Theoretical Study of Tearing in Hydroforming Deep Drawing Process

V. ARBABI, S.A. ZAHEDI
Department of Mechanical Engineering
University of Zabol
Zabol, P.O. Box. 98615-538
IRAN
ARBABI@UOZ.AC.IR, ABOLFAZLZAHEDI@YAHOO.COM

Abstract: - An axisymmetric analysis was developed to investigate the tearing phenomenon in cylindrical Hydroforming Deep Drawing (HDD). By theoretical method, the critical fluid pressures which result rupture in the workpiece were studied. The effects of anisotropy, drawing ratio, sheet thickness and strain hardening exponent on tearing diagram were also investigated.

Key-Words: - Hydroforming Deep Drawing (HDD)- Pressure Path- Axisymmetric analysis- Finite element simulation

1 Introduction
Hydroforming Deep Drawing (HDD) is one of the metal forming processes that is used in industry to produce complex sheets with high Limiting Drawing Ratio (LDR). Schematic of cylindrical cup drawing with HDD process is shown in Fig.1. A pressurized fluid is employed in front of the workpiece. As the punch travels, the workpiece begins to deform into a cylindrical cup [1].

Some of the advantages of sheet hydroforming are, improving the material formability, reduction of friction force, the accuracy of the forming part and the reduction of forming stages because of improvement of Limiting Drawing Ratio LDR[2-4]. Analysis of tearing phenomenon in hydroforming was studied by many researchers [5-8]. Generally, two kinds of material failure caused by inappropriate fluid pressure were identified. The failure by wrinkling at the lip area (The area that the blank is in contact with die and blank holder) results from insufficient fluid pressure, and the failure by rupture on the top of the cup results from excessive fluid pressure [9].

In this paper, a suitable punch-stroke pressure path was obtained theoretically that avoids rupture in HDD process. Finally effects of anisotropy, drawing ratio, sheet thickness and strain hardening in tearing diagram investigated.

2 Problem Formulation
A number of assumptions are made in this analysis that are:
• The thickness of the workpiece remains constant through out the process.
• The principal strain axes do not rotate.
• The tresca yield criterion is satisfied, and the fluid pressure $p$ is smaller than the radial stress $\sigma_r$ and the tangential stress $\sigma_\theta$. This assumption yields $\sigma_r - \sigma_\theta = \sigma_v$.

For axisymmetric problems the polar equilibrium equation in the rim area is [10].

$$\frac{d}{dr}(r(\sigma_r)) + \frac{l}{r}((\sigma_r - \sigma_\theta)) + f(p) = 0$$

(1)
where \( f(p) \) is the friction force in the rim area. By using the tresca criterion and a power law for the material properties, Eq. (1) can be rewritten as

\[
\frac{d}{dr}(\tau \sigma_r) + \frac{t}{r} (\sigma_0(\varepsilon_r)^n) + f(p) = 0 \tag{2}
\]

By using the normal anisotropy of the material in the formulation, the equivalent strain rate is denoted as

\[
\varepsilon_r^a = \frac{1+R}{\sqrt{1+2R}} \left[ \varepsilon_r^{*2} + \frac{2R}{1+R} \varepsilon_0^a + \varepsilon_0^{a*2} \right] \tag{3}
\]

Since it was assumed that the axis of strain does not rotate and by considering that the material follows volume constancy in the plastic deformation, the effective strain \( \varepsilon_r^a \) is obtained by integrating Eq. (3)

\[
\varepsilon_r^a = \sqrt{R_e \varepsilon_r} \tag{4}
\]

By substituting Eq. (4) into Eq. (2), we have

\[
\frac{d}{dr}(\tau \sigma_r) + \frac{t}{r} \sigma_0(\varepsilon_r)^n \sigma_r^a + f(p) = 0 \tag{5}
\]

Referring to Fig. 2 and substituting Eq. (5) for tension in area 1 we have

\[
\sigma_r^{(1)}(r) + \int_{r_0}^{r} \sigma_0(R_r)^{n/2}(\varepsilon_r^{(1)})^n \frac{dr}{r} + \frac{t}{r} f(p) dr = 0 \tag{6}
\]

By simplifying Eq. (6) for \( \sigma_r^{(1)} \) we have

\[
\sigma_r^{(1)}(r) = \frac{1}{r} \int_{r_0}^{b} \sigma_0(R_r)^{n/2}(\varepsilon_r^{(1)})^n \frac{dr}{r} + \frac{t}{r} f(p) dr \tag{7}
\]

The value of strain in area 1 is obtained by

\[
\varepsilon_r^{(1)} = \ln \left( \frac{G(r,h,p)}{r} \right) \tag{8}
\]

where

\[
G(r,h,p) = a \left[ 1 + \left( \frac{L}{a} \right)^2 - (1 + \frac{L}{a})^2 + \frac{\pi p}{a} (1 + \frac{L}{a}) \right]^{\frac{1}{2}} - 2(\frac{h}{a})^2 + 2(\frac{h}{a}) \frac{p}{a} H_v \cdot \left( \frac{h}{a}, \varepsilon_r \right) \tag{9}
\]

Here \( H_v \) is heaviside unit function. In the same manner, the stress in area 2 is as follows

\[
\sigma_r^{(2)} = \sigma_r^{(1)}(r = a + \rho) + \int_{r}^{a + \rho} \sigma_0(\varepsilon_r^{(2)})^n R_e a/2 dr \tag{10}
\]

in which the strain in area 2, \( \varepsilon_r^{(2)} \), is

\[
\varepsilon_r^{(2)} = \ln \left( \frac{F(r,h,p)}{r} \right) \tag{11}
\]

where \( F(r,h,p) \) is

\[
F(r,h,p) = a \left[ 1 + 2(\frac{L}{a})^2 + (\frac{\pi p}{a}) \frac{1}{2} - \beta - 2(\frac{h}{a}) \cos \beta + 2(\frac{h}{a}) \frac{p}{a} H_v \cdot \left( \frac{h}{a}, \varepsilon_r \right) \right]^{\frac{1}{2}} \tag{12}
\]

In Eq. (12), \( \beta \) is an angle shown in Fig. 3 and is obtained by

\[
\beta = \sin^{-1} \left( \frac{d + \rho - r}{\rho} \right) \tag{13}
\]

If the bending stress is neglected and the radial stress \( \sigma_r \) is calculated for the pure radial drawing case, equilibrium of area (2) become
\[ \sigma_r^{(2)}(2\pi at) = P \pi [(a + \rho)^2 - (a)^2] \]  \hspace{1cm} (14)

### 2.1 Analysis of tearing in hydroforming deep drawing

Tearing occurred at the upper part of the punch; just at the beginning of draw workpiece. It is caused by the firm contact between the workpiece and punch due to circumferential compressive fluid pressure [11]. This area is the transition between regions 2 and 3. The rate of tangential strain at the wall of the punch is zero. It mines

\[ \varepsilon_\theta^* = 0 \]  \hspace{1cm} (15)

By combining Eq. (15) and Eq. (3) and solving, then we have

\[ d\varepsilon_r^{(3)} = Z d\varepsilon_r^{(3)} \]  \hspace{1cm} (16)

In [12] predicted the instability of anisotropic material under biaxial plane strain conditions as follows:

\[ \frac{d\sigma_r}{d\varepsilon_r} = \frac{\sigma_x}{Z} \]  \hspace{1cm} (17)

Where

\[ Z = \frac{(1 + R)}{\sqrt{(1 + 2R)}} \]  \hspace{1cm} (18)

So, if Eq.(17) is in Eq.(10) will have

\[ \sigma_{r critical} = \sigma_0 n Z^{n+1} \]  \hspace{1cm} (19)

Obviously, necking and rupture occur when the radial load reaches a maximum value.

\[ F_{max} = \sigma_r^{(3)}(r = a) t \]  \hspace{1cm} (20)

By equating Eqs. (10) and (19) we have

\[ \sigma_0 n Z^{n+1} = \sigma_r^{(1)}(r = a) + \int \frac{\sigma_0 (\varepsilon_r^*) R^{n/2}}{r} dr \]  \hspace{1cm} (21)

By considering Eqs. (14) and (21) we can obtain the fluid pressure causing rupture in the workpiece. The material properties and the process parameters are given in Tables 1 and 2, respectively.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Material properties of the Analytical Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>St 14</td>
</tr>
<tr>
<td>Thickness, t (mm)</td>
<td>1</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Young’s modulus, E (GPa)</td>
<td>210</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Material processes of Analytical Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Punch diameter, a (mm)</td>
<td>35</td>
</tr>
<tr>
<td>Blank diameter, b (mm)</td>
<td>75</td>
</tr>
<tr>
<td>Friction coefficient, ( \mu )</td>
<td>0.08</td>
</tr>
<tr>
<td>Strain hardening exponent, ( n )</td>
<td>0.27</td>
</tr>
<tr>
<td>( \sigma_0 ) (MPa)</td>
<td>625</td>
</tr>
</tbody>
</table>

### 3 Results

The tearing diagrams obtained by the analytical equations shown in Fig.4.

![Tearing diagram](image)

Analytical method suggested that a counter-pressure history with a somewhat smaller pressurization at the initial stage and a larger one at the later stage would normally result in a proper cylindrical cup product. By using analytical equations, the effects of parameters in tearing diagram were understood easily that is shown in Fig. 5. This picture investigates the effects of anisotropy, drawing ratio, sheet thickness and strain hardening component on tearing diagram.
4 Conclusion

The results of theoretical of the HDD showed for product proper cylindrical cups we should use smaller pressurization at the initial stage and a larger one at the later stage.

References: