A Genetic Algorithm for Optimization in Conceptual Robot Manipulator

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Abstract: - An important criterion of the industrial robots is the dynamic performance of manipulators. The correlations between the parameters which determine the dynamic performance of robot manipulators are complex and highly non-linear. The gearbox size (mass) and the lengths of the arms are parameters that have a large impact on the performance and the cost of robots. In order to perform optimization, a mathematical programming model is developed. An objective function is defined to determine optimal gearbox mass and arm lengths from an acceleration capability perspective. This paper presents a Genetic algorithm procedure which shows how optimization can be used in the early phases of a development process in order to evaluate the potential of a concept. This study considers a three degree of freedom robot manipulator. The mathematical model is coded in the Matlab language and optimized using the Genetic algorithm. The results obtained with optimum values based on Genetic algorithm clearly shows the advantages of the proposed method.

Key-Words: - Optimization, Genetic Algorithm, Robot Arm, Manipulator

1 Introduction
The problems in robot design are characterized by complexity and the presence of several conflicting design objectives. In this paper the focus is on the conceptual design stage and on how rather simple mathematical models could be applied together with modern technique based on Genetic algorithm in order to support the designer. With the help of the presented technique the designer could investigate different conceptual designs and evaluate how changing requirements affect the optimal design. The optimization is carried out on a three degree of freedom (DOF) serial manipulator with an offset of the lower arm from axis one. The robot is mathematically modeled and coded in the Matlab language program based on the proposed Genetic algorithm all and calculations are done in this package. The optimization problem is formulated as minimizing the weight of the gearbox subjected to a set of constraints. These constraints are amongst others, a reach of 2.5 meters, a payload of 100 kg, and minimum acceleration of 0.75 G in the x-y-z directions of the base frame of the robot at predefined points. In most industrial robots the actuators consist of an electric motor and a gearbox. In this paper the motors are assumed to have a constant mass and are always able to deliver sufficient amount of torque to the gearbox. The size of the gearbox has large impact on the overall cost of the robot and is therefore an interesting parameter at the early phases of robot design. The gearbox is treated as discrete choices with a given mass and output torque.

For given bounds of joint torques, the corresponding acceleration radius is defined by the minimum upper bound of the magnitude of end-effector acceleration over the whole workspace. The method presented in this paper merely measures the dynamic performance by investigating the acceleration capability in some of the points within the workspace.

2 Presentation of the model
The dynamic model is presented using the Lagrangian equations of motion. Formulating the equation of motion using a set of independent generalized coordinates i.e. the joint variables leads to Lagrangian equations of the second type. Unlike the Lagrangian equation of the first type and the Newton-Euler formulation, the second type does not consist of any forces of constraint between adjacent links i.e. non redundant variables. The Lagrangian function is defined as the difference between the kinetic (K) and potential energy (U) of a mechanical system:

\[ L = K - U \] (1)

If generalized coordinates are \( q_1, q_2, q_3 \), the general form of Lagrange’s equation may be expressed as follow:

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial q_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \] (2)
If the coordinate systems to each link in the robot are placed according to the Denavit-Hartenberg method [3], it is easy and straight forward to write the transformation and rotation matrices between the different link frames. Together with the inertia matrices for every link and the jacobian for the manipulator, it is now possible to rewrite the equation of motion as:

\[ M(q)\ddot{q} + V(q, \dot{q}) + G(q) = Q \]  

(3)

This equation is called the general form of dynamical equations. The vector V is called the velocity coupling vector. There are two distinct types of velocity coupling between joints. The velocity squared terms correspond to centripetal terms, and the velocity product terms correspond to the Coriolis terms. The manipulator inertia matrix M is symmetric and positive definite and therefore always invertible. The offdiagonal elements of M represent acceleration coupling effects between joints [4].

### 3 The optimization of the model

The optimization problem are formulated as to minimize the weight of the gearbox, by choosing different discrete gearbox, and changing the lengths of the arms continuously, subjected to a few requirements on acceleration capability, reach and payload capacity. This formulation can be interpreted as to design the cheapest possible robot that still meets the acceleration demands. The acceleration should be achieved in the x- y-and z-direction of the base frame.

The base frame is a coordinate system connected to the arm which is connected to the ground, i.e. arm one in figure 2. The constraints are formulated as penalty functions added to the objective function. The optimization problem is thus formulated according to equation (4).

\[
\min F = f (m) + g_1 (\tau_{av}, \tau_{ned}) + g_2 (\text{offset}, a_2, a_3) + g_3 (m, \text{offset}, a_2, a_3) \]

\[
x_i^{\text{lower}} \leq x_i \leq x_i^{\text{upper}} \quad i = 1...4 \]  

(4)

where, the four optimization variables, \(x_i\), are bounded between lower and upper limits. The first optimization variable represent the size of the gearbox whereas the other three represent the offset and the arm lengths, \(f\) is the weight of the gearbox. The \(g_1\) is a penalty function in case if the torque needed to achieve the required acceleration exceeds the torque available. The \(g_2\) is a penalty function in case the reach requirement is not fulfilled. The last function \(g_3\), merely sums up the torques at all joints. Each penalty function is zero when the corresponding constraint is not violated and then grows exponentially with the degree of constraint violation.

### 3.1 Calculation Steps

As an argument to the penalty function \(g_1\), the necessary torque to achieve the required acceleration in the end-effector space (Cartesian space) needs to be calculated. In order to do this, one needs to map the end effector acceleration into the joint space. The relation between the velocity of the end effector and the corresponding angular velocities of the joints can be written as:

\[
\dot{x} = J(q) \cdot \dot{q} \]

(5)

where \(J\) is the Jacobian.

The acceleration is obtained by differentiating

\[
\ddot{x} = J(q) \cdot \ddot{q} + \frac{dJ(q)}{dt} \cdot \dot{q} \]

(6)

Assuming that the manipulator has zero velocity before evaluation at each predefined point in the workspace, the last term in equation (6) also turns zero. Also assuming that \(J(q)\) is a nonsingular and square matrix, one obtains an expression for the corresponding angular accelerations by multiplying both sides with:

\[
\ddot{q} = J^{-1}(q) \cdot \ddot{x} \]

(7)

Since the Jacobian \(J(q)\) is highly dependent on the lengths of the arms, equation (7) needs to be calculated for every set of arms. To calculate the required torque one also has to map the position of the robot in Cartesian space into the joint space in order to achieve the \(q\) vector that corresponds to the Cartesian position. The \(q\) vector also has to be calculated for every new set of arm lengths. By letting the result in equation (7) together with the \(q\) vector corresponding to the current position entering (3) the necessary torques \(Q\) are obtained. The mass of gearbox is 2.5 kg. The penalty \(g_2\) function has the lengths of the arms as arguments and the function merely sums the lengths and gives a penalty in case the sum is smaller than the required reach. The last penalty function \(g_3\), calculates the current torque at every joint in order to make the optimization algorithm search for arm lengths which minimize the required torque once a lightest set of gearbox is found.
3.2. The Genetic Algorithm

The contribution of current research is to solve this optimization problem using Genetic algorithm. While variables corresponded to gearbox are discrete and those corresponded to arms length are continuous, applying Genetic algorithm seems to be more promising rather than other optimal or heuristic algorithms.

To encode the solutions, consider each solution as a D-dimensional parameters vector (D is the number of variables in the model, in this typical problem, D=4). The first parameter is discrete parameter which is corresponding with gearbox’s mass. Other three parameters are continuous parameters which are corresponding to offset, the length of first arm and the length of second arm, respectively. Each parameter taking value from within user-defined bounds, as mentioned before:

\[ x_{i}^{\text{lower}} \leq x_{i} \leq x_{i}^{\text{upper}} \quad i = 1...4 \]

3.3. Application

In this section the proposed Genetic algorithm is applied to a three DOF manipulator with revolute joints and an offset of the lower arm from axis one. A scheme of the robot is shown in Fig. 2. The arms are modeled as rigid bodies with a density per length unit and extension in one direction i.e. the directions describing the cross section of the arms are neglected. The geometry of the cross section has large impact on the inertia of arm one, but less influence on the inertia of the lower and upper arm. The gearbox are modeled as point masses located at every joint and the payload is modeled as a point mass located at the end of the upper arm. The densities of the arms are considered constant. The values for each arm are shown in Table 1.

![Fig. 2 Scheme of the robot manipulator](image)

The arms are allowed to vary within the intervals shown in Table 2.

![Fig. 1 Scheme of genetic algorithm](image)

### Table 1. Density for the different arms of the robot.

<table>
<thead>
<tr>
<th>Robot arm</th>
<th>Density [kg/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>base link</td>
<td>1500</td>
</tr>
<tr>
<td>offset</td>
<td>400</td>
</tr>
<tr>
<td>lower arm</td>
<td>100</td>
</tr>
<tr>
<td>upper arm</td>
<td>75</td>
</tr>
</tbody>
</table>

### Table 2. Intervals for the lengths of the arms

<table>
<thead>
<tr>
<th>Arm</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offset</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Lower arm</td>
<td>0.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Upper arm</td>
<td>0.6</td>
<td>1.7</td>
</tr>
</tbody>
</table>
4. Computational Results
This section discusses the performance and the computational efficiency of the proposed Genetic algorithm. The objective function converged into an optimum corresponding to the set of parameters shown in table 3.
The optimization was carried out several times with different initial conditions and the same optimum was found every time.
The reason that the interval of the length of offset is not increased even though the genetic algorithm has chosen the upper boundary (offset = 0.399 m) is that a larger offset would decrease the workspace volume to much.
This is a constraint which is not included in the optimization and from an acceleration capability point of view it is natural to increase the offset as much as possible. Increased offset has little influence on joint one, but reduce the size of the gearbox on axis two and three.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gearbox</td>
<td>47 kg</td>
</tr>
<tr>
<td>Offset</td>
<td>0.399 m</td>
</tr>
<tr>
<td>Length of lower arm</td>
<td>1.36 m</td>
</tr>
<tr>
<td>Length of upper arm</td>
<td>0.71 m</td>
</tr>
</tbody>
</table>

Table 3. List of optimum values of the design variables.

Fig. 3. The mass of gearbox as function of the number of generations (genetic algorithm)

Fig. 4. The offset length as function of the number of generations (genetic algorithm).

Fig. 5. The length of the upper arm as function of the number of generations (genetic algorithm).

Fig. 6. The length of the lower arm as function of the number of generations (genetic algorithm).

Fig. 5 and 6 show how the length of upper arm changes during optimization of Genetic algorithms.
5 Conclusion
This paper presents a method for optimization of robot design in the conceptual design stage based on Genetic algorithm. The robot is mathematically modeled and solved using Genetic algorithm through Matlab language program. The optimization problem is formulated as to determine gearbox mass and arm lengths in order to minimize the weight of the gearbox and simultaneously obtain a prescribed acceleration at a set of points in the workspace. This formulation can be interpreted as to design the cheapest possible robot that still meets the acceleration demands. This could be time-consuming and difficult problem. The optimization method based on Genetic algorithm showed good capability in finding an optimum set of gearbox and arm lengths for a three DOF robot manipulator.

A more complex model of the gearbox should contain a speed-torque dependency and inertia properties. Furthermore the optimization would be more useful if one made the optimization cycle based and could take other characteristics such as lifetime and cycle time into account. With the electric motor included in the optimization additional design parameters such as gear ratio would be required likewise thermal aspects must be added to the characteristics included in the objective function. A problem when designing robots is to know for which cycles a robot design should be optimized. Cycle-based optimization is also a time-consuming task.

If it was possible to translate the result from an optimization without cycle-based evaluation, like the ones presented in this paper into a more global performance index it would be of benefit for the robot designer. A more complex model of the gearbox should contain a speed-torque dependency and inertia properties.

References: