Abstract: This paper deals with internal stability and related structural properties of a relatively broad class of finite dimensional strictly causal systems, which can be described in the state-space representation form. Dissipativity, instability, asymptotic stability as well as stability in the sense of Lyapunov are analyzed by a new approach based on an abstract state energy concept. The resulting energy metric function is induced by the output signal power and determines both, the structure of a proper system representation as well as the corresponding system state space topology. A special form of physically correct internal structure of an equivalent state space representation has been derived for both the continuous- and discrete-time signals as a natural consequence of strict causality, signal energy conservation, dissipativity and state minimality requirements. Several typical problems are solved in detail, and results of simulation examples are shown for illustration of fundamental ideas and basic attributes of the proposed method.

Key-Words: Signal power, signal energy, structure, state space velocity, state space metric, nonlinear system, internal, external, representation.

1 Introduction
Recall that from general point of view any collection of trajectories constitutes a dynamical system that, in principle, can be described either by external behavior, or by an internal structure. In the input-to-output framework the external behavior of a continuous-time causal system can be seen as a collection of all input-output trajectories satisfying the relation:

\[ F(t, y, y'..., y^{(m)}, u, \dot{u}, ..., u^{(m)}) = 0, \ m \leq n \] (1)

The input signals \( u(.) \) and output signals \( y(.) \), explicitly reflect a signal orientation property of causality relation and determine an external causality structure, which is important for external stability. Formally, we can write for an external stability property:

\[ \{ \text{System } S \text{ is stable} \} \Leftrightarrow \{|u(t)| < \delta \Rightarrow |y(t)| < \varepsilon\} \] (2)

In the present paper mainly the concepts of dissipativity and conservativity and their relationships with the internal stability problems will be examined. In such a case of the state-to-state framework, mainly an internal causality structure, reflecting a time orientation property of the causality relation and describing a collection of all state trajectories in, which no external signals are explicitly introduced seems to be appropriate:

\[ \dot{x}(t) = f[x(t)], \ x(t_0) \in X \subset \mathbb{R}^n \] (3)

Basic stability results concerning general internal stability problems are due to original work of A.M. Lyapunov. The main advantage of the Lyapunov’s approach is its generality. It applies for time-varying, linear and nonlinear systems as well. Notice that the resulting stability conditions based on the direct Lyapunov’s method are sufficient but not necessary in general. From practical point of view, main drawback of the direct method is lack of any systematic and universally applicable technique for generation of axiomatically defined Lyapunov functions \( V(x) \) having required properties [1 - 10].

Certainly, any abstract realizable system has to fulfill some causality and energy conservation requirements. Recall that existence of an abstract state space representation is necessary and sufficient for a system to be causal. On the other hand causality does not imply energy conservation. Consequently not every state space representation may be a priori considered as physically correct in the sense of a signal/state energy conservation law. This is one of crucial points of the proposed approach. Some theoretical and practical consequences of the new concepts will also be demonstrated [1, 2, 3, 11, 12, 13, 14].
2 Dissipativity and conservativity

Recall that according to Liouville’s theorem of vector analysis, dissipative systems have the important property that any volume of the state space strictly decreases under the action of the system flow. For continuous systems with the state velocity given by the nonlinear vector field \( f \), the property of dissipativity is defined by using the operation of divergence:

\[
\text{div} \ f(x) = \sum_{i=1}^{n} \frac{\partial f_i(x)}{\partial x_i} < 0
\]

Thus a linear system is dissipative if and only if its matrix \( A \) has negative trace. Nonlinear systems having a dissipative approximate linearization are locally dissipative, but need not to be globally dissipative. Vector fields for which it holds

\[
\text{div} \ f(x) = 0, \ f(x) = A(x)x \Leftrightarrow \text{Trace} \ A(x) = 0
\]

preserve volume along state trajectories, and are referred to as conservative.

3 Signal-energy-metric approach to dissipativity and conservativity

As an alternative to the well-known physical energy motivated method of Lyapunov functions a new conceptually different approach to dissipativity and stability problems has recently been proposed in [3], and called the signal-energy-metric approach. The key idea is that, in fact, it is not the (physical or abstract) energy by itself, but only a measure of distance from the system equilibrium to the actual state \( x(t) \), what is needed for stability analysis. Thus, instead of the concept of a system energy a form of the signal-energy-metric approach can be defined in a proper way, and the essence of the energy-metric approach can be then formally expressed by:

\[
E(x) = \frac{1}{2} \rho^2 \left[ x(t), x^* \right]
\]

where \( x^* \) denotes the equilibrium state.

Within the state space paradigm the concept of an abstract energy, as discussed in work of J.C.Willems seems to be one of the most natural means describing the internal system topology. A measure of distance of the actual state \( x(t) \) from an equilibrium point \( x^* \) or, more generally, from an invariant set, can be thought as a measure of energy accumulated in the state space of the given system. In this sense the concept of signal energy is closely related to storage functions, such as the available storage and required storage introduced by J.C.Willems in [6, 7], and discussed also in [8].

We start with a natural assumption that every real signal must be generated by a realizable system. Let us assume for a moment the signal generating system be given in the form:

\[
\mathcal{R}\{S\} : \quad x(t) = f(x(t)) + Bu(t), \quad x(t_0) = x^0, \quad y(t) = C x(t),
\]

(7)

It seems natural to postulate that besides the strict causality of \( \mathcal{R}\{S\} \) every real system has to satisfy some form of conservation law. Let the immediate value of the output signal power and corresponding value of the system energy accumulated in the state \( x(t) \) be defined by [15 - 20]:

\[
P(t) = \int \|u(t)\|^2 dt, \quad \|u(t)\|^2 \frac{dE(x)}{dt} = -P(t), \quad \delta > 0
\]

(8)

where the symbol \( \delta \) denotes an arbitrary energy scaling parameter.

For a proper equivalent state space representation \( \mathcal{R}\{S\} \) of the given system and for \( f(x) = Ax \) we get the signal power balance relation

\[
\frac{dE(x)}{dt} = \delta \dot{x}^2(t) - 2 \dot{x}^2 \dot{x}^T \dot{x} + \delta y^2(t)
\]

(9)

from which a special form of Lyapunov’s equation, expressing in fact the signal power balance, could be obtained. Hence, in case of zero future input signal \( u(t) = 0, \forall t \geq t_0 \), the total energy accumulated in the state at time \( t_0 \) must be equal to the amount of energy dissipated on the interval \([t_0; \infty)\) by the output:

\[
E(t_0) = \int_{t_0}^{\infty} \|y(t)\|^2 dt
\]

(10)

It is worthwhile to note that in general case the minimality of system representation \( \mathcal{R}\{S\} \) is equivalent to observability of \( (A, C) \) and controllability of \( (A, B) \), but for zero input only the observability is necessary. Thus the given representation must be in the state equivalence relation with a structurally observable representation called observability normal form.

In discrete-time case we proceed conceptually by exactly the same way as before. The signal generating system is now represented by:

\[
\mathcal{R}\{S\} : \quad x(k+1) = Ax(k) + Bu(k), \quad x(k_0) = x^0, \quad y(k) = Cx(k),
\]

(11)

and the immediate value of the output signal power and corresponding value of the system energy accumulated in the state, be defined by:

\[
P(k) = \|y(k)\|^2, \quad E(k) = \|y(k)\|^2, \quad P(k) = -\Delta E(k)
\]

(12)
Putting $u(k) = 0, \forall k \geq 0$, and computing the difference of the energy function $E(k)$ along any trajectory of the system representation, we get the signal power balance relation:

$$\Delta E[x(k)] = \delta^T (k) [A^T A + I] x(k) = -\gamma^T (k)$$

(13)

After some manipulations a special form of discrete-time Lyapunov’s equation, expressing in fact the signal energy conservation principle, could be obtained. Assuming zero future input $u(k) = 0, \forall k \geq 0$, the energy accumulated in the system in time $k=0$ is equal to the sum of energy quanta dissipated at the interval $[0; \infty)$ by the output signal, given by:

$$E(k = 0) = \sum_{i=0}^{\infty} \| y(k) \|^2$$

(14)

Again the given representation must be state equivalent to the observability normal form.

4 Structural consequences of dissipativity: Continuous-time signals

From the energy conservation principle for continuous-time signals in the form of the Eqns.(8), (9) and/or (10) it follows, that a special form of a structurally dissipative state equivalent system representation $\mathcal{R}\{S\}$, which is natural to call the dissipation normal form must exist and can be specified by the triplex of matrices $(A, B, C)$ as follows:

$$A = \begin{pmatrix} -\alpha & \alpha & 0 & 0 & \cdots & 0 & 0 \\ -\alpha & 0 & \alpha & 0 & \cdots & 0 & 0 \\ 0 & -\alpha & 0 & \alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -\alpha_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -\alpha_n \end{pmatrix}$$

(15)

$$C = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$B = [\beta_1 \beta_2 \cdots \beta_n]$$

The internal structure of the continuous-time dissipation normal form $\mathcal{R}\{S\}$ is shown in the Fig. 1.

It is easy to show that the set of real basic design parameters $\alpha, \gamma, \beta$ must satisfy the following fundamental consistency conditions having three important consequences:

1. $\alpha > 0 \iff$ structural dissipativity

2. $\forall i, \ i \in \{2,3,...,n\} : 0 \neq \alpha_i, \gamma \neq 0,$

3. $\exists i : \beta_i \neq 0 \iff$ structural minimality

3 structural asymptotic stability

5 Structural consequences of dissipativity: Discrete-time signals

From the energy conservation principle in form of the Eqns.(12), (13), and/or (14) it follows, that another special form of structurally dissipative state equivalent system representation called discrete-time dissipation normal form must exist and can be specified by the triplex of matrices $(A, B, C)$ as follows:

$$A = \begin{bmatrix} -\delta_{12} & \delta_{13} & 0 & \cdots & 0 & 0 & 0 \\ -\delta_{12} & -\delta_{13} & \delta_{14} & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\ -\delta_{n-1} \delta_{n-2} & \delta_{n-1} \delta_{n-2} & \delta_{n-1} \delta_{n-2} & \cdots & \cdots & \cdots & \cdots \\ \delta_{n-1} \delta_{n-2} & \delta_{n-1} \delta_{n-2} & \delta_{n-1} \delta_{n-2} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$B = [\beta_1 \beta_2 \cdots \beta_n]$$

The internal structure of the discrete-time dissipation normal form $\mathcal{R}\{S\}$ is shown in the Fig. 2.

Fig. 2: Structure of a discrete-time system representation $\mathcal{R}\{S\}$ in the dissipation normal form.

It is easy to show that the set of real basic (direct) design parameters $\delta_i$ and the set of real complementary parameters $\Delta_i$ must satisfy the following consistency conditions:

$$0 < \delta_i \leq 1, \delta_i^2 + \Delta_i^2 = 1, \ i \in \{1,2,...,n\}, \ \delta_n = \gamma,$$

(20)

having two important consequences:
6 Dissipativity and stability analysis: Linear continuous-time systems

Let a system $S$ for $n = 6$, and for the zero input $u(t)=0$, $t \geq t_0$, is given by the differential equation

$$y^{(6)} + a_6y^{(5)} + ... + a_2y^{(2)} + a_1y + a_0y = 0$$

The corresponding signal-energy-metric state space representation $R(S)$ in the dissipation normal form reads:

$$R(S): \dot{x}(t) = -A_1x(t) + A_2x(t)$$

and an internal structure of the matrix $A$ reduces to

$$A = \begin{bmatrix}
-a_1 & a_2 & 0 & 0 & 0 & 0 \\
-a_2 & 0 & a_3 & 0 & 0 & 0 \\
0 & -a_3 & 0 & a_4 & 0 & 0 \\
0 & 0 & -a_4 & 0 & a_5 & 0 \\
0 & 0 & 0 & -a_5 & 0 & a_6 \\
0 & 0 & 0 & 0 & -a_6 & 0 \\
\end{bmatrix}$$

with a characteristic polynomial

$$P(s) = s^6 + a_6s^5 + a_5s^4 + ... + a_2s^2 + a_1s + a_0$$

Hence, from the existence of a unique equilibrium state point of view, the dissipation parameter $\alpha_1$, as well as interaction parameters $\alpha_3$, $\alpha_5$ can be chosen arbitrary.

**Remark 1:** Notice that the dissipation parameter $\alpha_1$ is the only element of the matrix $A$, which sign determines the system dissipativity. The critical case $\alpha_1 = 0$ determines the system conservativity and separates stability of the equilibrium state from its instability.

Now, if the time derivative of the output signal energy function $E(x)$ along the system representation for the given matrix (25) will be computed, we get:

$$\frac{dE(t)}{dt} \bigg|_{R[S]} = -\alpha_1\dot{x}_1(t) = -\frac{\alpha_1}{\gamma^2}y^2(t)$$

where $\gamma$ is a positive real output scaling parameter

$$0 < \gamma < \infty$$

Thus, the signal energy conservation principle in form of the Eqn. (8) is valid if and only if it holds:

$$P(t) = y^2(t) \Leftrightarrow \alpha_i = \gamma^2$$

It is worthwhile to notice that for non-vanishing real output signal it holds $\dot{y} > 0$, and thus $\alpha_i > 0$. It means that the system is dissipative if and only if the dissipation power is positive or equivalently if at least one of the state variables is observable by the output.

On the other hand if $\alpha_i < 0$, then the system must be anti-dissipative; if the state variable $x_i(t)$ is real and the observed output signal is non-vanishing it must hold $\dot{y} < 0$, and consequently the dissipation power $P(t)$ must be negative. It means that in such a case for any real value of the state the observed output signal $y(t)$ must be imaginary. Such an interpretation of the signal power seems to be closely related to the concept of non-active power in a context of electrical engineering.

The last case $\alpha_i = 0$ occurs if the system is conservative in sense of the definition Eqn.(5); it corresponds to the situation when the system state is totally unobservable by the output. It means that no information about the internal state can be obtained by measurement, if the system under consideration is strictly conservative. The last observation seems to be closely related to the famous Heisenberg’s uncertainty principle of quantum physics.

What remains is to make clear the role of the interaction parameters $\alpha_3$, $\alpha_5$. Notice, that if we put $\alpha_3 = 0$, then the state variables $x_i$, $i = 5, 6$ become unobservable by the output $y$; thus only the first isolated subsystem with the state variables $x_i$, $i = 1, 2, 3, 4$, which is observable, will be asymptotically stable, while the second one will oscillate on the constant energy level, corresponding to its initial state energy with the frequency given by the natural frequency parameter $\alpha_6$. As a result the whole system is

\[\forall i, \ i \in \{1, 2, ..., n\}: |\Delta_i| < 1 \Leftrightarrow \text{structural asymptotic stability} \]

\[\forall i: 0 < \delta_i \leq 1, \ \gamma \neq 0, \ \beta_i \neq 0 \Leftrightarrow \text{structural minimality} \]
stable in the sense of Lyapunov, but not asymptotically. In such a case the characteristic polynomial takes the form:

\[
P(s) = (s^2 + \alpha_1^2) \prod_{i=1}^{4} (s-s_i)
\]

(34)

and well known algebraic stability conditions for roots of the characteristic polynomial follow:

\[
\text{Re } s_i < 0, \text{ for } i=1,2,3,4, \text{ and } \text{Re } s_5 = 0, \text{ Re } s_6 = 0
\]

(35)

Similarly, if we put \( \alpha_3 = 0 \), then the state variables \( x_i, i = 3,4,5,6 \) become unobservable by the output \( y \), and only the first isolated subsystem

\[
\dot{x}_i(t) = -\alpha_i x_i(t) + \alpha_i x_i(t)
\]

\[
\dot{y}_i(t) = \gamma_i x_i(t)
\]

(36)

which is observable, will be asymptotic stable, while the second one will oscillate on the constant energy level, corresponding to its initial state energy with frequencies given by natural frequency parameters \( \alpha_i, \alpha_6 \), modified by the interaction parameter \( \alpha_5 \). Again, the whole system is stable in the sense of Lyapunov, but not asymptotically. In such a case the characteristic polynomial takes the form:

\[
P(s) = (s^2 + \alpha_1 s + \alpha_2^2) \prod_{i=1}^{6} (s-s_i)
\]

(37)

and again the standard algebraic stability conditions for roots of characteristic polynomial follow:

\[
\text{Re } s_i < 0, \text{ for } i=1,2, \text{ and } \text{Re } s_i = 0, \text{ for } i = 3,4,5,6
\]

(38)

The time evolution of the output signal power is shown in the Fig.3 and the state energy in the Fig.4.

**Fig. 3.** Evolution of the output power \( P(t) \)

- a) conservative case \( \alpha_1 = 0, \alpha_k = 2,3,...,n \), arbitrary,
- b) stability \( \alpha_1 > 0, \alpha_3 = 0 \),
- c) stability \( \alpha_1 > 0, \alpha_5 = 0, \alpha_6 = 0 \),
- d) asymptotic stability \( \alpha_1 > 0, \alpha_k \neq 0, k = 2,3,...,n \).

If needed, we can determine the set of parameters \( \alpha_i, i = 1, 2, ..., n \) from the Eqn. (39). Then we get:

\[
\alpha_i = a_i = \Delta_i, \quad \alpha_i = \sqrt{\frac{a_i a_i - a_i^2}{a_i}} = \frac{\Delta_i}{\Delta_i}
\]

(40)

where the new parameters \( \Delta_k, k = 1, 2, ..., n \) can easily be recognized as diagonal minors of the well known Hurwitz determinant.
Using the Eqsns. (40), (31) together with the requirement \( a_i \in \mathbb{R} \), the following set of equivalent necessary and sufficient conditions of the asymptotic stability can be obtained:

\[
\begin{align*}
\alpha_i \in \mathbb{R}, & \quad \alpha_i > 0 \quad \Leftrightarrow \quad \Delta_i > 0 \\
\alpha_i \in \mathbb{R}, & \quad \alpha_i \neq 0 \quad \Leftrightarrow \quad \frac{\Delta_i}{\Delta} > 0 \\
\alpha_i \in \mathbb{R}, & \quad \alpha_i \neq 0 \quad \Leftrightarrow \quad \frac{\Delta_i \Delta_{i+1}}{\Delta_{i+1} \Delta_{i+2}} > 0 \quad \text{for} \quad i = 1, 2, \ldots, n \\
\alpha_i \in \mathbb{R}, & \quad \alpha_i \neq 0 \quad \Leftrightarrow \quad \frac{\Delta_i \Delta_{i+1} \Delta_{i+2} \cdots \Delta_{i+k}}{\Delta_{i+k} \Delta_{i+k+1} \Delta_{i+k+2} \cdots \Delta_{i+2k}} > 0
\end{align*}
\]

The resulting conditions are obviously equivalent to the set of the well-known Hurwitz conditions:

\[
\Delta_k > 0, \quad k = 1, 2, \ldots, n
\]

It means that existing linear algebraic methods for stability analysis can be seen as a special case of methods based on the proposed signal energy-metric approach. Moreover, the proposed state energy interpretation makes it possible to gain a better insight into the classical results of stability theory.

### 7 Dissipativity and generation of Lyapunov functions: Continuous-time systems

The proposed signal-energy-metric method can also be used as a systematic method of Lyapunov functions generation. Let’s consider a linear system given in the form

\[
y^{(i)}(t) + a_i y^{(i)}(t) + a_i y(t) + a_i y(t) + a_i y(t) = 0
\]

gained by an approximative linearization procedure, and let the state variables be defined by the standard way as follows

\[
x_i = y, \quad x_i = y, \quad x_i = y, \quad x_i = y
\]

Then the observability matrix is given by \( H_0 = I \), while the observability matrix \( H_0 \) of the state equivalent representation \( \mathfrak{S}(S) \) is triangular and invertible. It is easy to show that the Lyapunov function \( V \) can be computed by isometric transformations of the state space coordinate system:

\[
V(x(t)) = \frac{1}{2} x^T(t) \left[ H^T_0 . H_0 \right]^{-1} . x(t)
\]

and for the given system representation it can be explicitly expressed by

\[
V = \frac{1}{2} \left[ x_1^2 + \left( \frac{a_2}{a_3} x_3 + \frac{1}{a_2 a_3} x_1 \right) \right] + \left[ \frac{a_2}{a_3} x_3 + \frac{1}{a_2 a_3} x_1 \right] \ldots
\]

### 8 Dissipativity and stability analysis: Nonlinear continuous-time systems

Non-linear stability analysis will be demonstrated by a simple non-linear system described by:

\[
\dot{y}(t) + \varepsilon \left[ \alpha - \beta y^i(t) \right] y(t) + a_i y(t) = 0
\]

If the matrix \( C \) is defined by \( C = [y, 0] \), and the chosen structure of the matrix \( A(x) \) is defined by

\[
A(x, y) = \begin{bmatrix}
-\varepsilon \left[ \alpha - \beta x^i \right], & \sqrt{a_2} \\
-\varepsilon, & 0
\end{bmatrix}
\]

then the representation is locally observable if \( \gamma \neq 0, \quad \alpha > 0 \) and the energy conservation law implies

\[
\frac{dE(t)}{dt} \bigg|_{\mathfrak{S}(x)} = -P \leq 0, \quad P = \varepsilon \left[ \alpha - \beta x^i \right] x^i
\]

It follows that the unique equilibrium state \( x^* = 0 \) is asymptotically stable in the region D given by

\[
D = \left\{ x, y : |x| < \sqrt{\frac{3\alpha}{\beta}} \quad \text{and} \quad y < \frac{3\alpha}{\beta} \right\}
\]

if \( \varepsilon > 0, \quad \alpha > 0, \beta > 0, \quad a_i > 0 \)

Let us now consider the same nonlinear system but instead of the matrix structure \( A(x) \) the state \( x(t) \) be defined by the standard way as \( x_1 = y, x_2 = dx/dt \), then the corresponding system representation is structurally observable with the observability matrix \( H_0 = I \) and from the signal energy conservation principle we get

\[
\frac{dE(t)}{dt} \bigg|_{\mathfrak{S}(x)} = -P \leq 0, \quad P = \varepsilon \left[ \alpha - \beta x^i \right] x^i
\]

Lyapunov function \( V(x) \) can be determined by isometric transformations of the energy function (8)
\[ E(x) = \frac{1}{2} \rho \left[ x(t), 0 \right] \]

For \( \alpha = \beta = a_2 = 1 \) we get [8]

\[ V(x) = \frac{1}{2} \left[ \frac{1}{2} \rho x_1^2 + \frac{1}{2} \rho x_2^2 + (1 + \rho) x_2^2 \right] \]

### 9 State Energy Evolution in Nonlinear System

Let us consider another well known non-linear system given by

\[ \dot{y}(t) + a_1 \left[ 1 - \beta y(t) \text{sign}[y(t)] \right] \dot{y}(t) + a_2 y(t) = 0 \]

\[ A(x_1, x_2) = \begin{bmatrix} -a_1 \left[ 1 - \beta x_1(t) \text{sign}[x_1(t)] \right] & \sqrt{a_2} \\ -\sqrt{a_2} & 0 \end{bmatrix} \]

with \( a_2 > 0 \), and \( \alpha, \beta \) as arbitrary real design parameters.

A unique Lyapunov function \( V(x) \) can be determined by isometric transformations as the state energy \( E(x) \).

We get (for \( a_2 = 1 \)):

\[ 2E\left[ x_1, x_2 \right] = \frac{1}{2} a_1^2 \beta x_1^3 - a_1^2 \beta x_1 \text{sign} \left[ x_1(t) \right] - \]

\[ -a_1 \beta x_1 x_2 \text{sign} \left[ x_1(t) \right] + (a_1^2 + 1)x_2^2 + 2a_1 x_1 x_2 + x_2^2 \]

The dissipation power \( P(t) \) is given by

\[ \forall a_2 \geq 0, \quad \frac{dE}{dt} = -2a_1 x_1 \left[ 1 - \frac{1}{2} \beta x_1 \text{sign}[x_1(t)] \right] \]

For linear case \( (\beta = 0) \) and for \( a_2 = 1 \) the energy reduces to

\[ E\left[ x_1, x_2 \right] = 0.5(a_1^2 + 1)x_2^2(t) + a_1 x_1(t)x_2(t) + 0.5x_2^2(t) \]

### 10 Conclusion

In the contribution a new unifying, systematic and constructive approach to non-linear phenomena, based on a new concept of the signal-energy-metric state space system representation has been presented. Main features of the method are illustrated by typical examples and simulation results.

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**References:**


Massachusetts, 1970.


