The Performance of Macrodiversity System in the Presence of Long-term Nakagami-*m* Fading and Short-term Gamma Fading

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Abstract: - Probability density function (PDF) ,moments of signal and amount of fading (AF) at the output of macrodiversity system in closed form are obtained. Dual maximal-ratio combining (MRC) is implemented at the micro level (single base station) and selection combining (SC) with two base stations (dual diversity) is implemented at the macro level. This model assumes a Nakagami-*m* density function for the envelope of the received signal and a gamma distribution to model the average power to account for the shadowing. The results are shown graphically for different signal and fading parameters values.

Key-Words:- Shadowing, Nakagami-*m* Fading, Microdiversity, Macrodiversity, Probability Density Function, Amount of Fading

1 Introduction

Transmissions in wireless communications systems are influenced by various effects such as multipath and shadowing. Short-term fading is the result of multipath propagation while shadowing is the result of large obstacles between transmitter and receiver.

The reliability of communication over the wireless channels can be improved using diversity techniques, such as space diversity [1], [2]. Diversity techniques at single base station (microdiversity) reduce the effects of short-term fading. Impairments due to shadowing can be mitigated using macrodiversity techniques which employ the processing of signals from multiple base stations. The use of composite micro- and macrodiversity has recently received considerable interest due to the fact that it simultaneously combats the both short-term fading as well as shadowing. A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal or Nakagami-lognormal is considered in [3], [4].

The use of lognormal distribution to model the average power which is random variable due to shadowing doesn't lead to a closed form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) at the receiver. A compound fading model uses a gamma distribution to account for shadowing instead of the lognormal distribution [6], [7]. This model incorporates short-term fading and shadowing and provides an analytical solution for the PDF of the SNR facilitating the analysis of wireless systems.

In this paper, system following micro- and macrodiversity reception in correlated gamma shadowed Nakagami-*m* fading channels is considered. Microdiversity system was used to reduce the effects of short-term fading to the system performance. Macrodiversity system was used to reduce the effects of long-term fading to system performance. Closed form eexpressions for probability density function (PDF) at the output of system is obtained and used to derive moments and amount of fading (AF) of the proposed system. Numerical results are shown graphically.

2 Model of the System

The model of the macrodiversity system is shown in the Fig. 1.

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Dual-branch maximal-ratio combining (MRC) is implemented at the micro level (single base station) and selection combining (SC) with two base stations (dual diversity) is implemented at the macro level. Signals at antennas in single base station are independent. The two base stations are treated to have nonzero correlation.

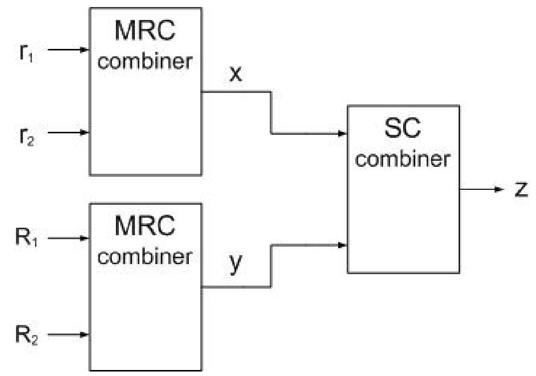

Fig.1. Macrodiversity system model

The probability density functions (PDFs) of the combiner input signals, r_1, r_2 and signals R_1, R_2 in the presence of Nakagami-*m* fading, are [5]:

$$p_{r_i}(r_i) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega_1}\right)^m r_i^{2m-1}\exp\left(-\frac{m}{\Omega_1}r_i^2\right),$$
$$r_i \geq 0 \;\; for \;\; i = 1,2$$

$$p_{R_i}(R_i) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega_2}\right)^m R_i^{2m-1}\exp\left(-\frac{m}{\Omega_2}R_i^2\right), \quad (1)$$
$$R_i \geq 0 \;\; for \;\; i = 1,2$$

where Ω_1, Ω_2 are signals power, on the first and second dual MRC receiver respectively, m is Nakagami-*m* fading parameter (m≥0.5). After replacement $x_i = r_i^2$ and $y_i = R_i^2$, $i = 1,2$ in (1), the following expression can be obtained by transformation of random variables:

$$p_{x_i}(x_i) = \left(\frac{m}{\Omega_1}\right)^m x_i^{m-1}\exp\left(-\frac{m}{\Omega_1}x_i^2\right), \; i = 1,2$$

$$p_{y_i}(y_i) = \left(\frac{m}{\Omega_2}\right)^m y_i^{m-1}\exp\left(-\frac{m}{\Omega_2}y_i^2\right), \; i = 1,2 \quad (2)$$

The probability density functions (PDFs) at the outputs of MRC receivers are respectivly:

$$p_x(x) = \int_0^x p_{x_1}(x - x_2) p_{x_2}(x_2)\, dx_2$$

$$p_y(y) = \int_0^y p_{y_1}(y - y_2) p_{y_2}(y_2)\, dy_2 \quad (3)$$

Furthermore, the probability density functions (PDFs) at the output of the macro diversity system is:

$$p_z(z) = \int_0^\infty d\Omega_1 \int_0^{\Omega_1} d\Omega_2 p_x(z/\Omega_1) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2)$$
$$+ \int_0^\infty d\Omega_1 \int_{\Omega_1}^\infty d\Omega_2 p_y(z/\Omega_2) p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \quad (4)$$

where $p_{\Omega_1\Omega_2}(\Omega_1\Omega_2)$ is the joint probability density function of Ω_1 and Ω_2 which has gamma density distribution and can be expressed as [7], [8]:

$$p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) = \frac{\rho^{-\frac{c-1}{2}}}{\Gamma(c)(1-\rho)s_0^{c+1}}(\Omega_1\Omega_2)^{\frac{c-1}{2}} \cdot$$
$$\cdot \exp\left(-\frac{\Omega_1 + \Omega_2}{(1-\rho)s_0}\right) I_{c-1}\left(\frac{2\sqrt{\rho\Omega_1\Omega_2}}{(1-\rho)s_0}\right) \quad (5)$$

where ρ is the correlation between Ω_1 and Ω_2, c is the order of gamma distribution and integer, s_0 is related to the average power of Ω_1 and Ω_2, $I_{c-1}(\cdot)$ is the modified Bessel function of the first kind of order (c-1), where c is integer, and $\Gamma(\cdot)$ is gamma function.

3 System Performances and Numerical Results

The determination of the probability density of the combiner output signal is important for the macrodiversity performances determination.

The integral (4) can be evaluated in closed form using Eq. (2), (3), (5) and using [9, Eqs (3.381/2) and (3.471/9)] with the result:

$$p_z(z) = \frac{4m^{2m}}{\Gamma(m)^2} \cdot \frac{1}{\Gamma(c)(1-\rho)s_0^{c+1}} z^{2m-1} \cdot$$
$$\cdot \sum_{i=0}^{m-1}\sum_{p=0}^{\infty}\sum_{k=0}^{\infty}\left((-1)^i \binom{m-1}{i}\frac{1}{m+i}\right) \cdot$$
$$\cdot P\left(\frac{1}{(1-\rho)s_0}\right)^{p-1}\left(\frac{mzs_0(1-\rho)}{2}\right)^{\frac{2c+2p-2m+k}{2}} \cdot$$
$$\rho^p \frac{1}{p!\Gamma(p+c)}\left(\frac{1}{(1-\rho)s_0}\right)^{p+c+k} \cdot$$

$$\cdot K_{2c+2p-2m+k}\left(2\sqrt{\frac{2mz}{\left(1-\rho\right)s_0}}\right) \qquad (6)$$

where $P=\frac{1}{\prod_{j=0}^{k}\left(c+p+j\right)}$ and $K_n(\cdot)$ is modified Bessel function of the second kind of order n.

Expression (6) requires summation of an infinite number of terms. Table I summarize the number of terms for both sums (p=k), needed for PDF, in order to achieve an accuracy better then ±2% after the truncation of the infinite series. As table 1 indicates, an increase in ρ leads to an increase of the number of terms that are needed to be summed in order to achieve the target accuracy. Furthermore, an increase of z increases the number of terms that are required to be summed.

z	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.7
5	2	3	4	6
10	4	5	6	9
15	5	6	7	11
20	6	7	8	13
25	7	8	9	15
35	8	9	11	17

Table 1 Number of terms for both sums for convergence of the PDF of macrodiversity system in range of ±2% (PDF, m=3, c =2, s_0= 4, p=k)

The probability density functions (PDFs) at the output of the macrodiversity system are given in Figs. 2-5 for diferent parameters m,ρ,c and s_0.

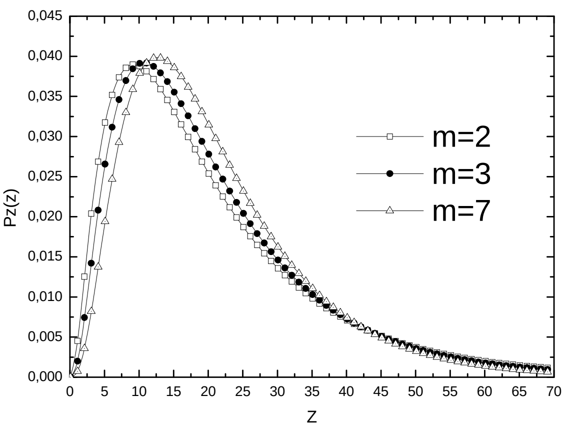

Fig.2. Probability density function at the output of macrodiversity system for different integer m parametars and constant following parametars: c = 2, ρ = 0.2, s_0 = 4

The moments at the output of the macrodiversity system can be obtained by following expression :

$$m_n=\int_0^\infty z^n p_z(z)dz \qquad (7)$$

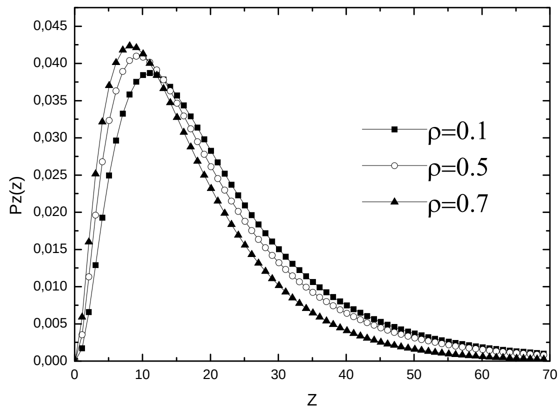

Fig.3. Probability density function at the output of macro diversity system for different ρ parametars and constant following parametars: c = 2, m = 3, s_0 = 4

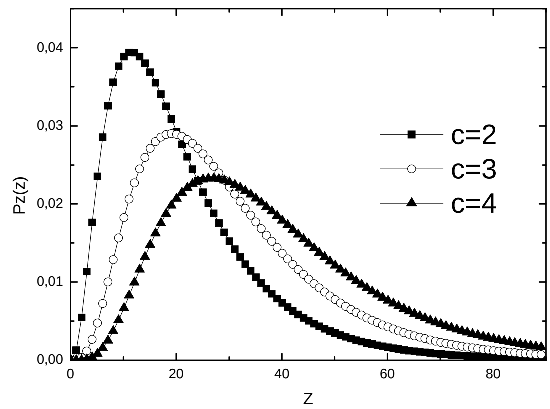

Fig.4. Probability density function at the output of macro diversity system for different c parametars and constant following parametars: ρ = 0.2, m = 4, s_0 = 4

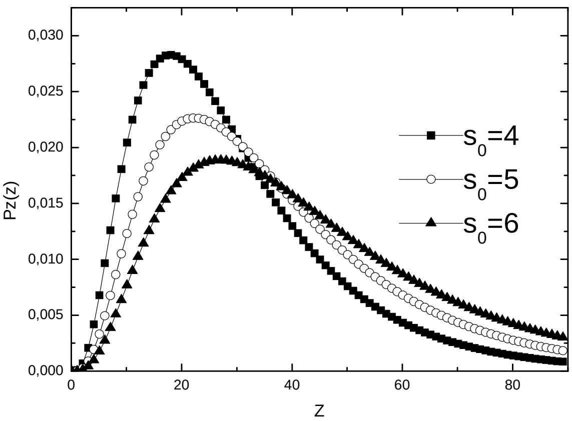

Fig.5. Probability density function at the output of macrodiversity system for different s_0 parametars and constant following parametars: $\rho = 0.2$, $m = 3$, $c = 3$

Combining expression (6) and (7) and using [9, Eqs (6.561/16)], the moment expression at the output of macrodiversity system is derived as:

$$m_n = \frac{4m^{2m}}{\Gamma(m)^2} \cdot \frac{1}{\Gamma(c)(1-\rho)s_0^{c+1}} \cdot$$

$$\cdot \sum_{i=0}^{m-1} \sum_{p=0}^{\infty} \sum_{k=0}^{\infty} \left((-1)^i \binom{m-1}{i} \frac{1}{m+i} \right) \left(\frac{1}{(1-\rho)s_0} \right)^{p-1} P \cdot$$

$$\cdot \rho^p \frac{1}{p!\Gamma(p+c)} \left(\frac{1}{(1-\rho)s_0} \right)^{p+c+k} 2^{2c+2p+2m+k+2n-2} \cdot$$

$$\cdot 2 \left(\frac{ms_0(1-\rho)}{2} \right)^{\frac{2c+2p-2m+k}{2}} \Gamma(2c+2p+k+n) \cdot$$

$$\cdot \left(2\sqrt{\frac{2m}{(1-\rho)s_0}} \right)^{-2c-2p-2m-k-2n} \Gamma(2m+n) \qquad (8)$$

Expression (8) also requires summation of an infinite number of terms. Table 2 and 3 are derived for n=1 and n=2 respectively. Tables indicate that an increase in ρ leads to an increase of the number of terms, and an increase of s_0 increases the number of terms that are required to be summed.

S_0 [dB]	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.7
-5	6	10	13	23
0	9	11	13	23
5	9	11	13	23
10	9	11	13	23

Table 2. Number of terms for both sums for convergence of the first moment of macrodiversity system in range of ±2% (First moment, m=3, c=3, p=k)

S_0 [dB]	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.7
0	2	3	4	6
5	4	5	6	9
10	5	6	7	11

Table 3. Number of terms for both sums for convergence of the second moment of macrodiversity system in range of ±2% (Second moment, m=3, c=3, p=k)

Moments for n=1 and n=2 have particular significance, where m_1 represents average value of the signal at the output of macrodiversity system and m_2 represent average square value of the signal at the output of macrodiversity system. Moments m_1 and m_2 are shown in Figs. 6. and 7. for different parameter values.

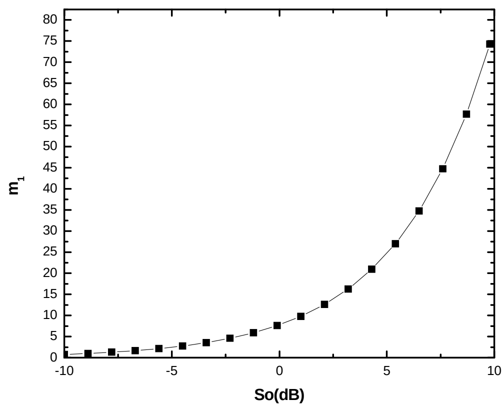

Fig.6. First moment at the output of macrodiversity system for constant following parametar values: $\rho = 0.1$, $m = 3$, $c = 3$

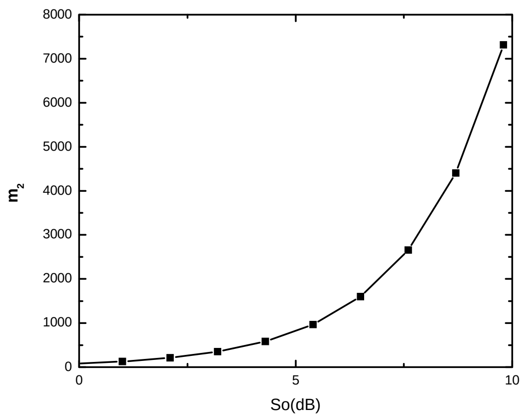

Fig.7. Second moment at the output of macrodiversity system for constant following parametars: $\rho = 0.1$, $m = 5$, $c = 3$

Amount of fading (AF), which is a measure of the performance of the entire system, can be obtained by following expression:

$$AF = \frac{m_2}{m_1^2} - 1 \tag{9}$$

Amount of fading (AF), at the output of the macrodiversity system are given in Figs. 8-9 for different parameters. According to the numerical results shown in the figures, it can be concluded that improvement of the performance of the system can be obtained for increasing parametars m and c, and decreasing parametar ρ.

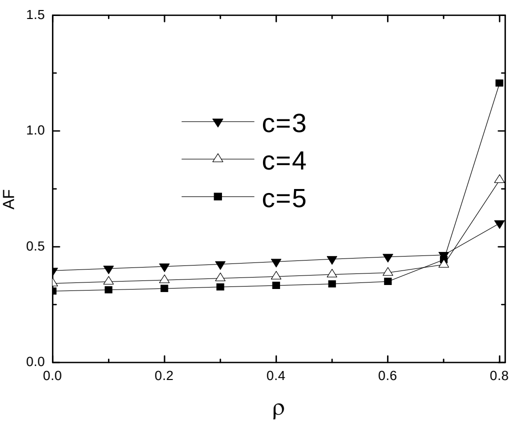

Fig.8. Amount of fading at the output of macro diversity system for differnet *c* parametars and constant following parametars: $m = 3$, $s_0 = 4$

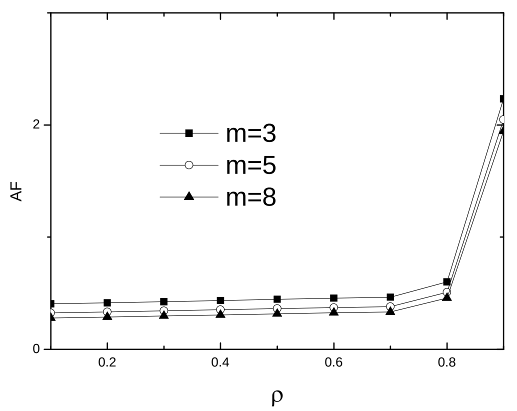

Fig.9. Amount of fading at the output of macro diversity system for differnet integer *m* parametars and constant following parametars: $c = 3$, $s_0 = 4$

4 Conclusion

Using a compound PDF model, system with micro- and macrodiversity reception in gamma shadowed Nakagami-*m* fading channels has been analyzed. Closed form eexpression for the PDF after diversity combining at the micro and macro level is obtained and used to study the moments and AF of proposed system. Numerical results have been graphically presented, showing the effect of correlation coefficient, Nakagami-*m* factor and order of gamma distribution on the system performance. It was also shown that composite micro- and macrodiversity provides significantly performance improvement which was the foreground task of this paper.

References:

[1]M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 1st ed. New York: Wiley, 2000.

[2]A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[3] F. Hansen and F.I. Mano, "Mobile Fading-Rayleigh and Lognormal Superimposed", *IEEE Trans. Vehic. Tech.*, vol. 26, pp. 332–335, 1977.

[4] E. K.Al-Hussaini, A.M. Al-Bassiouni, H. Mourad and H. Al-Shennawy, "Composite Macroscopic and Microscopic Diversity of Sectorized Macrocellular and Microcellular Mobile Radio Systems Employing RAKE Receiver over Nakagami Fading plus Lognormal Shadowing Channel", *Wireless Personal Communications*, vol. 21, pp. 309–328, 2002.

[5] W. C. Jakes, *Microwave Mobile Communication*, 2nd ed. Piscataway, NJ: IEEE Press, 1994.

[6] P. M. Shankar, "Analysis of microdiversity and dual channel macrodiversity in shadowed fading channels using a compound fading model", *Int. J. Electron. Comm.*, vol. 62, pp. 445-449, 2007.

[7] S. Yue, TBMJ Ouarda, B. Bobee, "A review of bivariate gamma distributions for hydrological application", *J. Hydrol.* vol. 246, pp. 1-18, 2001.

[8] E. Xekalaki, J. Panaretos and S. Psarakis, "*A Predictive Model Evaluation and Selection Approach- The Correlated Gamma Ratio Distribution*", *STOCHASTIC MUSINGS: PERSPECTIVES FROM THE PIONEERS OF THE LATE 20TH CENTURY*, J. Panaretos, ed., Laurence Erlbaum, Publisher, USA, pp. 188-202, 2003. Available at SSRN: http://ssrn.com/abstract=947067]

[9] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, Academic, New York, 5th edn., 1994.