A Note on Chaos Conversion in the Frequency Domain

JIRI PETRZELA, ZDENEK HRUBOS
Department of Radio Electronics
Brno University of Technology
Purkynova 118, Brno 612 00
CZECH REPUBLIC

Abstract: This paper deals with the simplest driven dynamical system capable to generate complex irregular behavior realized as voltage-mode circuit. The structural similarity of the final network if compared to the state-feedback multifunctional filters is obvious and consequences are briefly discussed. It is shown that the output signal energy expressed by means of frequency spectrum strongly depends on the input signal amplitude and frequency even if the two-port parameters are the same. The possible solutions of the associated mathematical model are calculated using the concept of the so-called Lyapunov exponents.

Key-Words: Driven systems, frequency spectrum, chaos, state-feedback filter

1 Introduction
Lately, the research of many scientists and engineers is focused onto relations between the real physical systems and its mathematical models from the viewpoint of study of the associated nonlinear dynamical behavior. This is important also in the case of various electronic circuits since chaos has been observed in the oscillators with frequency dependent feedback, oscillators with negative resistance elements, PLL, DC/DC converters, the circuits with analog and digital parts which work together and even in some power systems. The problems covered by chaos theory are universal and can be observed in the non-autonomous nonlinear dynamical systems with at least two degrees of freedom. Many building blocks in the common radio-communication channel can be considered as two-port which is always generally nonlinear and includes not only two but several energy storage elements. The location and number of these elements can boost the order of the describing differential equation leading to the increased chance for complex long-term unpredictable motion. Thus the active building blocks and its parasitic capacitances also play a significant role. There exist many examples where chaos is unwanted phenomenon and can be observed in the networks which are basically linear. It is clear that some precise enough but easy to establish chaos quantifier [1] should be used to distinguish chaotic signal from noise. To determine spectrum of the Lyapunov exponents (LE) is indeed a good choice how to do this. For such purpose Mathcad and build-in fourth-order Runge-Kutta method has been employed. During the process of numerical integration Gram-Smith orthogonalization has been performed after each step. For further details or questions please do not hesitate to write the authors. Other possibility to quantify chaos is calculation of the metric dimension of attractor.

2 Describing Equation
It seems that the simplest forced dynamical systems with possible chaotic behavior [2] can be expressed as
\[ \dot{x} = x - x^3 + \alpha \cdot \sin(\beta t), \]
where \( \alpha \) and \( \beta \) are real numbers, serve as the natural bifurcation parameters. Fig. 1 gives XY plane projections of the typical attractors for (1) if amplitude is fixed \( \alpha=1 \) and normalized angular frequency varies over the narrow range \( \omega = 0.68, 0.7, 0.72 \) and \( 0.74 \). Analogically Fig. 2 provides the attractors if the same driving force is fed also to the node marked as y. Such cases are interesting since there are universal filters with multiple inputs and single output. The two-dimensional topographically scaled contour-surface plot of the largest \( LE_{\text{max}} \in (-0.01, 0.397) \) is shown in Fig. 3 where second picture represents magnified area from the previous one.

Fig. 1: The typical state space trajectories for given dynamical system with single node input, see text.
3 Circuitry Implementation
Assume the most straightforward method for the circuit implementation of the nonlinear dynamical system [3], i.e. method based on the integrator block schematic. Only three functional blocks are necessary - inverting integrator, differential or summing amplifier and at least one two-port with desired nonlinear transfer function. In practice, cheap and off-the-shelf operational amplifier TL084 has been used. The individual state variables are voltages at the outputs of integrators as it is shown in Fig. 4. Note that there is a significant similarity of this network if compared to the well-known state variable multifunctional filter [4], [5]. The concrete realization of the nonlinear part of the vector field does not represent problem. Moreover the cubic type of the polynomial can be roughly considered as the approximation of transfer function of the active devices, especially if operational transconductance amplifiers or the current conveyors are utilized. From the circuit synthesis point of view the biggest problem is how to implement the nonlinear part of the vector field by using minimum active devices. Due to the fact that there are much more commercially available devices working in the so-called voltage-mode (high-impedance input nodes) this approach will be adopted. The whole analog oscillator will be fed by the symmetrical ±15V voltage source. The ranges of the state variables in Fig. 1 and Fig. 2 are narrow enough for circuit realization. The nonideal properties of the active blocks should be also taken into account. For example, finite input and output impedances of DTs brings the error terms into the original differential equations.

4 Experimental Verification
The circuitry provided in Fig. 4 together with nonlinear two-port formed by a connection of two four-quadrant analog multipliers AD633 has been verified by a number of laboratory measurements. The corresponding results obtained by using the spectrum analyzer Agilent 4395A are given in Fig. 5. It is obvious that even in such small input signal frequency range from 1 kHz up to 3.5 kHz the chaotic windows alternates with periodic motions several times and this is definitely not complete scenario.

Fig. 2: The typical state space trajectories for given dynamical system with two node input, see text.

Fig. 3: Largest LE plotted as function of input signal’s main parameters, see text.

Fig. 4: Circuitry implementation of the given mathematical model.
Fig. 5: Frequency spectrum for different driving force frequency and single input node.

Fig. 5: Frequency spectrum for different driving force frequency and single input node, continued.
5 Conclusion

It has been demonstrated that even two-port circuits with very simple topology can exhibit complex behavior including multiple-periodicity and chaos. Moreover, this motion is reproducible and in accordance with mathematical investigation. It is proved in [6] that there exist even simpler mathematical models with associated chaotic behavior. Based on the results presented in this paper the authors believe that analog circuits are upper-frequency-limited not only by finite gain-bandwidth product of the active building blocks but also by the higher-order chaotic-like dynamical events. Several interesting informations about chaos and circuit synthesis can be found in the publication [7].

Acknowledgement:
Research presented in this brief paper has been supported by Grant Agency of the Czech Republic under project number 102/09/P217.

References: