i+1 \) components with different levels of resistance \( r_{i,j} < r_{i+1,j} \) and subjected to the same set of demands \( j \), defected at different times \( t_{i-1} < t_i < t_{i+1} \).

In reality the problem is complex because of repeated constrain and a combination of different nature to whom the components are submitted. A deeper comprehension of a multidimensional spectrum requires that the phenomenon of catalectic failure to be considered random, undetermined, thus statistical and probabilistic approached. Such a model-based technique is provided by through Bayes' theorem and the resulting consequences.

Given a random variable \( x \) whose probability distribution depends on a set of parameters \( P = (P_1, P_2, ..., P_p) \). Exact values of the parameters are not known with certainty, Bayesian reasoning assigns a probability distribution of the various possible values of these parameters that are considered as random variables. Bayes’ theory is generally expressed through probabilistic statements as following:

\[
P(A / B) = P(A) \times \frac{P(B / A)}{P(B)}
\]

(19)

\( P (A / B) \) is the probability of \( A \) given the event \( B \) occurs or the posteriori probability. Using Bayes' theory may be recurring, that if exist an a priori distribution (\( P (A) \)) and a series of tests with experimental results \( B_1, B_2, ..., B_n \), expressed according to successive equations:

\[
P(A / B_1) = P(A) \frac{P(B_1 / A)}{P(B_1)}
\]

\[
P(A / B_1, B_2) = P(A) \frac{P(B_1 / A)}{P(B_1)} \frac{P(B_2 / A)}{P(B_2)}
\]

(20)
\[ P(A / B_1, B_2, \ldots B_n) = P(A / B_1, B_2, \ldots B_{n-1}) - \frac{P(B_n / A)}{P(B_n)} \]

A posteriori distribution is used as the test results are known, being obtained as a new function a priori. The start of operations sequences in the Bayesian method regards the transformation \( \gamma \).

In any transformation is looking to find invariant terms. The determination of \( \lambda \), invariants, Bayesian distribution is gamma. Admitting that for each individual component \( i \) of a batch, a resistance \( r_{i,j} \) for the requests spectrum \( j \), it is recognized that there is a statistical distribution \( g(r_{i,j}) \) of the quantities \( r_{i,j} \). As a consequence of univocal relationship \( r_{i,j} \rightarrow \lambda_i \), the distribution \( g(r_{i,j}) \) implies a distribution \( h(\lambda_i) \) of the components failure intensities of considered batch. The distribution \( h(\lambda_i) \) is a random distribution of values and reflects the statistical distribution \( \lambda \) of components that survive from batch at time \( t \). The components tend to defect faster, the mean distribution \( h(\lambda_i) \) move to the left as in fig. 1.

\[
\frac{\sigma'(\lambda)}{\sigma(\lambda)} = \frac{\beta' \sqrt{\alpha'} + 1}{\beta \sqrt{\alpha} + 1} = \frac{\beta'}{\beta} = 1 + \beta t
\]

The aging phenomenon has a general model given by the equation:
\[
f(t) = \frac{(\alpha + 1)^\beta}{1 + \beta t} e^{-\beta t}
\]

If it accepts the general pattern given by the relation (24) the obtain curve is a three-dimensional diagram (fig.2).

The period of decline that characterizes hyperbolic useful life of components is followed by an exponential growth that characterizes aging components at time \( t \) in operational regime.

**Fig. 2 Failure rate depending on a posteriori distribution**

**4. Case study. Experimental results**

In order to establish the period of time in which by an artificial aging of a batch of components is achieved a desired level of reliability, we assume the condition:
\[
z(kt^*) = \lambda^*
\]

where: \( \lambda^* \) is the reached target of components failure rate in operational conditions;

\( k \) is acceleration coefficient;

\( t^* \) is artificial aging duration.

A method for setting \( \alpha \) and \( \beta \) parameters based on available information, for a situation where prior information is limited to the estimated failure rate consider the empirical matching of random variables moments, consisting of observations performed at theoretical moments calculated using unconditioned probability density. The results show that in some cases reach an inadequate sensitivity distribution from the experimental results so that the uncertainty is either over or under estimated.

Assuming a distribution of gamma a priori:
\[
\frac{\lambda_1}{\lambda} = 1 + \beta t
\]

Also \( 1 + \beta t \) characterize the time restriction a posteriori distribution:

\[
\frac{\sigma'(\lambda)}{\sigma(\lambda)} = \frac{\beta' \sqrt{\alpha'} + 1}{\beta \sqrt{\alpha} + 1} = \frac{\beta'}{\beta} = 1 + \beta t
\]

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Relationship (27) relates parameters $\alpha$ and $\beta$ of a priori distribution:
\[
I^* = \frac{(\alpha + 1)\beta - \lambda^*}{k\beta\lambda^*}
\]
(27)

The good correlation between experimental and theoretical results which confirms the adequacy of the law gamma for a priori distribution presented syntactically in Table 1.

<table>
<thead>
<tr>
<th>Time intervals $(t_1 - t_2)$</th>
<th>Mean Time $tn = \frac{t_1 + t_2}{2}$</th>
<th>Number of failed component $(c)$</th>
<th>Number of functional component $(n)$</th>
<th>$z \times lo (ore)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 500</td>
<td>250 h</td>
<td>19</td>
<td>893</td>
<td>2,35</td>
</tr>
<tr>
<td>500 – 1000</td>
<td>750 h</td>
<td>9</td>
<td>874</td>
<td>4,86</td>
</tr>
<tr>
<td>1000 – 2000</td>
<td>1500 h</td>
<td>17</td>
<td>857</td>
<td>5,04</td>
</tr>
<tr>
<td>2000 – 3000</td>
<td>2500 h</td>
<td>12</td>
<td>845</td>
<td>7,04</td>
</tr>
<tr>
<td>3000 - 5000</td>
<td>4000 h</td>
<td>18</td>
<td>827</td>
<td>9,19</td>
</tr>
</tbody>
</table>

5. Conclusions

The checking up of simulation, experimental and analytical, shows a good concordance, the estimated parameters ($\alpha$ and $\beta$) obtain by two ways have rigorous same values. Estimated values of the parameters are: $\hat{\alpha} = -0,940$ and $\hat{\beta} = 6,19 \times 10^{-4}$.

The instant failure rate is:
\[
z(t) = \frac{3,71 \times 10^{-5}}{1 + 6,19 \times 10^{-4} xt}
\]

By extrapolation, within the limits of validity of this model can be obtain intensities of failure for any number of hours.

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