Modeling of the Heat Pipe Heat Exchangers for Heat Recovery

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Abstract: - Research has been carried out on the theory, design and construction of heat pipes, especially their use in heat pipe heat exchangers for energy recovery, reduction of air pollution and environmental conservation. Heat pipe heat exchangers are widely used for heat recovery in various ranges of applications because of their simple structure, special flexibility, high efficiency, good compactness and excellent reversibility. Heat recovery from fire heaters or turbine flue gases is an important application for this type of heat transfer devise. In this paper a computer simulation was developed by MATLAB to design the heat pipe heat exchangers. This program can be considered as a simple tool for modeling and designing heat pipe heat exchangers.

Key-Words: - Heat Pipe, Heat Recovery, MATLAB, Modeling.

1 Introduction

A heat pipe heat exchanger in which groups of heat pipes are arranged within a box, the center of which is partitioned, a high temperature fluid flows on one side, a low temperature fluid flows on the other side, thereby transferring the heat of the high temperature fluid to the low temperature fluid via fluid sealed in heat pipes. Since heat pipes have the capability to transport heat over appreciable distances virtually isothermally, it is not necessary to subdivide the original flow streams into interspersed multiple flow passages in a heat pipe heat exchanger. Instead, the original flow streams remain intact and separated. Heat transfer is accomplished via multiple small heat pipes extending through the common wall of the hot and cold side flow streams. Such an arrarigement for a counterflow heat pipe heat exchanger is shown in Fig. 1.

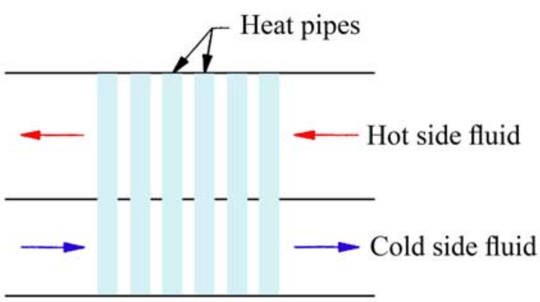

Fig. 1. Heat pipe heat exchanger- counterflow configuration.

As in conventional heat exchangers, fins may be added to the external surface of the heat pipes to increase the effective surface area. However, the contribution of fins in augmenting heat transfer area must be balanced against their increased weight. Since it is not necessary to intersperse the hot and cold side flow streams in a heat pipe heat exchanger, heat transfer is accomplished with little or no disruption or diversion of the flow streams. Inlet and outlet flow distribution plenums will be considerably simplified, or may not be needed at all. Complete separation of the flow streams can be particularly advantageous when the hot and cold side fluids are chemically reactive, and mixing of the streams in the event of a leak must be avoided.
The heat pipes in a heat pipe heat exchanger are arranged in stages, each stage consisting of a single row of heat pipes, all of which are at the same temperature. The heat pipe temperature changes from stage to stage, varying from a minimum value at one end of the heat exchanger to a maximum value at the other end. If the temperature range is large enough, more than one heat pipe fluid could be required to assure adequate heat transport capability in all stages. The required heat transfer area for a given heat transfer rate decreases as the number of stages is increased, but the pressure drop of the hot and cold side fluids also increases with the number of stages. Therefore, pressure drop limitations may establish the maximum number of stages and the minimum heat transfer area.
In a heat pipe heat exchanger, the core volume and weight are proportional to the sum of the hot side (evaporator section) area and the cold side (condenser section) area. In an unfinned conventional heat exchanger in which one fluid flows inside tubular flow passages and the other fluid flows outside these passages, the hot and cold

side areas are virtually equal and extend over the same length. The core volume and weight are then proportional to either the hot or the cold side areas, rather than their sum. Therefore, in comparing the heat transfer area of unfinned heat pipe and conventional heat exchangers, the outside surface area of the heat pipes should be compared to the hot or cold side area of the conventional heat exchanger flow passages. Depending on specific design circumstances, a heat pipe heat exchanger may be larger or smaller than a conventional heat exchanger with the same heat transfer rate.

2 Mathematical model description

The analysis of the heat transfer aspects of HPHE's is based on the heat transfer rate equation obtained by an energy balance of the heat, exchanger:

$$Q = UA(T_h - T_c) \tag{1}$$

where Q is the heat transferred, U is an overall heat transfer coefficient, S is the heat transfer area, and T_h and T_c are the temperatures of the high and low-temperature fluids. To determine the overall heat transfer coefficient, the heat exchanger can be modeled as a thermal resistance network shown in Fig. 2.[1]

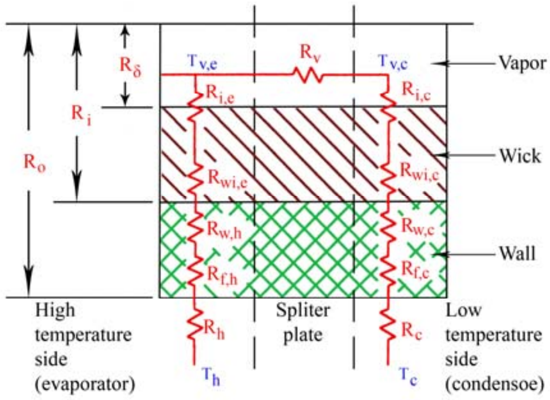

Fig. 2. Equivalent resistance network for a heat pipe in a heat exchanger.

During heat exchanger operation, fluid impurities, rust formation, or other reactions between the fluid and wall or fin material can cause the heat transfer surfaces to foul. This fouling can greatly increase the resistance to heat transfer between the fluids and is dependent on the operating temperature, fluid velocity, and length of service of the heat exchanger. By introducing a thermal resistance to account for this fouling, R_f, and including the effect of finned surfaces, the overall heat transfer coefficient can be written as :

$$\frac{1}{US} = \frac{1}{U_c S_c} = \frac{1}{U_h S_h} = \frac{1}{(\eta_0 h S)_c} + R_{f,c} + R_{hp} + R_{f,h} + \frac{1}{(\eta_0 h S)_h} \tag{2}$$

where η_o is the fin effectiveness, h is the heat transfer coefficient, and R_{hp} is the thermal resistance of the heat pipe, which includes the resistances due to heat pipe wall and wick as well as the resistance due to the evaporation and condensation of the heat pipe working fluid. The convective heat transfer coefficients are highly dependent on surface geometry, flow conditions and fluid properties.

There are two main approaches used in the design of a heat pipe heat exchanger: the log-mean temperature difference model (LMTD), and the effectiveness-number of transfer units (ε-NTU) model.

2. 1 Effectiveness-number of transfer units (ε-NTU) method

The ε-NTU method is based on the heat exchanger effectiveness, ε, which is defined as the ratio of the actual heat transfer in a heat exchanger to the heat transfer that would occur in a heat exchanger with infinite surface area. With infinite surface area, the exit temperature of the low-temperature fluid would equal the inlet temperature of the high-temperature fluid. Therefore, the effectiveness can be given as

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})} \tag{3}$$

Q is the actual heat transfer and C_{min} is the minimum heat capacity. Applying conservation of energy relationships, the general exponential function for a counter-flow heat exchanger is

$$\varepsilon = \frac{1 - \exp\left[\frac{-U_t S_t}{C_{\min}}\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]}{1 - \frac{C_{\min}}{C_{\max}} \exp\left[\frac{-U_t S_t}{C_{\min}}\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]} \tag{4}$$

The ratio $U_t S_t / C_{min}$ is defined as the number of transfer units (or NTU) as

$$NTU = \frac{U_t S_t}{C_{\min}} \tag{5}$$

The minimum heat capacity is defined as

$$C_{\min} = (\dot{m} c_P)_{\min} \tag{6}$$

where the fluid with the smaller value of the product of mass flow rate and specific heat is chosen to have the minimum heat capacity. The number of heat transfer units, NTU, is a nondimensional expression of the heat transfer area of the heat exchanger. For co-current flow, a similar relation for thermal effectiveness can be derived

$$\varepsilon = \frac{1-\exp\left[\frac{-U_t\,S_t}{C_{\min}}\left(1+\frac{C_{\min}}{C_{\max}}\right)\right]}{1+\frac{C_{\min}}{C_{\max}}} \qquad (7)$$

In a heat pipe heat exchanger, heat is transferred from the high and low temperature fluid by the evaporation and condensation of the working fluid in the individual heat pipes or thermosyphons. With this configuration, the maximum heat capacity is due to the phase change of the heat pipe working fluid. This fact leads to the analysis of heat pipe heat exchangers as two separate heat exchangers coupled by the heat pipe working fluid, which is similar to a liquid-coupled indirect-transfer heat exchanger.

Since the maximum heat capacity is several orders of magnitude larger than the minimum due to the phase change, C_{min}/C_{max}, the expressions for effectiveness in eqns. (4) and (7) reduce to

$$\varepsilon = 1-\exp(-NTU) \qquad (8)$$

The effectiveness of the evaporator and condenser sections of the heat pipe heat exchanger can thus be defined as

$$\varepsilon_h = 1-\exp(-NTU_h) \qquad (9)$$

$$\varepsilon_c = 1-\exp(-NTU_c) \qquad (10)$$

Where

$$NTU_h = \frac{U_h\,S_h}{C_h} \qquad (11)$$

$$NTU_c = \frac{U_c\,S_c}{C_c} \qquad (12)$$

and U_h and U_c are the overall heat transfer coefficients in the high- and low-temperature sides, S_h and S_c the heat transfer surface areas of the evaporator and condenser sections including any finned surfaces, and C_h and C_c are the heat capacities of the high- and low-temperature fluid streams. For an individual heat pipe (or thermosyphon), the effectiveness is defined as

$$\varepsilon_p = \left(\frac{1}{\varepsilon_{\min}}+\frac{C^*}{\varepsilon_{\max}}\right)^{-1} \qquad (13)$$

where ε_{min} and ε_{max} are the minimum and maximum values of ε_h and ε_c respectively. The heat capacity ratio C^* is

$$C^* = \frac{C_{\min}}{C_{\max}} \le 1 \qquad (14)$$

where C_{max} is the larger of the heat capacities of the high- and low-temperature fluid streams and C_{min} is the smaller. This expression can also be used to calculate the effectiveness of a single row or stage of heat pipes or thermosyphons. The only differences occur in the NTU expressions where the heat transfer areas, S_h and S_c , are based on the total heat transfer area in a row or stage.

For a multistage heat pipe heat exchanger in which there are a number of columns each containing a row of heat pipes (normal to the high-and low-temperature fluid streams), the effectiveness can be determined by considering the rows of heat pipes as a separate heat exchangers connected in series similar to that of a multipass heat exchanger.[1]

The effectiveness of a multistage heat pipe heat exchanger in counter flow is [2]

$$\varepsilon = \frac{\left(\frac{1-C^*\varepsilon_p}{1-\varepsilon_p}\right)^n - 1}{\left(\frac{1-C^*\varepsilon_p}{1-\varepsilon_p}\right)^n - C^*} \qquad (15)$$

where n is the number of stages and C^* is defined in eqn. (6.18). For the special case of C_{min}/C_{max} = 1, eqn. (15) reduces to

$$\varepsilon = \frac{n\varepsilon_p}{1+(n-1)\varepsilon_p} \qquad (16)$$

In eqns. (15) and (16) as the number of rows or stages in a HPHE increases, it can be seen that $\varepsilon \to 0$.

The two methods of HPHE analysis each have specific advantages and disadvantages. The ε-NTU method is the more direct of the two, but requires simplifying assumptions in the expression for the effectiveness. The LMTD does not have these simplifications, but is often requires an iterative solution procedure based on a guessed temperature distribution.

2. 2 Thermal resistance

By definition, the overall heat transfer coefficient for a single heat element is [1]

$$U_p = \frac{1}{R_p} \qquad (17)$$

where R_p is the thermal resistance of a single heat transfer element in the HPHE, which is defined as the sum of the individual resistances in the element. An equivalent resistance network for an individual heat pipe in a HPHE is shown in Fig. 2. From this figure, the total thermal resistance of a single heat pipe can be written as

$$R_p = R_h + R_{f,h} + R_{w,h} + R_{wi,e} + R_{i,e} + R_v + R_{i,c} + R_{wi,c} + R_{w,c} + R_{f,c} + R_c \qquad (18)$$

where R_h and R_c are the convective resistances at the outer surface of the evaporator and condenser sections,

$$R_h = \frac{1}{(\eta_0 hS)_h} \qquad R_c = \frac{1}{(\eta_0 hS)_c} \tag{19}$$

and for extended surfaces η_o is the fin efficiency and h is the convective heat transfer coefficient. $R_{f,h}$ and $R_{f,c}$ are the the fouling resistances due to corrosion or oxidation at the outer surfaces of the heat pipes or thermosyphons, and are defined as

$$R_{f,h} = \frac{R'_{f,h}}{(\eta_0 S)_h} \qquad R_{f,c} = \frac{R'_{f,c}}{(\eta_0 S)_c} \tag{20}$$

where $R'_{f,h}$ and $R'_{f,c}$ are fouling factors which are dependent on the high-and low-temperature fluids. These resistances are typically neglected, but in cases when the HPHE is operating in a corrosive environment, they can be significant and should be accounted for with an additional conduction resistance through the corrosion layer. $R_{w,h}$ and $R_{w,c}$ are the resistances due to the pipe wall. Heat transfer through the pipe wall, assuming a conventional circular heat pipe, can be written as

$$R_{w,h} = \frac{1}{2\pi k_w L_e}\ln\left(\frac{R_o}{R_i}\right), R_{w,c} = \frac{1}{2\pi k_w L_c}\ln\left(\frac{R_o}{R_i}\right) \tag{21}$$

Similarly, $R_{wi,h}$ and $R_{wi,c}$ are resistance terms which occur due to heat transfer through the liquid saturated wick

$$R_{wi,h} = \frac{1}{2\pi k_{eff} L_e}\ln\left(\frac{R_i}{R_\delta}\right), R_{wi,c} = \frac{1}{2\pi k_{eff} L_c}\ln\left(\frac{R_i}{R_\delta}\right) \tag{22}$$

where k_{eff} is the effective thermal conductivity of the liquid saturated wick, R_δ is the vapor core radius, and R_i is the inner pipe radius. The thermal resistance terms $R_{i,e}$ and $R_{i,c}$ are the resistances which occur due to the phase change of the working fluid at the liquid-vapor interface. These resistances can be defined in the evaporator and condenser sections as

$$R_{i,e} = \frac{1}{(h_{i,e} S_{i,e})}, R_{i,c} = \frac{1}{(h_{i,c} S_{i,c})} \tag{23}$$

where $h_{i,c}$ and $h_{i,e}$ are the convective heat transfer coefficients at the liquid-vapor interface of the evaporator and condenser sections, respectively. The remaining thermal resistance, R_v, is the resistance associated with the temperature drop in the vapor flow. Since the vapor in the heat pipe is saturated, the pressure drop associated with the vapor flow results in a temperature drop across the heat pipe. In normal heat pipe operation, this term is usually very small.

2. 3 Pressure drop analysis

The fluid flow configuration in the core of a heat pipe heat exchanger is flow normal to either a bare or finned bank of tubes. The fractional pressure drop for flow normal to tube banks is given by [2]

$$\frac{\Delta p}{p_{in}} = \frac{G^2}{2 p_{in}\rho_{in}}\left[\left(1+\left(\frac{A_{fr}}{A_{ft}}\right)^2\right)\left(\frac{\rho_{in}}{\rho_{out}}-1\right)+\hat{f}\frac{A}{A_{fr}}\frac{\rho_{in}}{\rho_{out}}\right] \tag{24}$$

where Δp is the pressure drop through the tube bank, p_{in} is the inlet pressure, G is the mass velocity, ρ_{in} and ρ_{out} are the fluid density at the inlet and outlet, A_{fr} is minimum free-flow area in the core, A_{ft} is the total frontal area of the heat exchanger, and f is the Fanning friction factor.

For individually circular finned tubes, eqn. (24) can be written as [3]

$$\Delta p = 2n\,\hat{f}'\frac{G^2}{\rho_{in}} + G^2\left(\frac{1}{\rho_{out}} - \frac{1}{\rho_{in}}\right) \tag{25}$$

2.4 External convective resistances

The heat transfer characteristics for the external resistances are generally expressed in terms of the Colburn factor, j_H. Colburn [4] extended the Reynolds analogy between energy and momentum transfer which states

$$\frac{f}{2} = St \equiv \frac{h}{\rho\omega c_p} \tag{26}$$

where f is the friction coefficient, St is the Stanton number, and w is the fluid velocity. Equation (26) is only applicable when the Prandtl number is equal to 1 (Pr = 1). However, Colburn determined As with the pressure drop analysis, the heat transfer characteristics of a heat pipe heat exchanger is highly dependent on the geometry of the tube banks. An empirical correlation for individually circular finned tube bank is that of Briggs and Young [5]

$$j_H = 0.134\,\mathrm{Re}_D^{-0.139}\left(\frac{L_f}{s'}\right)^{0.2}\left(\frac{L_\delta}{s'}\right)^{0.11} \tag{27}$$

where

$$s' = \frac{1}{N_f} - \delta_f \tag{28}$$

and Re_D is the Reynolds number based on the outside tube diameter, L_f is the fin height, δ_f is the fin thickness, and N_f is the number of fins per unit length.

2. 5 Internal convective resistances

Unlike the outer surface convective resistances, the inner resistance formulations are not similar in the evaporator and condenser section. Additionally, the internal resistances have completely different formulations for heat pipes and thermosyphons. The internal resistance in a heat pipe is governed by conduction through the liquid saturated wick and the phase change at the liquid-vapor interface. In thermosyphon evaporators, the most commonly used correlation is the Rohsenow correlation for nucleate pool boiling [6]

$$\frac{C_P(T_w - T_{sat})}{\lambda} = C_{sf}\Pr^{1.7}\left[\frac{h_{i,e}(T_w - T_{sat})}{\mu\lambda}\left(\frac{\sigma}{v' - v'_v}\right)^{0.5}\right]^{0.33} \quad (29)$$

where σ is the liquid surface tension, v' is the specific volume, and C_{sf} is a correlation constant based on the boiling liquid and surface combination. Once the heat transfer coefficient, h, is found in eqn. (29), the internal evaporator resistance, $R_{i,e}$ can be found using eqn. (30). Stulc et al. [7] proposed a modified Nusselt correlation which defined the inner surface evaporative resistance for vertical thermosyphons

$$R_{i,e} = \frac{\Delta T_i}{Q_i} \quad (30)$$

where

$$Q = 0.56\pi D_i\left(\frac{2L_e}{L_e + L_c}\sqrt{\frac{L_e + L_c}{L_e}} - 1\right)L_c^{0.75}(\lambda\rho_l^2 k_l^2 g\mu_l^{-1})^{0.25}\Delta T_i^{0.75} \quad (31)$$

and

$$\Delta T_i = \Delta T_{i,e} - \Delta T_{i,c} \quad (32)$$

$\Delta T_{i,e}$ is the temperature difference between the liquid-vapor interface and the vapor in the evaporator and $\Delta T_{i,c}$ is the temperature difference between the vapor and the liquid-vapor interface in the condenser. This formulation has a more direct solution than the Rohsenow correlation, but requires a knowledge of the temperature drop across the thermosyphon. Lee and Bedrosian [8] roposed the use of the Martinelli correlation [9] for condensing vapor in a thermosyphon

$$\frac{h\ D_i}{k} = \frac{1}{\beta}\left[\frac{1}{4} - Y^2 + \frac{3}{4}Y^4 - Y^4\ln Y\right] \quad (33)$$

where

$$Y = \frac{R_\delta}{R_i} \quad (34)$$

and

$$\beta = Y^4\ln Y\left(\frac{\ln Y}{2} - \frac{1}{2}\right) + \frac{Y^4}{8} + Y^2\left(\frac{\ln Y}{2} - \frac{1}{4}\right) + \frac{1}{8} \quad (35)$$

R_δ is the vapor space radius. This correlation has the advantage of not requiring temperature information.

Azad and Geoola [10] proposed a correlation for condensing water vapor in thermosyphons as a function of Reynolds number.

$$Nu_c = 5.03\mathrm{Re}^{1/3}\Pr^{1/3} \quad \text{for Re} \prec 50000$$
$$Nu_e = 0.0265\mathrm{Re}^{0.8}\Pr^{1/3} \quad \text{for Re} \prec 50000 \quad (36)$$

where $Nu = hD/k$. Once h is found, the internal condensation resistance can be calculated using eqn. (23). However, this correlation is only valid for condensing water vapor.

The above correlations are general formulations based on simple geometries. In reality, the internal resistances of both heat pipes and thermosyphons are significantly more complex. In heat pipes, the internal resistance is strongly influenced by external heat distribution and orientation. In thermosyphons, the internal resistance in the evaporator is governed by factors such as liquid fill and applied external heat distribution. In thermosyphon condensers, the thermal resistance is governed by heat sink conditions, heat load, and many other factors. However, using the above correlations, estimates of both the internal and external convective resistances can be found, which will enable the entire heat pipe heat exchanger to be evaluated using either the LMTD or the ε-NTU method.

3 Matlab program

To design heat pipe heat exchangers the program "Heat Pipe Heat Exchanger Designer" has been developed by Matlab.

Software has been developed for thermosyphon heat pipe heat exchanger design and the application of software has been executed for four industrial case studies. Some shots of the graphical user interface of the program are shown in following figures.

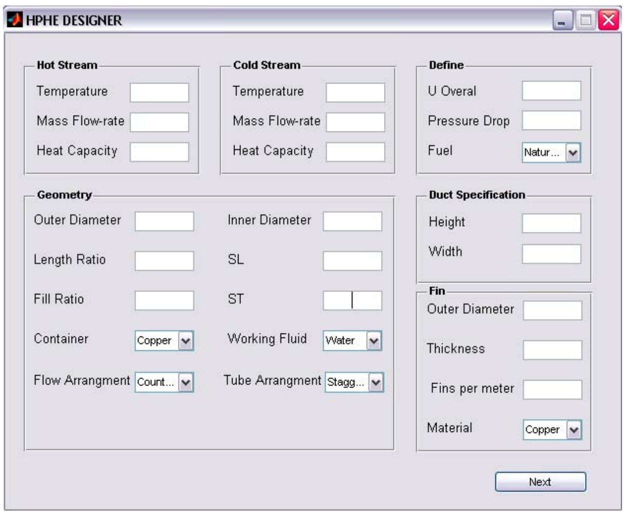

Fig. 3. HPHE designer program.

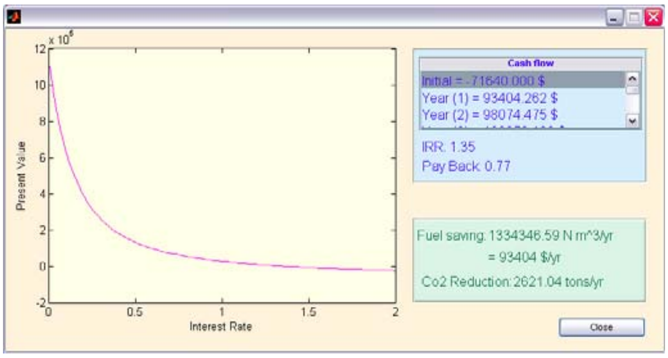

Fig. 4. Economical evaluations.

4 Results discussion

This section presents the comparison between the program results and the experiment. The results of program were compared to the data from a heat pipe heat exchanger that has been made in Ferdosi University.[11]

Table 1 shows the data of pilot plant and Table 2 shows the program results.

Table 1. Pilot plant data.

Cold stream inlet temperature	298 (K)
Hotstream inlet temperature	523 (K)
Cold stream mass flow rate	1.0125 (kg/s)
Hot stream mass flow rate	1.0764 (kg/s)
Hotstream outlet temperature	478 (K)
Cold stream heat capacity	1008 (J/kg K)
Hot stream heat capacity	1030 (J/kg K)
Exchanger dimension (length×width×heigth)	0.27×0.43×1.2 (m)
Tube material	Copper
Fins material	Aluminum
Number of fins	300 fin/m
Tube arrangement	In line, S_T=S_L=30mm
Overall heat transfer coefficient	17.24 W/m^2K
Pressure drop (hot stream)	1745.4 Pa

Table 2. Program results.

Exchanger dimension (length×width×heigth)	0.21×0.50×1.3 (m)
Overall heat transfer coefficient	17.24 W/m^2K
Pressure drop (hot stream)	1621.845 Pa
Pressure drop (cold stream)	1487.984 Pa

5 Conclusions

In this paper, a computer program has been developed to design heat pipe heat exchangers. The results of the program compared to a pilot plant and show good agreement with experimental data. This program is also capable to figure out economical evaluations for designed heat exchanger such as: cash flow, pay back, IRR and fuel saving.

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