Variance-Based Sensitivity Analysis of Stability Problems of Steel Structures using Shell Finite Elements and Nonlinear Computation Methods

ZDENEK KALA and JIRI KALA
Department of Structural Mechanics
Brno University of Technology, Faculty of Civil Engineering
Veverí Str. 95, Brno
CZECH REPUBLIC
kala.z@fce.vutbr.cz; kala.j@fce.vutbr.cz http://www.vutbr.cz

Abstract: - The present paper deals with an analysis of the influence of initial imperfections on the ultimate limit state of slender steel members under axial compression. The theoretical analysis is based on a geometrical and material non-linear finite element method; the steel structure is modelled using SHELL elements. The ANSYS program was applied. Sensitivity analysis is used to determine the sensitivity of strut load-carrying capacity with respect to the variance of initial imperfections. The methodology based on Sobol’s variance is used for the sensitivity analysis. Input imperfections are of random origin. Majority of the input imperfections were measured and their random realizations were computed using their histograms whilst employing the LHS method. The LHS method was used in the evaluation of sensitivity indices. The computation model elaborated is unique with regard to its numerically demanding character.

Key-Words: steel, imperfections, shell elements, structures, stability, sensitivity analysis, reliability.

1 Introduction
The yield strength is a basic strength characteristic, which is fundamentally limited by the ultimate limit state of steel structures. Characteristic values of yield strength are listed in the standards for design of steel structures. Yield strength is a dominant strength characteristic of struts under tension, bending and torsion when elastic behaviour is presumed. Real yield strength values are obtained through experimental research [3, 7, 8] and represent the basic input data of a number of reliability analysis [5], [15]. The majority of compressed steel struts are long and slender and the increasing strain does not result in the attainment of yield strength due to loss of stability and collapse of the strut. In comparison with other types of strain, the stable collapse is not characterised by increase in deformation, and occurs suddenly. The strut load carrying capacity is limited by initial imperfections, including:

- Geometric deviations: initial strut axis curvature, non-adherence to theoretical profile shape (despite being within limits of permitted tolerances) etc.
- Structural deviations: namely the residual stress in an unloaded strut arising during welding, straightening or cooling of rolled products.
- Construction imperfections: such as improperly acting joints, or on the contrary fixed ends in real structures resulting in structural behaviour deviation from the ideal state.

2 Initial Imperfections
In the limit state method acc. to the EUROCODE standard the design load carrying capacity value $R_d$ is considered as the lower quantil (for reliability index $\beta=3.8$ it is the 0.1 percentile) evaluated from the random load carrying capacity $R$, See Fig. 1.

![Fig. 1: Ultimate limit state](image)

The variance of the load carrying capacity of a structure is influenced by the variance of input imperfections. The variance of input imperfections is influenced by production quality, which should be monitored in all countries of the European Union. Availability of these data presents a problem. The basic methodology of reliability assessment utilizing random input characteristics is listed in EN1990.

2.1 Yield Strength
Mechanical characteristics of steel S235 and geometrical characteristic of IPE profile were published in [3, 7, 8]. Mechanical characteristics of U profile of steel S355
were published in [9]. Current mechanical characteristics of steel are published in [6]. Statistics of real mechanical properties of structural steel in the above listed publications were evaluated from characteristic and design values and verified partial safety factors of reliability. Histograms of yield strength of steel S235 and S355 are depicted in Fig. 2 and Fig. 3.

Fig. 2: Histogram of yield strength of steel S235

Fig. 3: Histogram of yield strength of steel S355

2.2 Geometrical Imperfections

The observance of geometric dimensions and the weight of bars are eminently checked in production. Tolerances on geometrical shape and dimensions are listed in the Tolerance Standard EN 10034:1993. The permitted limit deviation of the actual weight from the theoretical weight is given as + 4 % for individual bars. The tolerance limits of weight and geometry are in discordance. The variables $h$, $b$, $t_1$, $t_2$, see Fig. 4, were statistically evaluated from experimentally obtained data. One of the main geometrical characteristics is the relative cross-sectional area, which is given as the ratio of the real cross-sectional area obtained from the measurement of the cross-sectional geometry to the nominal value of the cross-sectional area.

Fig. 4: IPE 220 Cross-section and FEM model

The results of the relative area, which were evaluated for hot-rolled steel profiles IPE 160 to IPE 240, are depicted in Fig. 5.

Fig. 5: IPE 220 Cross-section and FEM model

The initial curvature of member axis was modelled in the form of one half sine curve. According to results of experimental research [2], the dominant shape of initial curvature is given as one half-wave of the sine function. It was considered with initial curvature in the plane of primary bending in the direction of axis $y$. The amplitude $e_0$ of maximal initial imperfection of the axis was introduced as the random quantity with Gauss density function. In the event that the amplitude $e_0$ is measured for a higher number of struts, the positive and negative realizations should occur with the same frequency.

2.3 Residual Stress

For hot-rolled steel girders, the unsteady cross-section cooling down takes place after hot rolling. The regions at the flange edge and at the web middle cool down more
rapidly, the primary tendency being the stabilization of volume changes during shrinkage. The following, slower cooling down and shrinkage of thermally more exposed internal cross-section parts in the contact region of flanges and web causes the compressive stress in the regions previously cooled down and volume stabilized. Contemporarily, shrinkage stress takes place at the flange edge and at the web middle. These primary technology states of stress components can continue redistributing due to beam loading and unloading.

Fig. 6: Model of residual stress

Residual stress was introduced with mean value 80 MPa and standard deviation 40 MPa, with triangular distribution both on flanges and web [1].

2.4 Young’s modulus
According to experimental measurements, the Young’s modulus $E$ can be considered with the Gaussian distribution, mean value being 210 GPa, and standard deviation, 12.6 GPa [2, 11].

3 Computation Model
3.1 Shell Finite Elements
The beam was meshed in the programme ANSYS. In general, the beam was modelled of thin-walled elements, type SHELL 181, i.e., the thin-walled effect was taken into account. The symmetry was used with regard to the very demanding character of the problem solved. In the bar half in the symmetry plane, we supposed the shift prevention in all cross-section nodes in direction of axis $X$, and rotation around axes $Y$ and $Z$. On the second edge of the bar half solved, we prevented the shifts of nodes in direction of the axis $Y$ on the flange of profile IPE240. On the lower flange of that edge, we prevented the shifts in the direction of the axis $Z$. The upper flange was left free. Within the framework of each run of the LHS method [10], the load-carrying capacity was solved by non-linear computation by means of the programme system ANSYS. A very detailed FEM model was used, See Fig. 7. Also the influence of local imperfections which can contribute to the load-carrying capacity loss was thereby taken into consideration.

Fig. 7: Strut under compression (elements SHELL 181)

3.2 Nonlinear Computation Methods
In geometrically and material non-linear FEM solution, the Euler method was applied based on proportional loading in combination with the Newton-Raphson method. We determined the load-carrying capacity as the loading constant at which the matrix determinant of tangential stiffness $K_t$ of the structure would approach zero with certain accuracy. As we required the determination of load-carrying capacity with accuracy 0.1 %, it was necessary to use, with the Euler method, automatic control of the loading step. We supposed bilinear kinematic material strengthening. Further on, we also have supposed that the initial steel plastification occurs when Mises stress exceeds yield strength.
3.3 Input random variables

All the input random quantities were considered with the Gaussian density function, their statistical characteristics being described in the chapter 2. For geometrical characteristics of cross-section dimensions, it was supposed that the nominal (characteristic) value was equal to the mean value.

Table 1: Statistic characteristics of the input variables

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Mean value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength of S235 $f_y$</td>
<td>297.3 MPa</td>
<td>16.8 MPa</td>
</tr>
<tr>
<td>Yield strength of S355 $f_y$</td>
<td>394.5 MPa</td>
<td>19.809 MPa</td>
</tr>
<tr>
<td>Young’s modulus $E$</td>
<td>210 GPa</td>
<td>12.6 GPa</td>
</tr>
<tr>
<td>Cross-sectional depth $h$</td>
<td>220 mm</td>
<td>0.975 mm</td>
</tr>
<tr>
<td>Cross-sectional width $b$</td>
<td>110 mm</td>
<td>1.093 mm</td>
</tr>
<tr>
<td>Web thickness $t_w$</td>
<td>5.9 mm</td>
<td>0.247 mm</td>
</tr>
<tr>
<td>Flange thickness $t_f$</td>
<td>9.2 mm</td>
<td>0.421 mm</td>
</tr>
<tr>
<td>Residual stress $rs$</td>
<td>80 MPa</td>
<td>40 Mpa</td>
</tr>
<tr>
<td>Amplitude of initial axis curvature $e_0$</td>
<td>0 mm</td>
<td>0.767 L/1000</td>
</tr>
<tr>
<td>Residual stress $rs$</td>
<td>80 MPa</td>
<td>40 Mpa</td>
</tr>
</tbody>
</table>

It has been assumed that 95% of the realizations of random imperfection $e_0$ of the strut of profile IPE 220 are found within the tolerance limits 0.15% $L$ ($L$ is length of the strut) mm of the standard EN 10034.

The aim of the studies is a comparison of the influence of imperfections from Table 1 on the load-carrying capacity of struts made of steels S235 and S355 with identical non-dimensional slenderness. The non-dimensional slenderness is given in EUROCODE 3 by:

$$\lambda = \sqrt{\frac{A \cdot f_y}{F_{cr}}}$$  \hspace{1cm} (1)

where $A$ is the cross-sectional area, $f_y$ is the characteristic value of yield strength, and $F_{cr}$ is the Euler critical force of bilaterally hinged strut.

4 Sensitivity Analysis

The coherent concept of sensitivity analysis enabling an analysis of the influence of arbitrary subgroups of input factors (doubles, triples, etc.) on the monitored output was worked out by the Russian mathematician Ilja M. Sobol [12, 13, 14].

Sobol’s first order sensitivity indices may be written in the form:

$$S_i = \frac{V\{E(Y|X_i)\}}{V(Y)}$$  \hspace{1cm} (2)

Sobol proposed an alternate definition $S_i = corr\{Y, E(Y|X_i)\}$ based on the evaluation of the correlation between output random variable $Y$ and the conditional random arithmetical mean $E(Y|X_i)$. Analogously, we can write the second order sensitivity indices:

$$S_{ij} = \frac{V\{E(Y|X_i,X_j)\}}{V(Y)} - S_i - S_j$$  \hspace{1cm} (3)

Sensitivity index $S_{ij}$ expresses the influence of doubles on the monitored output. Other Sobol’s sensitivity indices enabling the quantification of higher order interactions may be expressed similarly.

$$\sum_i S_i + \sum_{i < j} S_{ij} + \sum_{i < j < k} S_{ijk} + \ldots + S_{123 \ldots M} = 1$$  \hspace{1cm} (4)

The number of members in (4) is $2^M - 1$, i.e., for $M=3$, we obtain 7 sensitivity indices $S_1, S_2, S_3, S_{12}, S_{13}, S_{123}$; for $M=10$, we obtain 1023 sensitivity indices; it is excessively large for practical usage. The main limitation in the determination of all members of (4) is the computational demanding character.

Possible motivations for sensitivity analysis are [10]:

- Model corroboration. Is the inference robust? Is the model overly dependent on fragile assumptions?
- Research prioritization. Which factors are most deserving of further analysis or measurement? \Rightarrow Factor prioritization setting.
- Model simplification. Can any factors or compartments of the model be fixed or simplified? \Rightarrow Factor fixing setting.
- Identifying critical or otherwise interesting regions in the space of the input factors. Identifying the factors which interact and which may thus generate extreme values. This is important, for example, in structure reliability.
- Prior to parameter estimation, to help set up the (actual or numerical) experiment in those conditions in which the output sensitivity to the factor to be estimated is the greatest.

Note that the model simplification underpinned by the “factor fixing” setting can become very important when models need to be audited, for example, in the face of scientific controversy or for use in policy assessment [10]. The LHS (Monte Carlo type) method was applied to calculation of sensitivity indices [10]. The model output $Y$ is the load-carrying capacity calculated in each run of the LHS method.
The calculation process by the LHS method can be practically clarified on the calculation of the first order sensitivity indices (2). In the first step, $N$ realizations of the quantity $X_i$, i.e. $X_i(1), \ldots, X_i(N)$ were generated. After that, $K$ realizations of vector $X_i$ (all except for the $i$th one), i.e. $X_i(j, 1), \ldots, X_i(j, K)$ were generated for each realization $X_i(j), j = 1, \ldots, N$. Let us notice that $K$ can but need not be equal to $N$. Further on, $E(Y|X_i)$ must be determined for each $j$:

$$E(Y|X_i) \approx m(j) = \frac{1}{K} \sum_{k=1}^{K} f(X_i(j), X_{i-1}(j, k))$$  \hspace{1cm} (5)

Approximately, the numerical value $V(E(Y|X_i))$ can be obtained according to the relation:

$$V(E(Y|X_i)) \approx \frac{1}{N-1} \sum_{j=1}^{N} (m(j)-\overline{m})^2$$  \hspace{1cm} (6)

where $\overline{m}$ is an assessment of arithmetical mean. In this study, $N=K=30000$ simulation runs of the LHS method were applied. The variance $V(Y)$ of load-carrying capacity is calculated under the assumption that all the input imperfections are considered to be random ones; 30000 simulation runs were applied, as well. It was proceeded similarly when calculating the second order indices (3).

5 Sensitivity Analysis Results

The results of the load-carrying capacity sensitivity analysis are presented in Fig. 8, Fig. 9, Fig. 10 and Fig. 11. The results of sensitivity analysis of struts with slenderness $\overline{\lambda}=0.6$ and $\overline{\lambda}=1.0$ are compared. The partial variances are applied to compute sensitivity indices which, in turn, are used to measure importance of each factor.

Fig. 8: Sensitivity analysis for $\overline{\lambda}=0.6$, S235

In connection with the nonlinear FEM, extremely demanding calculations are concerned requiring very much computer time even on the most advanced multiprocessor computers.

6 Conclusion

For the struts with slenderness $\overline{\lambda}=0.6$, yield strength is the dominant quantity; for the beams with slenderness $\overline{\lambda}=1.0$, the amplitude of initial curvature of the strut axis represents the dominant imperfection. The flange
The article was elaborated within the framework of projects of GACzR 103/07/1067, AVČR IAA201720901 and MSM0021630519.

References: