Optimizing the Controlled Production Flows

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Abstract: - In this paper, we consider the discrete event systems (DES) modeled via a state – space representation. The control objective is the avoidance of a given set of states. We achieve this goal by verifying that certain predicates, specified in terms of states, are always false. We model the DES involved in physical systems with a class of controlled state machines, respectively the formalism of the controlled Petri nets. A model of a railway system illustrates the theoretical approach.

Key-Words: - Controlled Petri nets, discrete event systems, production flows, railway systems.

1 Introduction
In this paper, we consider discrete event systems (DES) modeled via a state space representation. The model objective is the avoidance of a given set of states, or equivalently the fact that certain predicates, specified in terms of states are always false. This problem was addressed in [1], [2] for DES modeled as cyclic marked graphs, and in [3] for DES, which can be modeled as controlled state machines, another class of controlled Petri nets. We address the state space controlled Petri nets and a technique to reduce the complexity of these nets, by taking into account the fact that the complexity of the considered nets depends mainly on the representation of the control design, respectively on the forbidden sets of places. Our goal is to describe in an efficient way the forbidden sets of the modeled system, e.g. to describe in an efficient way the DES modeled with controlled Petri nets. An example from the railway systems illustrates this approach: the Petri net model of railway station incompatible tracts. We mention that every railway station has such a table, given to the general traffic rules allocated to a certain station.

Petri nets graphically describe the system behavior in terms of the successive states realized after the occurrence of events. Events correspond to a change from one state to the next state. The number of tokens in each place represents the state of the Petri net. When a transition fires, tokens are moved from one place to another place. The firing of a transition corresponds to an event. In a controlled Petri net, the transitions fire only when all their input control places are marked. The control objective is to prevent a set of forbidden states from being reached, while at the same time enabling a maximal set of achievable state sequences. The Petri nets property to allow a graphical and analytical representation, closely related to the technological process being modeled, makes them a very useful tool for enabling the application of the theory.

2 Controlled Petri Nets

2.1 Basic properties
A controlled Petri net is defined [1] as a five-tuple
\( G=(P,T,F,C,B) \), where \( P \) and \( T \) are finite sets of state
places and transitions, respectively; \( F \subset (PxT) \cup (TxF) \) is a set of directed arcs connecting
state places and transitions; \( C \) is a finite set of control
places; \( B \subset (CxT) \) is a set of directed arcs associating
control places with transitions. A state place \( p \in P \) is said
to be an input to (respectively output from) transition \( t \in T \)
if \( (p, t) \in F \) (respectively \( (t, p) \in F \)). Denote by \( \{p\}_t \),
respectively \( \{p\}_t \) the sets of these input, respectively output
places. The sets \( \{p\}_t, \{p\}_t, \{c\}_t \), and \( \{c\}_t \) are defined similarly.
Each controllable transition is influenced by one control
place and this control place influences only this one
single transition. A control \( u:C \rightarrow \{0,1\} \) indicates whether
control place \( c \) is marked or not (\( u(c)=1 \) or 0). Two
special controls are \( u_1(c)=1 \) \( \forall c \), enabling all transitions,
and \( u_0(c)=0 \) \( \forall c \), disabling all controllable transitions.
A transition, in a controlled Petri net, is enabled if it is
both state and control enabled, that means that under
marking \( m \) must have: \( m(p) \geq 1, \forall p \in \{p\}_t \), if we assume
that there is at most one arc in \( B \) and \( F \) between any place
and any transition. When an enabled transition \( t \) fires, a
token can be removed from each state place, which has
an arc to \( t \). Then, the token will be added to any place,
which has an arc from \( t \). We assume that several
transitions can fire simultaneously provided they are
always enabled, and the new state, after the occurrence of
these events is:
One denotes by a cycle [4] a sequence of places \((p_1, p_2, \ldots, p_n)\) which does not contain the same place twice and where there exists a transition connecting the first place to the last place. Cycles are uncontrollable if, for each pair of consecutive places in \(T\), there is at least one uncontrollable transition connecting them.

### 2.2 Sets of forbidden states

As in most papers on forbidden state problem [1], [5], we represent the set of forbidden states via unions and intersections of simpler sets. In the language of predicates this means that complicated predicates are decomposed into simple predicates via conjunction and disjunction. This leads to a simple, modular control design. The simplest possible forbidden set \(M_{p,k}\) corresponds to a particular place containing \(k\) or more tokens:

\[
M_{p,k} = \{m \in M | m(p) \geq k\}
\]  

(2)

If we consider simple forbidden sets (one-dimensional specification in [5]) where the total token load in a set of places is limited: for \(r : P' \subseteq P \rightarrow R_+\), we have:

\[
M_{p,k,r} = \{m \in M | \sum_{p \in P'} r(p).m(p) \geq k\}
\]  

(3)

Equation (2) represents a buffer where at most \(k-1\) pieces can be stored. Equation (3) represents a case where a finite buffer contains different types of work-pieces, where different work-pieces require a different amount of space, so that the buffer overflows if the weighted sum \(\sum_{p \in P'} r(p).m(p)\) became \(k\) or more. In a railway line, for example, if \(k=1\), \(r(p)=1\) and \(P'\) represents neighboring sections, then \(M_{p,k,r}\) represents the fact that two trains being simultaneously in neighboring section is a dangerous situation.

In intersections of simple forbidden sets, a state \(m \in M\) is forbidden when it belongs to each of the simple forbidden sets \(M_{p,ki,ri}\) at the same time. This corresponds to a conjunction in the predicate formulation. Besides conjunctions, one can also take disjunction of predicates. That means that a state is forbidden if it belongs to at least one of a group of simple forbidden sets or of intersections of simple forbidden sets:

\[
M = \bigcap_{j=1}^{w} \bigcup_{i=1}^{v} M_{p_{ij},k_j,a_j}
\]  

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Where \(v,w,k_j\) are positive integers. One may say [4] that a marking \(m\) can be reached uncontrollably from an initial marking \(m_0\), if, even if \(u_{\text{zero}}\) is used and thus all controllable transitions are disabled, the initial marking \(m_0\) can be transformed via enabled transitions into the final marking \(m\). An influence path \(inf_p\) of a transition \(t\) is a sequence of places \(inf_p=(p_1,p_2,\ldots,p_m)\) such that [5]:

- \(r(t)=r(p_1)\)
- For \(j=2,3,\ldots,n\), there exists an uncontrollable transition \(t_j\) leading from \(p_{j-1}\) to \(p_j\).
- All transition following \(p_n\) are either controllable or lead to a place \(p_j\) already encountered earlier in the influence path \(inf_p\).

Once a token has entered in an influencing path \(inf_p\), it can uncontrollably reach \(p\). The complexity of the control synthesis algorithms depends strongly on the number of places in the influencing zones for the different simple forbidden sets. It is thus useful to try to reduce the size of the network, before starting the control design. We can replace several parallel transitions between a pair of places, by one single transition. This transition should be uncontrollable since one of the parallel transitions is uncontrollable. If in a controlled Petri net, in certain condition, exist influence paths, where a sequence of uncontrollable transitions moves the tokens from any upstream place in \(inf_p\) to \(p\). This implies that places of an uncontrollable path are indistinguishable from a control viewpoint. Thus, from a control point of view, it is allowed to replace all the places in \(inf_p\), and the transitions connecting these places, by one single place. The reduced net is characterized by a new finite marking set \(M'\). We call this equivalence between functional properties the functional abstraction equivalence (f.a.e.). An example will facilitate the understanding of this concept.

### 3 An Example of Controlled Production Flows Optimization

Two trains (two railway engines) are placed in the railway station given in Figure 1, composed of five lines (sections) \(l_1, l_2, \ldots, l_5\) and eight switches \(sw_1, sw_2, \ldots, sw_8\). In the given railway (as in the most railway stations) it is forbidden to shunt wagons by pushing them, which means that a train enters a section on an end (odd switches, for example) and leaves it on the other end (even switches).

The discrete events are generated by line sensors indicating when a line belongs to a train route generate the discrete events. Controlled Petri net models the system made up of union of two independent controlled Petri nets: one net represents the train number 1 (with wagons, for example) path, and the other one represents the train number 2 (with carriages) path.
Our Petri net can, therefore, be used to represent concurrent systems with no shared events. The different places $P_w=(P_{wi}, i=1,…,5)$ for the train number 1 and $P_{ca}=(P_{cai}, i=1,…,5)$ for the train number 2 identify possible locations of those two trains. Analogous we have different transitions $T_w=(t_{wi}, i=1,…,8)$, and $T_{ca}=(t_{cai}, i=1,…,6)$ and different control places $C_w=(c_{wi}, i=1,…,6)$, and $C_{ca}=(c_{cai}, i=1,…,6)$ allocated to the above given transitions. The controlled Petri nets are given in Figure 2, and in Figure 3.

Fig. 1. A railway station with five lines and eight switches

Fig. 2. Controlled Petri nets for movements of wagons

Fig. 3. Controlled Petri nets for movements of carriages
The reduced net without uncontrollable cycles is given in Figure 4.

\[ M = \bigcup_{j=1}^{5} \{ m \in M \mid m(p_{u_j}) + m(p_{ca_j}) \geq 2 \} \quad (5) \]

The Petri net model of the table of the incompatible tracts of the railway station given in Figure 1 contains the states representing the forbidden tracts of two different trains. For example, suppose we prevent two trains (wagon and carriage) to go simultaneously to line 1, the set of forbidden markings \( M_{p1} \) is:

\[ M_{p1} = \{ m \in M \mid m(p_{w1}) \geq 1 \} \bigcap \{ m \in M \mid m(p_{ca1}) \geq 1 \} \quad (6) \]

The influencing path \( \text{inf.}_{p1} \) the sets of neighboring sections of section 1; \( P_{1} \) and the control places \( C_{p1} \) are given in Table 1.

<table>
<thead>
<tr>
<th>i</th>
<th>( P_i )</th>
<th>Inf. ( p_i )</th>
<th>( C_{p1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((P_{w1}, P_{ca1}))</td>
<td>((P_{w3}, P_{w5})) ((P_{ca2}, P_{ca4}))</td>
<td>((C_{w4}, C_{w7})) ((C_{ca4}, C_{ca7}))</td>
</tr>
</tbody>
</table>

Following the example given in Table 1 we can build the table of all the incompatible tracts of the railway station in Figure 1.

4 Conclusion

This paper has resumed our model for the forbidden state problem of DES modeled by controlled Petri nets. Our goal is to present, by an adequate control, a given set of forbidden states from being reached. Forbidden markings characterize constrain sets expressing that some places of the Petri net cannot contain more than a certain amount of tokens. This concept allows us to obtain a maximally permissive control law. An example, dealing with a railway system, respectively the railway station incompatible tract of the Petri net model illustrates the theoretical concepts. Future study will extend the results to wider class of timed controlled Petri nets.

References: