Conformal Mapping in Naval Architecture

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Abstract: In this paper, we present how to extend the Lewis transformation for obtain the contour of the ship’s cross section of different types of ships. Moreover, we present how a more accurate transformation of the cross sectional hull shape can be obtained by using a bigger number of parameters.

Key–Words: hydrodynamic ship cross section; Lewis Transformation; area of the cross section.

1 Introduction

During the last twenty years, in the area of naval hydrodynamics as well as in other domains of mechanical engineering, a growing interest has occurred towards the algorithmic methods of solving some definite problems.

The shapes of the ships have been described around a point, as it is difficult to describe them according to analytical mappings.

It is necessary to approximate the ship’s shape by continuous functions, in order to get some practical results. A method, which has imposed itself during the last few years, is that of multi-parameter conformal mappings, with good results also in the case of extreme bulbous forms.

A more accurate transformation of the cross sectional hull form can be obtained by using a greater number of parameters n. Somewhat better approximations will be obtained by taking into account also the first order moments of half the cross section about the x and y axes. The ship hull forms have been described by the well-known classic Lewis Transformation [3], and by an Extended-Lewis Transformation with three parameters, as given by Athanassoulis and Loukakis [1], with practical applicability for any types of ships. We already have studied [2] an algorithmic method solving directly the problems that appear in naval architecture domain concerning the contour of ship’s cross-section.

The general transformation formula is given by:

\[ f(Z) = \mu_s \sum_{k=0}^{n} a_{2k-1} Z^{-2k+1}, \]  

where: \( f(Z) = z \), \( z = x + iy \) is the plane of the ship’s cross section. \( Z = ie^\alpha e^{-i\varphi} \) is the plane of the unit circle, \( \mu_s = 1 \), \( a_{2k-1} \) are the conformal mapping coefficients \( (k = 1, ... n) \), \( n \) is the number of parameters.

Therefore we can write in turn:

\[ x + iy = \mu_s \sum_{k=0}^{n} a_{2k-1} (ie^\alpha e^{-i\varphi})^{-(2k-1)}, \quad (2) \]

\[ x + iy = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} e^{-(2k-1)\alpha} \]

\[ [icos(2k-1)\varphi - sin(2k-1)\varphi]. \quad (3) \]

From the relation between the coordinates in the Z - plane (the ship’s cross section) and the variables in the z - plane (the circular cross section), it follows:

\[ x = -\mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} e^{-(2k-1)\alpha} \sin(2k-1)\varphi, \quad (4) \]

\[ y = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} e^{-(2k-1)\alpha} \cos(2k-1)\varphi. \quad (5) \]

Now by using conformal mapping approximations, the contour of the ship’s cross section follows from putting \( \alpha = 0 \) in (1.4) and (1.5). We get:

\[ x_o = -\mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} \sin(2k-1)\varphi, \]

\[ y_0 = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} \cos(2k-1)\varphi. \]

The breadth on the waterline of the approximate ship’s cross section is defined by

\[ B_0 = 2\mu_s \beta, \quad \beta = \sum_{k=0}^{n} a_{2k-1}, \]
and the draft is defined by

\[ D_0 = 2\mu(s)\delta, \text{ with } \delta = \sum_{k=0}^{n} (-1)^k a_{2k-1}. \]

The breadth on the waterline is obtained for \( \varphi = \pi/2 \), that means:

\[ x_{\pi/2} = -\mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} \sin(2k - 1)\pi/2, \]

hence

\[ x_{\pi/2} = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1}, \text{ and } B_0 = 2x_{\pi/2}. \]

The scale factor is \( \mu_s = B_0/2\beta \) and the draft is obtained for \( \varphi = 0 \):

\[ y_0 = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} \cos(2k - 1)0, \]

hence

\[ y_0 = \mu_s \sum_{k=0}^{n} (-1)^k a_{2k-1} \]

and

\[ D_0 = y_0, \]

with \( \mu_s = D_0/\delta \).

## 2 Extended Lewis Conformal Mapping

We can obtain better approximations of the cross sectional hull form by taking into account the first order moments of half of the cross section about the \( x \) and \( y \) - axes. These two additions to the Lewis formulation were proposed by Reed and Nowacki [4] and have been simplified by Athanassoulis and Loukakis [1] by taking into account the vertical position of the centroid of the cross section. This has been done by extending the Lewis Transformation from \( n=2 \) to \( n=3 \) in the general transformation formula.

The three-parameter Extended Lewis Transformation of a cross section is defined by:

\[ z = f(Z) = \mu_s a_{-1} Z + \mu_s a_1 Z^{-1} + \mu_s a_3 Z^{-3} + \mu_s a_5 Z^{-5}, \]

where \( a_{-1} = 1, \mu_s \) is the scale factor and the conformal mapping coefficients \( a_1, a_3, a_5 \) are called Lewis coefficients.

Then, for \( z = x + iy \) and \( Z = ie^{\alpha}e^{-i\varphi} \), that is \( Z = ie^\alpha [\cos(-\varphi) + isin(-\varphi)] \), we have:

\[ x = \mu_s (e^\alpha \sin\varphi + a_1 e^{-\alpha} \sin\varphi - a_3 e^{-3\alpha} \sin3\varphi + a_5 e^{-5\alpha} \sin5\varphi) \]

and

\[ y = \mu_s (e^\alpha \cos\varphi - a_1 e^{-\alpha} \cos\varphi + a_3 e^{-3\alpha} \cos3\varphi - a_5 e^{-5\alpha} \cos5\varphi) \]

For \( \alpha = 0 \) we obtain the contour of the so-called Extended Lewis form expressed as:

\[ x_0 = \mu_s (\sin\varphi + a_1 \sin\varphi - a_3 \sin3\varphi + a_5 \sin5\varphi) \]

and

\[ y_0 = \mu_s (\cos\varphi - a_1 \cos\varphi + a_3 \cos3\varphi - a_5 \cos5\varphi) \]

where the scale factor \( \mu_s \) is:

\[ \mu_s = B_s/(1 + a_1 + a_3 + a_5) \]

or

\[ \mu_s = D_s/(1 - a_1 + a_3 - a_5), \]

in which \( B_s \) is the sectional breadth on the waterline and \( D_s \) is the sectional draught. The half breadth to draft ratio \( H_0 \) is given by:

\[ H_0 = \frac{B_s}{2D_s} = (1 + a_1 + a_3 + a_5)/(1 - a_1 + a_3 - a_5). \]

An integration of the Extended Lewis form delivers the sectional area coefficient \( \sigma_s \):

\[ \sigma_s = \frac{A_s}{B_sD_s} = \frac{\pi}{4} \left[ 1 - a_1^2 - a_3^2 - a_5^2 \right]' \]

in which \( A_s \) is the area of the cross section,

\[ A_s = \pi/2 : \mu_s^2 (1 - a_1^2 - 3a_3^2 - 5a_5^2) \]

and

\[ B_sD_s = 2[(1 + a_3)^2 - (a_1 + a_5)^2]. \]

Now the coefficients \( a_1, a_3, a_5 \) and the scale factor \( \mu_s \) will be determined in such a manner that the sectional breadth, the draft and the area of the approximate cross section and of the actual cross section are identical. We have studied [2] a typical and realistic form. More precisely, we have considered a dry bulk carrier of 55,000 tone deadweight capacity. That application was made in Java language and created both a text file and a graphical chart.

The figure was obtained with a software package specially developed for this purpose [2].

The graphical representation of the points shows the contour of the ship’s cross section of the dry bulk carrier.
3 Close-Fit Conformal Mapping

A more accurate transformation of the cross sectional hull form can be obtained by using a greater number of parameters n. A very simple and straightforward iterative least squares method to determine the Close-Fit conformal mapping coefficients will be described here shortly.

The scale factor \( \mu_s \) and the conformal mapping coefficients \( a_{2k-1} \), with a maximum value of n varying from n which are 2 until 10, have been determined successfully from the offsets of various sections in such that the sum of the squares of the deviations of the actual cross section from the approximate described cross section is minimised.

The general transformation formula is again given by

\[
z = \mu_s \sum_{k=0}^{n} a_{2k-1} Z^{-2k+1}, \text{ with } a_{-1} = 1
\]

Then the contour of the approximated cross section is given by

\[
x_0 = -\mu_s \sum_{k=0}^{n} (-1)^{k} a_{2k-1} \sin(2k-1) \varphi,
\]

\[
y_0 = \mu_s \sum_{k=0}^{n} (-1)^{k} a_{2k-1} \cos(2k-1) \varphi.
\]

with scale factor:

\[
\mu_s = \frac{B_2/2}{\sum_{k=0}^{n} a_{2k-1}} = \frac{D_s}{\sum_{k=0}^{n} (-1)^{k} a_{2k-1}}
\]

The procedure starts with initial values for \( [\mu_s a_{2k-1}] \). The initial values of \( \mu_s, a_1 \) and \( a_5 \) are obtained with the Lewis method as has been described before, while the initial values of \( a_5 \) through \( a_{2k-1} \) are set to zero. With these \( [\mu_s a_{2k-1}] \) values, \( \varphi_i \)-values are determined for each offset in such a manner that the actual offset \( (x_i, y_i) \) lies on the normal of the approximated contour of the cross section in \( (x_{0i}, y_{0i}) \).

Now \( \varphi_i \) has to be determined. Therefore a function will be defined by the distance from the offset \( (x_i, y_i) \) to the normal of the contour to the actual cross section through \( (x_{0i}, y_{0i}) \).

These i offsets \( (i = 0, 1, ..., I) \) have to be selected at approximately equal mutual circumferential lengths, eventually with somewhat more dense offsets near sharp corners. Then \( \varphi_i \) is defined by

\[
\cos \varphi_i = \frac{x_{i+1} - x_i - 1}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}},
\]

\[
\sin \varphi_i = \frac{-y_{i+1} + y_i}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}.
\]

With this \( \varphi_i \)-value, the numerical value of the square of the deviation of \( (x_i, y_i) \) from \( (x_{0i}, y_{0i}) \) is calculated: \( e_i = (x_i - x_{0i})^2 + (y_i - y_{0i})^2 \).

After doing this for all \( I+1 \) offsets, the numerical value of the sum of the squares of deviations is known: \( E = \sum_{i=0}^{I} e_i \)

Then, new values of \( [\mu_s a_{2k-1}] \) have to be determined such that E is minimised. This means that the derivative of this expression to each coefficient \( [\mu_s a_{2k-1}] \) is zero, so:

\[
\frac{\partial E}{\partial (\mu_s a_{2j-1})}, \text{ for } j = 0, ..., n
\]

This provides \( n+1 \) equations:

\[
\sum_{k=0}^{n} (-1)^n [\mu_s a_{2k-1}] \sum_{i=0}^{l} \cos((2j - 2k)\varphi_i) = \sum_{i=0}^{l} [-x_i \sin((2j - 1)\varphi_i) + y_i \cos((2j - 1)\varphi_i)]
\]

for \( j = 0, ..., n \).

To obtain the exact actual breath and draught, the last two equations are replaced by the equations for the breath at the waterline and the draught:

\[
\sum_{k=0}^{n} (-1)^n [\mu_s a_{2k-1}] \sum_{i=0}^{l} \cos((2j - 2k)\varphi_i) = \sum_{i=0}^{l} [-x_i \sin((2j - 1)\varphi_i) + y_i \cos((2j - 1)\varphi_i)],
\]

for \( j = 0, ..., n-2 \),

\[
\sum_{k=0}^{n} (-1)^n [\mu_s a_{2k-1}] = B_s/2,
\]

\[
\sum_{k=0}^{n} (-1)^n [\mu_s a_{2k-1}] = D_s.
\]
These n+1 equations can be solved numerically, so that new values for $[\mu_s, a_{2k-1}]$ will be obtained. These new values are used instead of the initial values to obtain new $\phi_i$-values of the I+1 offsets again. This procedure will be repeated several times and stops when the difference between the numerical E-values of two subsequent calculations becomes less than a certain threshold value $\Delta E$, depending on the dimensions of the cross section; for instance:

$$\Delta E = (I + 1)(0, 0.0005 \sqrt{b_{\text{max}}^2 + d_{\text{max}}^2})$$

in which, $b_{\text{max}}$ is maximum half breadth of the cross section and $d_{\text{max}}$ is maximum draught of the cross section.

Because $a_{-1} = 1$, the scale factor $\mu_s$ is equal to the final solution of the first coefficient ($k = 0$). The n other coefficients $a_{2k-1}$ can be found by dividing the final solutions of $[\mu_s, a_{2k-1}]$ by this $\mu_s$-value.

4 Conclusions

The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrary shape of a cross section in a complex plane can be derived from the more convenient circular section in another complex plane. In the future we hope to obtain the graphical representation which will be much better because we will consider more than five coefficients.

References:


