The Performance Analysis of MRC Combiner Output Signal in the presence of Weibull and Log-normal fading

PETAR NIKOLIĆ
Tigartyres, Pirot
Nikole Pašića 213
SERBIA
p.nikolic@tigartyres.com

DRAGANA KRSTIĆ, GORAN STAMENOVIĆ, DUŠAN STEFANOVIĆ,
MIHAJLO STEFANOVIĆ
Faculty of Electronic Engineering
University of Niš
Aleksandra Medvedeva 14
SERBIA
dragana.krstic@elfak.ni.ac.rs

Abstract: - The performance of dual diversity systems in the presence of Weibull and log-normal fading over two uncorrelated branches will be presented in this paper. The performance measures of fading communication systems such as Probability density function (PDF) of Signal to noise ratio (SNR), Amount of fading (AF) and Outage probability (P_{out}) will be calculated and graphically represented for Maximal Ratio Combining. The results will be shown graphically for different signal and fading parameters values.

Key-Words: - Log-Normal fading, Weibull fading, Maximal Ratio Combining, Amount of fading, Outage probabilty

1 Introduction

In wireless communications, fading causes difficulties in signal recovery. When a received signal experiences fading during transmission, its envelope and phase both fluctuate over time. The overall fading process for land mobile satelite systems is a comlex combination of multipath fading and a log-normal shadowing. Multipath fading caused by the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components. This type of fading is relatively fast and is responsible for the short-term signal variation. In terrestrial and satellite land-mobile systems, the link quality is also affected by slow variation of the mean signal level due to the shadowing from terrain, buildings and trees.

One of the methods used to mitigate these degradation are diversity techniques, such as space diversity [1], [2]. Diversity combining has been considered as an efficient way to combat multipath fading and improve the received signal-to-noise ratio (SNR) because the combined SNR compared with the SNR of each diversity branch, is being increased. In this combining, two or more copies of the same information-bearing signal are combining to increase the overall SNR.

A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal or Nakagami-lognormal are considered in [3-5]. Up to now, composite multipath/shadowed fading environment modelled as Weibull-lognormal, has been considered only in several papers [6, 7]. The Weibull distribution plays an important role in several scientific fields, but it has become recently the topic of wireless communications theory [8], particularly with mobile radio systems operating in the 800/900 MHz frequency range. The Weibull model exhibits an excellent fit to experimental fading channel measurements, for both indoor [9] and outdoor [10] environments.

The use of log-normal distribution [1], [11] to model the average power which is random variable due to shadowing doesn't lead to a closed form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) at the receiver. This pdf can be evaluated numerically using some of software tools as Matlab and Mathematica. Obtain numerical results will be shown graphically.

This paper presents Maximal-Ratio Combining procedure for communication system where the diversity combining is applied over two uncorrelated branches (*ρ=0*), which are given as channels with Weibull and log-normal fading.

In this environment the receiver does not average the envelope fading due to multipath but rather, reacts to the instantaneous composite multipath/shadowed signal. This is often the scenario in congested downtown areas with slow-moving pedestrians and vehicles. This type of composite fading is also observed in land mobile satellite systems that are subjected to vegetative or urban shadowing

Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining schemes whose SNR is the sum of the SNR's of each individual diversity branch. MRC is the optimal combining scheme, but its price and complexity are high, since MRC requires cognition of all fading parameters of the channel.

2 System and Channel Models

A dual-branch diversity system over two uncorrelated channels in the presence of Weibull and log-normal fading is being considered. Under these conditions, instantaneous SNR, *p(γ)* is obtained by averaging the instantenous Weibull fading average power over the conditional pdf (probability density function) of the log-normal shadowing, which results with combination of with combination of Weibull distribution (for fast multipath feding) and log-normal distribution (for shadowing) [1,12]:

$$p(\gamma/\Omega)=\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{c/2}\gamma^{c/2-1}\exp\left[-\left(\frac{\gamma}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right],\quad \gamma\geq 0 \qquad (1)$$

$$p_{\Omega}(\Omega)=\frac{\xi}{\sqrt{2\pi}\sigma_i\gamma_i}e^{-\frac{(10\log_{10}\gamma_i-\mu_i)^2}{2\sigma_i^2}} \qquad (2)$$

$$p(\gamma)=\int_0^{\infty}p_{\gamma}(\gamma/\Omega)p_{\Omega}(\Omega)d\Omega \qquad (3)$$

Substituting Eq. (1) and Eq. (2) in Eq. (3), *p(γ)* can be written as:

$$p(\gamma)=\int_0^{\infty}\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{c/2}\gamma^{c/2-1}\exp\left[-\left(\frac{\gamma}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right]\cdot \cdot\left\{\frac{\xi}{\sqrt{2\pi}\sigma\Omega}\exp\left[-\frac{(10\log_{10}\Omega-\mu)^2}{2\sigma^2}\right]\right\}d\Omega,\quad \gamma\geq 0 \qquad (4)$$

where $\xi=10/\ln 10=4.3429$, μ_i (db) is mean of $10\log_{10}\gamma$, σ_i(db) is standard deviation of $10\log_{10}\gamma$, and *c* is Weibull shape parameter.

In Fig 1, 2. and 3. the pdf curves *p(γ)* are shown for various values of the factors μ, σ and *c*. For the case where *c=2*, an overall fading consists of Rayleigh and log-normal.

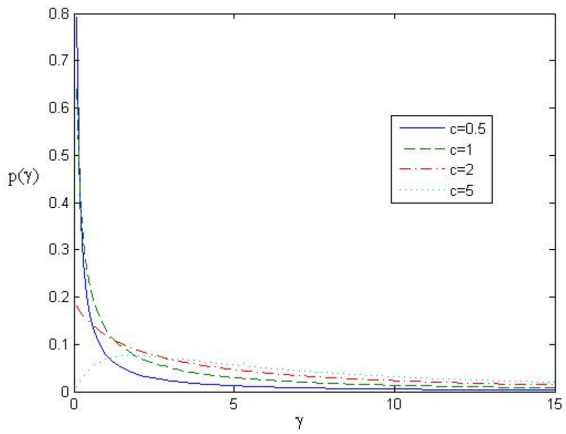

Fig 1. The PDF *p(γ)* for $\mu=10db$, $\sigma=5db$, $c=0.5,1,2,5$

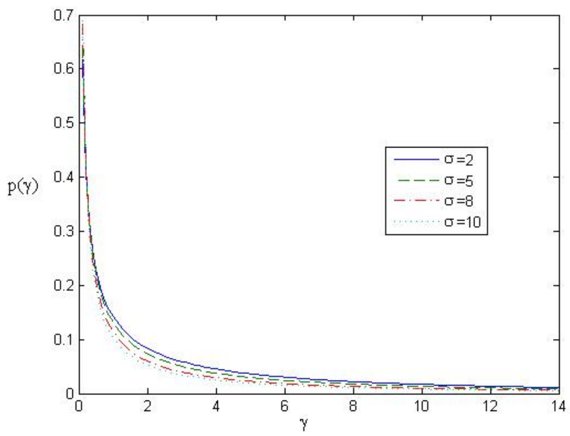

Fig.2. The PDF *p(γ)* for $\mu=10db$, $\sigma=1,5,8,10db$, $c=1$

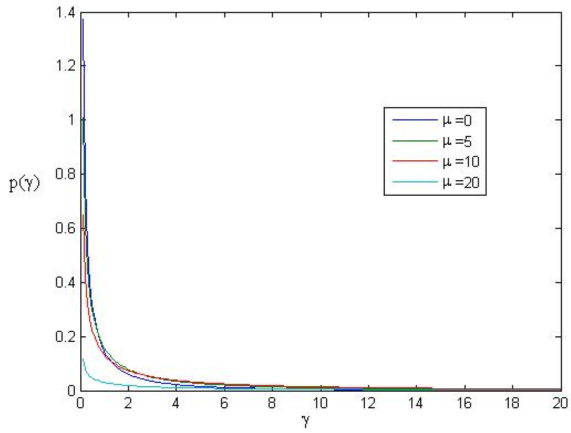

Fig. 3. The PDF *p(γ)* for $\mu = 0{,}5{,}10{,}20db$, $\sigma = 5db$, $c = 1$

For further evaluations it is important to find moments of combined SNR.

N-th moment of the output SNR is given by [11]:

$$E[\gamma^n] = \int_0^\infty \gamma^n p_\gamma(\gamma) d\gamma \tag{5}$$

N-th moment for distribution in Eq. (4) can be written as

$$E[\gamma^k] = \frac{\Gamma\left(1+\frac{2k}{c}\right)}{\Gamma\left(1+\frac{2}{c}\right)^k} \exp\left[\frac{k}{\xi}\mu + \frac{1}{2}\left(\frac{k}{\xi}\right)^2 \sigma^2\right] \tag{6}$$

Amount of fading (AF) is a unified measure of the severity of fading for particular channel model and is typically independent of the average fading power, but it is dependent of the instantaneous SNR.

Amount of fading is defined by:

$$AF = \frac{\mathrm{var}(\alpha^2)}{(E[\alpha^2])^2} = \frac{E[(\alpha^2 - \Omega)^2]}{\Omega^2} \tag{7}$$

$$AF = \frac{E[\gamma^2] - (E[\gamma])^2}{(E[\gamma])^2} = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1 \tag{8}$$

where α is fading amplitude, Ω is average fading power, $E[]$ denotes statistical average and *var( )* denotes variance.

The joint probability density function in the case of two uncorrelated fading channels is given by [13]:

$$p_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2) = p_{\gamma_1}(\gamma_1) p_{\gamma_2}(\gamma_2) \tag{9}$$

3 Maximal Ratio Combining

The total SNR at the output of the MRC combiner is given by:

$$\gamma_{MRC} = \sum_{l=1}^{L} \gamma_l \tag{10}$$

where *L* is number of branches.

The average combined SNR at the MRC (maximal ratio combining) output with two branches is given by:

$$\gamma_{MRC} = \gamma_1 + \gamma_2 \tag{11}$$

Probability density function of the sum of the first and second branch $\gamma_{MRC} = \gamma_1 + \gamma_2$ can be written as

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} p_{\gamma 2}(\gamma_{MRC} - \gamma_1) p_{\gamma 1}(\gamma_1) d\gamma_1 \tag{12}$$

Substituting Eq. (4) in Eq. (11), $p_{\gamma_{MRC}}(\gamma_{MRC})$ can be obtained as

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} \left[\int_0^{\infty} \frac{c_2}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_2}\right)}{\Omega} \right)^{c_2/2} (\gamma_{MRC} - \gamma_1)^{c_2/2-1} \cdot \right.$$

$$\cdot \exp\left[-\left(\frac{\gamma_{MRC} - \gamma_1}{\Omega_2} \Gamma\left(1+\frac{2}{c_2}\right) \right)^{c_2/2} \right] \cdot$$

$$\cdot \left\{ \frac{\xi}{\sqrt{2\pi}\sigma_2\Omega_2} \exp\left[\frac{10\log_{10}\Omega_2 - \mu_2)^2}{2\sigma_2{}^2} \right] \right\} d\Omega_2 \cdot$$

$$\cdot \int_0^\infty \frac{c_1}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_1}\right)}{\Omega_1} \right)^{c_1/2} \gamma_1{}^{c_1/2-1} \exp\left[-\left(\frac{\gamma_1}{\Omega_1} \Gamma\left(1+\frac{2}{c_1}\right) \right)^{c_1/2} \right] \cdot$$

$$\left. \cdot \left\{ \frac{\xi}{\sqrt{2\pi}\sigma_1\Omega_1} \exp\left[\frac{10\log_{10}\Omega_1 - \mu_1)^2}{2\sigma_1{}^2} \right] \right\} d\Omega_1 \right] d\gamma_1 \tag{13}$$

Relatively simple closed form expressions to represent $p_{\gamma_{MRC}}(\gamma_{MRC})$ can not be derived, because Eq. (13) is too complex for tractable communication system analyses. This pdf can be evaluated numerically using some of software tools (Matlab, Mathematica).

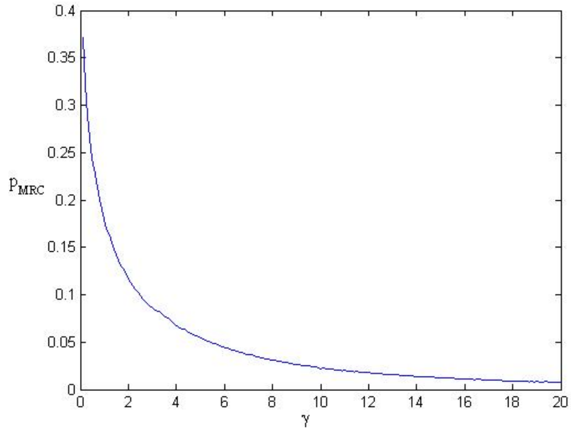

Fig 4. The PDF $p_{\gamma_{MRC}}(\gamma_{MRC})$ for $\mu_i = 5db$, $\sigma_i = 2db$, $c_i = 1$

Very often it is assumed for performance analysis of communication systems, that channel coefficients are uncorrelated and identically distributed. Fig 4. depicts pdf of γ_{MRC} at the output of MRC combiner for two uncorrelated identically distributed channels with identical parameters $\mu_i = 10db$, $\sigma_i = 5db$, $c = 1$.

N-th moment of γ_{MRC} can be expressed as [1]

$$E\left[\gamma^n{}_{MRC}\right] = E\left[(\gamma_1 + \gamma_2)^n\right] \tag{14}$$

Using binomial expansion, (14) can be written as

$$E\left[\gamma^n{}_{MRC}\right] = E\left[\sum_{k=0}^{n}\binom{n}{k}\gamma_1{}^k\gamma_2{}^{n-k}\right] = \sum_{k=0}^{n}\binom{n}{k}E\left[\gamma_1{}^k\gamma_2{}^{n-k}\right] \tag{15}$$

The average combined SNR $\bar{\gamma}_{MRC}$ at the MRC output can be written as

$$\bar{\gamma}_{MRC} = E\left[\gamma^1{}_{MRC}\right] = \bar{\gamma}_1 + \bar{\gamma}_2 \tag{16}$$

The second moment of γ_{MRC} is given by

$$E\left[\gamma^2{}_{MRC}\right] = E\left(\gamma_1{}^2\right) + 2\bar{\gamma}_1\bar{\gamma}_2 + E\left(\gamma_2{}^2\right) \tag{17}$$

The amount of fading can be calculated using Eq. (6), Eq. (16) and Eq. (17) in Eq. (8) as

$$AF_{MRC} = \left[\frac{\Gamma\left(1+\frac{4}{c_1}\right)}{\Gamma\left(1+\frac{2}{c_1}\right)^2}\exp\left[\frac{2}{\xi}\mu_1 + \frac{1}{2}\left(\frac{2}{\xi}\right)^2\sigma_1{}^2\right] + \right.$$
$$+\frac{\Gamma\left(1+\frac{4}{c_2}\right)}{\Gamma\left(1+\frac{2}{c_2}\right)^k}\exp\left[\frac{k}{\xi}\mu_2 + \frac{1}{2}\left(\frac{2}{\xi}\right)^2\sigma_2{}^2\right] +$$
$$+2\exp\left[\frac{1}{\xi}\mu_1 + \frac{1}{2}\left(\frac{1}{\xi}\right)^2\sigma_1{}^2\right]\cdot$$
$$\left.\cdot\exp\left[\frac{1}{\xi}\mu_2 + \frac{1}{2}\left(\frac{1}{\xi}\right)^2\sigma_2{}^2\right]\right]\cdot$$
$$\cdot\left(\exp\left[\frac{1}{\xi}\mu_1 + \frac{1}{2}\left(\frac{1}{\xi}\right)^2\sigma_1{}^2\right] + \right.$$
$$\left. + \exp\left[\frac{k}{\xi}\mu_2 + \frac{1}{2}\left(\frac{k}{\xi}\right)^2\sigma_2{}^2\right]\right)^{-2} - 1 \tag{18}$$

The amount of fading for MRC has less values then single channel receiver for identically distributed channels that is shown in Fig 5.

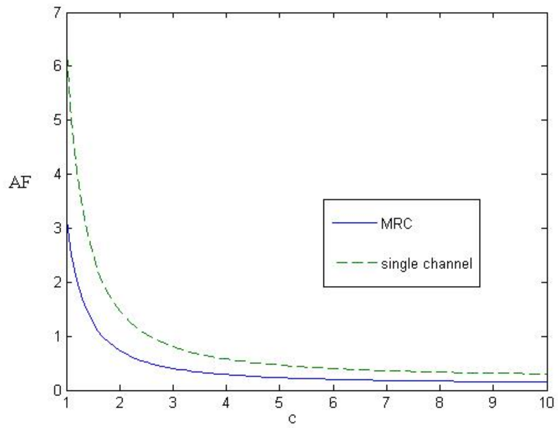

Fig 5. Amount of fading for single channel and MRC receiver for $\mu_i = 5db$, $\sigma_i = 2db$, $c_i = 1$

The outage probabilty is standard performance criterion of diversity systems opereting over fading chanels and it is defined as the probability that the instantaneous error rate exceeds a specified value, or equivalently, that combined SNR of MRC falls below a predetermined threshold γ_{th}.

$$P_{out}^{MRC}(\gamma_{th}) = P\left[\gamma_{MRC} = \gamma_1 + \gamma_2 \le \gamma_{th}\right] \tag{19}$$

$P_{out}^{MRC}(\gamma_{th})$ is defined in form of integral by

$$P_{out}^{MRC}(\gamma_{th}) = \int_0^{\gamma_{th}} p_{\gamma MRC}(\gamma_{MRC}) d\gamma_{MRC} \quad (20)$$

Substituting Eq. (4) in Eq. (20) $P_{out}^{MRC}(\gamma_{th})$ can be written as

$$P_{OUT}^{MRC}(\gamma_{th}) = \int_0^{\gamma_{th}} \int_0^{\gamma_{MRC}} \left[\int_0^{\infty} \frac{c_2}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_2}\right)}{\Omega} \right)^{c_2/2} \cdot \right.$$

$$\cdot (\gamma_{MRC} - \gamma_1)^{c_2/2-1} \exp\left[-\left(\frac{\gamma_{MRC} - \gamma_1}{\Omega_2} \Gamma\left(1+\frac{2}{c_2}\right) \right)^{c_2/2} \right] \cdot$$

$$\cdot \left\{ \frac{\xi}{\sqrt{2\pi}\sigma_2\Omega_2} \exp\left[\frac{10\log_{10}\Omega_2 - \mu_2)^2}{2\sigma_2^{\,2}} \right] \right\} d\Omega_2 \cdot$$

$$\cdot \int_0^{\infty} \frac{c_1}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_1}\right)}{\Omega_1} \right)^{c_1/2} \gamma_1^{\,c_1/2-1} \exp\left[-\left(\frac{\gamma_1}{\Omega_1} \Gamma\left(1+\frac{2}{c_1}\right) \right)^{c_1/2} \right] \cdot$$

$$\left. \cdot \left\{ \frac{\xi}{\sqrt{2\pi}\sigma_1\Omega_1} \exp\left[\frac{10\log_{10}\Omega_1 - \mu_1)^2}{2\sigma_1^{\,2}} \right] \right\} d\Omega_1 \right] d\gamma_1 d\gamma_{MRC} \quad (21)$$

Fig 6. shows the outage probability versus instantaneous SNR for the same factors as in Fig 5. From this figure we can see that MRC combiner has better performances then single channel receiver.

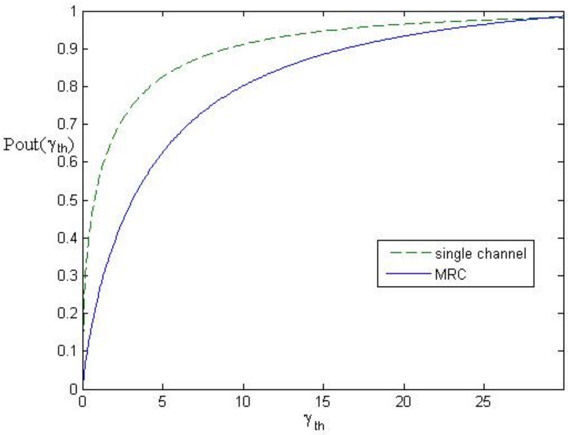

Fig 6. The outage probability for single channel and MRC receiver for $\mu_i = 5db$, $\sigma_i = 2db$, $c_i = 1$

4 Conclusion

In this paper a unified performance analysis for dual diversity MRC over uncorrelated Weibull and log-normal fading channels is presented. Probability density function of SNR, Amount of fading and Outage probability are derived in the form of multiple integral. That can not be obtained in relatively simple closed-form expression for evaluation of this parametars, because system structure is too complex and it was performed numerical calculation of them. As ilustration of this aproach, caracteristics of receiver are shown for MRC dual diversity case to point out the effect of the overall fading. The calculated curves are shown graphically for different signal and fading parameters values. It is shown that MRC receiver has better performances then single channel receiver.

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