Application of Fuzzy Analysis to Stability Problems of Steel-Concrete Structural Members

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Abstract: - The paper deals with the fuzzy analysis of ultimate limit state of a steel strut with encased web in compression. The first part of the paper lists the presumptions required for determination of the column’s theoretical load carrying capacity. Stresses in the concrete and steel sections are determined according to the principles of elasticity. The ultimate limit state is given as the limit stress attained in the most stressed section of either the steel or concrete section. General extended principle, which takes into account the epistemic uncertainty of input parameters, was utilized for the analysis.

Key words: concrete, steel, stability, compression, design, uncertainty, fuzzy, imperfection.

1 Introduction
Concrete and steel are materials generally used in the building industry. Structures to be built from high grade steel and concrete are more frequently used in the field of design of modern engineering structures. Steel is a high quality building material, its greatest merit being its high strength. Concrete carries the compressive stresses and protects the tensile steel from corrosion and high temperatures. The material combination of steel-concrete brings about the occurrence of special phenomena the influence of which on structural reliability is not commonly implemented in design at present. The basic problem lies in the determination of initial imperfections and in the subsequent analysis of their influence on limit states and other monitored structural properties.

2 Theoretical Model
The analysis subject is the ultimate limit state of a steel-concrete column in compression the system length of which is equal to its critical length, \( L = L_{cr} = 3 \text{ m} \). The column consists of steel profile HEA140 encased with high strength concrete, see Fig. 1.

The load \( F \) acting on the column consists of load \( F_S \), which is carried by the steel section, and load \( F_C \), which is carried by the concrete section, i.e. \( F = F_S + F_C \). Let us assume that the strut is produced in the shape affine to eventual buckling, with deflection at mid length denoted as \( e_0 \). The maximum deflection mid-span of a strut \( e \), which is loaded by axial force \( F \) in its elastic state, may be determined according to Timoshenko [4] as:

\[
e = \frac{e_0 F}{F_{cr} \left( 1 - \frac{F}{F_{cr}} \right)}
\]

(1)

where

\( F \) is the load acting on the column and \( F_{cr} \) is Euler's critical force \( F_{cr} = \pi^2 EI / L^2 \). In accordance with article 6.7.3.1 (3) of Standard EN 1994-1-1: 2006, the effective elastic flexural rigidity \( EI \) of the steel-concrete column which is given according to the formula listed below may be used for short term loading.

\[
EI = E_S \cdot I_S + K_E \cdot E_C \cdot I_C
\]

(2)

where

\( I_S \) and \( I_C \) are second moments of area in the plane of bending of structural steel and concrete (without consideration to cracking),

\( E_S \) is the modulus of the steel elasticity,

\( E_C \) is the tangent modulus of the elasticity of concrete,

\( K_E \cdot E_C \cdot I_C \) is the effective flexural rigidity of the concrete section.

\( E_S \cdot I_S \) is the effective flexural rigidity of the steel section.

The values of forces \( F_C \) and \( F_S \) can be obtained from the condition that deflection at strut mid span \( e \) equals the...
deflection of the steel section \( e_S \), which is equal to the deflection \( e_C \) of the concrete section, i.e. \( e = e_S = e_C \).

\[
e = \frac{e_0}{1 - \frac{F_S}{F_{cr,S}}} = \frac{e_0}{1 - \frac{F_C}{F_{cr,C}}}
\]

where \( F_{cr,S} \) is Euler's critical force of the steel section \( F_{cr,S} = \pi^2 E_S I_S / L_s^2 \), \( F_{cr,C} \) is Euler's critical force of the concrete section \( F_{cr,C} = \pi^2 \cdot K_E \cdot E_C I_C / L_s^2 \). The load carried by the steel section \( F_S \) and the load carried by the concrete section \( F_C \) with parameter \( K_E \) can be determined from the above listed mathematical dependencies:

\[
F_S = F \cdot E_C \cdot I_C / EI
\]

\[
F_C = F \cdot K_E \cdot E_C \cdot I_C / EI
\]

Stresses in the steel and concrete sections are determined according to the principles of elasticity. The load-carrying capacity of the steel member is given by the yield strength attained in the most stressed section. The load-carrying capacity of the concrete section is given as the cubic strength in the most compressed section or as 10% of the cubic strength in the most stressed section. The load-carrying capacity of the steel-concrete column is given as the minimum of the above listed values.

### 3 Input Quantities

The value of initial imperfection \( e_0 \) was considered according to the tolerance limits of Standard EN 10034 (1995). The steel grade S355 and the concrete grade C50/60 were used. The modulus of elasticity of concrete was considered in compliance with Standard Eurocode 2 (2006) according to the formula:

\[
E_c = 22 \cdot \left( \frac{5 \cdot f_{cc}}{6 \cdot 10} \right)^{0.3} \cdot \Theta_{Ec}
\]

where:

- \( E_c \) - tangent modulus of elasticity [GPa],
- \( f_{cc} \) - cubic strength [MPa],
- \( \Theta_{Ec} \) - non-dimensional coefficient of the (i) influence of concrete constituents (namely aggregates) and (ii) variance of cylindrical and cubic strength.

We have incomplete information at our disposal without corresponding mathematical description of parameters \( K_E \) and \( \Theta_{Ec} \).

### 4 Fuzzy Analysis of Carrying Capacity

One of the most important principles enabling the transformation of any arbitrary operation in the classic set into an operation in the fuzzy set is that of the general extension principle \([5, 6, 1]\). For fuzzy numbers \( K_E \) and \( \Theta_{Ec} \subseteq R \), the general extension principle may be written as: Let \( K_E, \Theta_{Ec} \) be convex fuzzy numbers and \( f: y = f(K_E, \Theta_{Ec}) \) be a given binary function. The degree of membership \( \mu_y \) of fuzzy number \( y \) may then be obtained acc. to the relation (7).

\[
\mu_y(K_E, \Theta_{Ec}) = \sqrt{\left( \mu_1(K_E) \wedge \mu_2(\Theta_{Ec}) \right)}
\]

The result obtained is the fuzzy number \( y \), containing elements with degree of membership \( \mu_y \), which is given as the supremum (maximum) from the minima (\( K_E, \Theta_{Ec} \)) for all \( K_E, \Theta_{Ec} \), for which \( f(K_E, \Theta_{Ec}) = y \).

Three basic operations are introduced for fuzzy sets – intersection, union and supplement (a number of other operations are sometimes listed, e.g., as limited sum, limited difference, power, probabilistic sum, Lukasiewicz’s operations of intersection and union).

Relation (7) is however quite inadequate for direct evaluation. Due to this fact, basic fuzzy arithmetic was worked out for simple problems, in which the response of the system (structure) may be expressed by a polynomial function. Fuzzy analysis in the form of the general extension principle (similarly to stochastic methods) has some limitations in more complex problems due to the need to perform a high number of combinations of input data. A solution of this problem is provided through the approximation of the structural response, the so-called surface response function in the simplest possible form. This makes the fuzzy analysis to be a very strong tool utilizable whenever provision for uncertainty, which is not of stochastic character, is
needed. No relevant information is available for the parameter $K_E$ because it cannot be determined from measurements on a higher number of steel-concrete columns. The parameter $K_E$ is given in EN 1994-1-1: 2006 by $K_E=0.6$. No relevant information is available for the parameter $\Theta_{E_c}$, either. For the fuzzy analysis, we shall consider that the parameters $K_E$ and $\Theta_{E_c}$ have fuzzy numbers with symmetrical triangular membership functions, see Fig. 2 and Fig. 3.

Fig. 2: Fuzzy number of parameter $K_E$

Fig. 3: Fuzzy number of parameter $\Theta_{E_c}$

The fuzzy number of the load-carrying capacity was determined using the general extension principle (7), see Fig. 4. The membership function has an asymmetrical form and the support of the fuzzy number is within the interval 875.8 kN to 1256.1 kN.

Fig. 4: Fuzzy number of load carrying capacity

The fuzzy number of deflection is singleton because the ultimate limit state is limited by yield strength.

5 Conclusion

The results of fuzzy analysis quantify the dependence of load-carrying capacity on the change of coefficients $K_E$, $\Theta_{E_c}$. The output asymmetric and low non-linear membership function vs. triangular symmetric membership functions of coefficients $K_E$, $\Theta_{E_c}$ are obtained. This information is very valuable because it quantifies the non-linear dependence between the coefficients of model uncertainties $K_E$, $\Theta_{E_c}$ and also the theoretical load-carrying capacity.

At present, the basic method for the evaluation and verification of limit states according to EUROCODE is the partial safety factors method. In this regard, the elaboration of uncertainty studies from the perspectives of transparency and verification of procedures implemented in practice is topical [2]. The probabilistic assessment of reliability enables comparison, generalization and further improvement of normative procedures. Recently, alternate approaches in the representation of uncertainty based on the theory of fuzzy sets, the theory of possibilities and the Dempster-Shafer theory are more frequently applied in addition to traditional probabilistic methods [3].

The article was elaborated within the framework of projects of GACzR 103/07/1067 and CIDEAS 1M68407700001(1M0579).

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