Determination of electrical conductivity of metal plates using planar spiral coils

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Abstract: - Planar spiral coils are used in the present paper to estimate electrical conductivity of a conducting plate by eddy current method. Formulas for the change in impedance are derived from the corresponding formulas for air-cored cylindrical coils in the limit where the height of the coil approaches zero. The difference between experimental and theoretical values of the change in impedance is minimized in order to estimate electrical conductivity of a conducting plate. Reasonable agreement between experimental and theoretical data is found.

Key-Words: - electrical conductivity, change in impedance, planar spiral coils

1 Introduction

Eddy current testing methods are widely used in practice for quality control of electrically conducting materials. In particular, electrical conductivity of metals is often measured by eddy current methods [1]. A suitably chosen norm of the difference between experimental and theoretical data is usually minimized in order to estimate unknown electrical conductivity of a conducting material. Mathematical models used in the analysis are based on closed-form solutions [2] – [6] developed for air-cored cylindrical coils located above a conducting medium to be tested.

In the present paper planar spiral coils are used to determine electrical conductivity of a metal plate with constant thickness. Planar spiral coils have some advantages in comparison with conventional air-cored eddy current coils [7]. For example, printed-circuit-board technology can be used to manufacture planar spiral coils. In addition, planar spiral coils can be potentially used for inspection of complex geometries. Relatively high frequencies for conductivity estimation are needed in some applications. In the present paper wide range of frequencies (from 10 kHz to 480 kHz) is used to determine electrical conductivity of conducting plates.

The paper is organized as follows. First, we develop formulas for the change in impedance of a planar spiral coil placed above a conducting plate.

Second, experimental setup is briefly described and the results of experimental measurements are presented. Third, the inverse problem is solved by minimizing the norm of the difference between the experimental and theoretical data and the conductivity of a conducting plate is estimated. All measurements are performed for plates with known conductivity. Reasonable agreement is found between actual and estimated values of electrical conductivity.

2 Theory

The impedance of an air-cored cylindrical coil in air is given by the formula [1]

$$Z_{air} = \frac{2i\pi\omega\mu_0 n^2}{l^2(r_o - r_i)^2}\int_0^\infty \frac{I^2(kr_o, kr_i)}{k^5}\left(l + \frac{e^{-kl}-1}{k}\right)dk, \tag{1}$$

where μ_0 is the magnetic constant, l is the height of the coil, r_o and r_i are the outer and inner radii of the coil, n is the number of turns, ω is the angular frequency and

$$I(a,b) = \int_a^b xJ_1(x)dx \ . \tag{2}$$

Here $J_1(x)$ is the Bessel function of the first kind of order 1.

A single-layer planar spiral coil (see Fig. 1) is characterized by the outer and inner radii (r_o and r_i) and the number of turns (n). Since the thickness of the windings (l) in this case is negligible we can calculate the impedance of a planar spiral coil in air by considering the limit in (1) as $l \to 0$ (it is assumed here that the windings are circular). It can easily be shown that

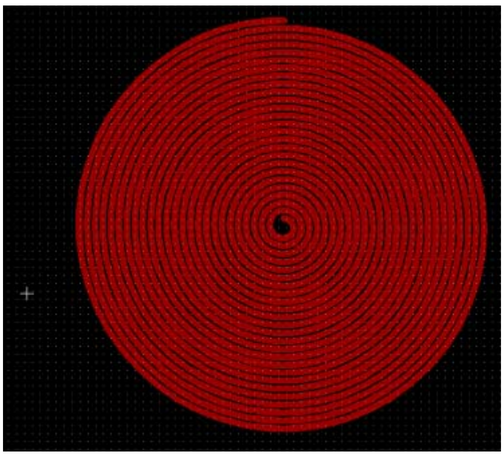


Fig.1. A planar spiral coil.

$$\lim_{l\to 0}\frac{1}{l^2}\left(l + \frac{e^{-kl}-1}{k}\right) = \frac{k}{2}. \tag{3}$$

Using (1) and (3) we obtain the impedance of a planar spiral coil in air in the form

$$Z_{air} = \frac{i\pi\omega\mu_0 n^2}{(r_o - r_i)^2}\int_0^\infty \frac{I^2(kr_o, kr_i)}{k^4}dk. \tag{4}$$

Note that the integral in (2) can be expressed in terms of the Bessel and Struve functions of orders zero and one [8]:

$$\int_a^b xJ_1(x)dx = g(b) - g(a), \tag{5}$$

where

$$g(x) = \frac{\pi}{2}\left[J_1(x)H_0(x) - J_0(x)H_1(x)\right]. \tag{6}$$

It follows from (4)-(6) that the impedance of a planar spiral coil in air can be written in the form

$$Z_{air} = \frac{i\pi\omega\mu_0 n^2}{(r_o - r_i)^2}\int_0^\infty \frac{[g(kr_o - g(kr_i]^2}{k^4}dk. \tag{7}$$

The change in impedance of a planar spiral coil situated at a height s above a conducting plate of thickness T with conductivity σ is obtained from the corresponding formula for an air-cored cylindrical coil [1] in the limit as $l \to 0$ and has the form

$$\Delta Z = \frac{i\pi\omega\mu_0 n^2}{(r_o - r_i)^2}\int_0^\infty \left[g(kr_o) - g(kr_i)\right]^2 \times \frac{e^{-2ks}(k^2-\lambda^2)\left(1-e^{-2\lambda T}\right)}{k^4\left[(\lambda+k)^2 - (\lambda-k)^2 e^{-2\lambda T}\right]}dk, \tag{8}$$

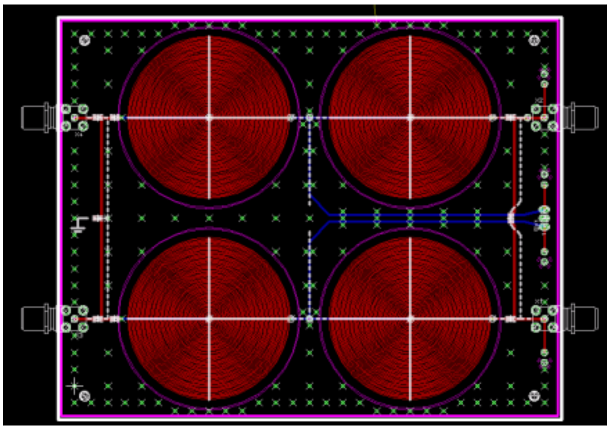
where $\lambda = \sqrt{k^2 + i\omega\sigma\mu_0}$.

Formulas developed in this section are based on the assumption that a conducting plate is of infinite dimensions in the two horizontal directions. No analytical solutions are available for the case of a conducting medium of a finite size. Numerical methods (such as finite difference or finite element methods) should be used in the case where the size of the conducting object is comparable to the size of the coil [10]. Such an approach is usually time-consuming in the case where the inverse problem should be solved. The solution of the inverse problem (for example, determination of electrical conductivity) requires a fast and robust method for the solution of the direct problem. Thus, it is desirable to know how large a sample should be so that one can use the approach based on an infinite medium instead of the model based on the finite size of the conducting sample. The answer to this question depends on frequency since the depth of penetration of the field into the sample at high frequencies is limited due to the skin effect. The estimates provided in [11] at low frequencies (up to 5 kHz) suggest that the area of placing the air-cored cylindrical coil must be 16 – 20 mm greater than the diameter of the coil.

3 Experimental Setup

Experiments were conducted in the laboratory of the company “Metrosert” in Tallinn, Estonia. The company is the Central Office of Metrology (equivalent to a National Metrology Institute) in Estonia (details can be found on the company’s website www.metrosert.ee).

The coils (see Figs. 1 – 2) were made at the Department of Electronics of the Tallinn University of Technology (www.ttu.ee).



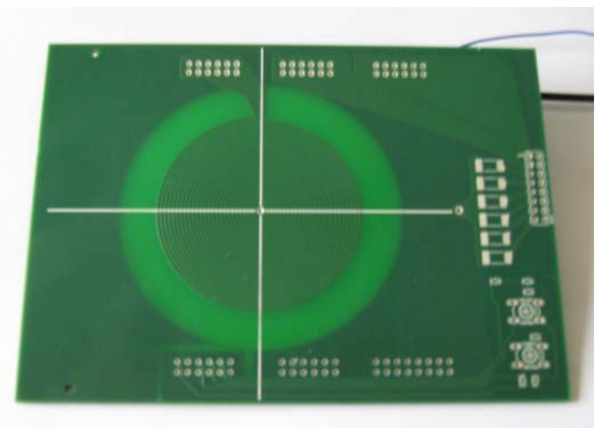


Fig.2. Planar spiral coils manufactured at TTU (Tallinn University of Technology).

Measurement setup is based on a single planar coil, fabricated on the top side of the double-sided standard PCB (FR4 material, 35 micrometers copper layer). Two spiral coils have been used, both with the diameter of 50 mm. One coil has the $n = 25$ turns and another $n = 80$ turns.

The measurements described in this paper were performed in a shielded laboratory with controlled relative humidity and air temperature within ranges of (40 ± 5) % and (23.0 ± 0.5) °C. Three nonferrous metal plates were used in the study: titanium, Nordic gold and brass with the conductivity values of 2.171 $MS\cdot m^{-1}$, 9.57 $MS\cdot m^{-1}$ and 14.27 $MS\cdot m^{-1}$ respectively. The conductivities of the plates were determined by the Van der Pauw (VdP) technique [9] at the DC current with the relative uncertainty of the measurement less than 0.3 % of the measured value. The temperature coefficients of the samples were determined in an air-thermostat and applied in all subsequent measurements.

In the eddy-current method, a planar coil was placed perpendicularly to the measurement surface with a constant load. The load was applied in order to reduce the effect of the lift-off (the distance from the coil to the metal plate) on the conductivity measurement results. The coil impedance in air and above the conducting plate was measured by a LCR-meter over the frequency range from 1 kHz to 480 kHz. The measurements were automated by specially developed software, taking several series in order to estimate repeatability of the measurements.

Two high-accuracy LCR-measuring instruments - an Agilent 4294A and a QuadTech 7600 were applied for the impedance measurement. The performance of the meters was checked by measuring the resistance, capacitance and inductance standards at 1 kHz and by comparing the operation of the devices with the resistance standards over frequencies from 100 Hz to 1 MHz. The relative measurement uncertainty estimates due to the difference U_{diff} in the measurement results obtained by two instruments are summarized in Table 1 where R and X are the real and imaginary parts of the impedance of the coil, respectively. The U_{diff} value was used as an experimental estimate of the uncertainty of the impedance measurement. The average measurement result obtained by two instruments was compared to the mathematical model in order to find the conductivity of the plate.

f, kHz	*X*	*R*
	U_{diff}, %	U_{diff}, %
1	0.1	23.4
10	0.3	4.7
60	0.2	0.2
120	1.0	1.3
240	3.9	1.9
360	9.1	3.4
480	18.2	4.7

Table 1. Uncertainty estimates due to LCR-meters taking into account sensitivity of the coil.

Table 2 shows experimentally measured impedance of Coil 1 ($r_o = 0$ mm, $r_i = 24$ mm, $n = 80$) in air at differerent frequencies.

f (kHz)	*X* (Ohm)	*R* (Ohm)
1	0.7001	24.0438
10	6.4545	24.4721
60	36.4647	25.0613
120	72.4308	25.3414
240	144.3014	25.6845
360	216.2062	25.9121
480	288.2037	26.0972

Table 2. Experimentally measured impedance of Coil 1 in air.

Experimental values for the impedance of Coil 1 above the three conducting plates are shown in Tables 3 – 5.

f (kHz)	*X* (Ohm)	*R* (Ohm)
10	2.6875	25.5440
60	8.0545	29.0945
120	12.5569	31.6856
240	19.9574	35.3967
360	26.4593	38.2575
480	32.4977	40.6627

Table 3. Experimentally measured impedance of Coil 1 above the metal plate with σ = 2.17 MS/m.

f (Khz)	*X* (Ohm)	*R* (Ohm)
10	1.4932	24.9654
60	4.8025	26.6982
120	7.8789	27.9416
240	13.2706	29.7153
360	18.2342	31.0745
480	22.9719	32.2160

Table 4. Experimentally measured impedance of Coil 1 above the metal plate with σ = 9.57 MS/m.

f (kHz)	*X* (Ohm)	*R* (Ohm)
10	1.2801	24.8426
60	4.2492	26.2677
120	7.0799	27.2971
240	12.1188	28.7832
360	16.8139	29.9480
480	21.3343	30.9543

Table 5. Experimentally measured impedance of Coil 1 above the metal plate with σ = 14.27 MS/m.

4 Determination of Electrical Conductivity

A planar spiral coil in air is modeled as an ideal inductor. It can be seen from (7) that the real part of Z_{air} is equal to zero while experimentally measured real part of Z_{air} is clearly different from zero (see Table 2). Several corrections are described in the literature [1], [7] in order to take into account non-ideal behavior of the coil. In particular, the concept of an effective outer radius of air-cored coil is found to be quite useful in reducing the total error in conductivity determination. In the present paper we used this approach for planar spiral coils. At each frequency the effective outer radius of the coil, r_o^{eff} is found as the value at which the calculated value from (7) coincides with experimental data from

Table 2. The average value of the outer radius of the coil is then calculated: $r_o^{eff} = 21.5$ mm. This value is used in all calculations below.

The following formula is used in the paper to estimate conductivity of a conducting plate:

$$\Delta = \sqrt{\frac{1}{N}\sum_{j=1}^{N} |\ \Delta Z_{\exp} - \Delta Z_{theor}\ |^2}\ , \qquad (9)$$

where $\Delta Z_{\exp}$ and ΔZ_{theor} are the experimental and theoretical values of the change in impedance of the coil in the presence of the conducting plate, and N is the number of frequency points. Six frequencies (f = 10, 60, 120, 240, 360 and 480 kHz) are used in all calculations. The conductivity of the plate is varied until (9) is minimized. All calculations are done using software package "Mathematica". From the user's point of view there are at least two advantages of using "Mathematica" in comparison with other packages. First, numerical integration is effectively performed in "Mathematica". In addition, built-in functions can be used to calculate the Bessel and Struve functions in (7) and (8).

Two examples of calculations are shown in Figs. 3 and 4. The points represent the values of Δ in the specified interval of σ. The cubic polynomial is used to fit the data (solid curve). The estimated value of σ is obtained as the minimum of the cubic polynomial in the specified interval of conductivity values.

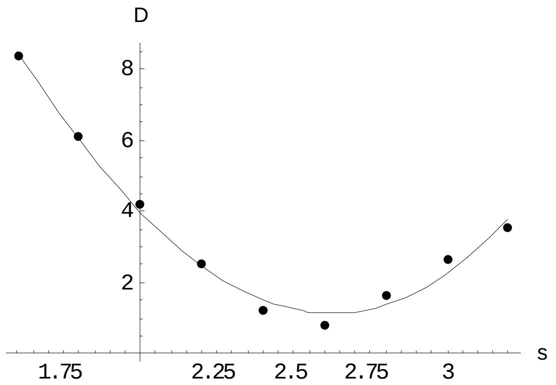


Fig.3. The graph of Δ versus σ for titanium plate (conductivity 2.171 MS·m^{-1}). The estimated lift-off is $s = 0.2094$ mm.

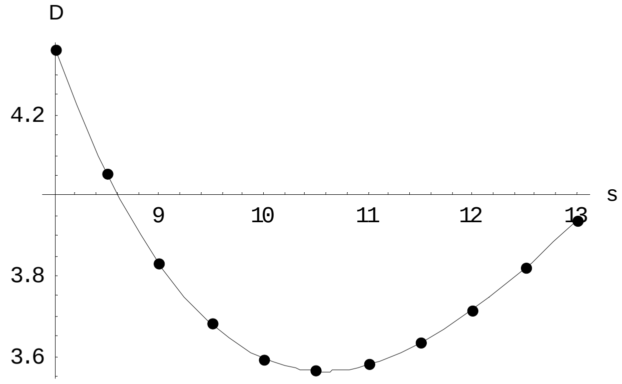


Fig.4. The graph of Δ versus σ for Nordic gold plate (conductivity 9.57 MS·m^{-1}). The estimated lift-off is $s = 0.1975$ mm.

The estimated conductivity values are 2.38 MS·m^{-1} and 10.567 MS·m^{-1}, respectively (the relative errors in conductivity determination are 9.6% and 10.4%, respectively).

5 Conclusion

Several conclusions can be drawn from the analysis. First, the graphs in Figs. 3 and 4 show that there is a well-defined minimum of the function Δ with respect to σ. Second, the inductance of the coil in air varies with frequency. As a result, some correction procedures are necessary to reduce errors. In the present paper we estimated the effective radius of the coil as the average of the effective radii for different frequencies. Perhaps, better results can be achieved if one uses effective radius of the coil at each frequency. In addition, relatively high errors in conductivity determination can be (at least partially) explained by experimental errors at high frequencies (see Table 1). The results presented in Table 1 can also be used to specify frequency range where the measurements should be made (in order to reduce the effect of experimental errors). Finally, calculations show that the conductivity estimates are quite sensitive to the variation of the lift-off. Future work is needed to improve the accuracy of conductivity estimation for wide range of frequencies.

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