Symbol Error Rate of Quadrature Subbranch Hybrid Selection/Maximal-Ratio Combining in Rayleigh Fading Under Employment of Generalized Detector

VYACHESLAV TUZLUKOV
School of Electrical Engineering and Computer Science
Kyungpook National University
1370 Sankyuk-dong, Buk-gu, Daegu 702-701
SOUTH KOREA
Email: tuzlukov@ee.knu.ac.kr

Abstract: - The symbol-error rate (SER) of a quadrature subbranch hybrid selection/maximal-ratio combining scheme for 1-D modulations in Rayleigh fading under employment of the generalized receiver, which is constructed based on the generalized approach to signal processing in noise, is investigated. At the generalized receiver, $N$ diversity branches are split into $2N$ in-phase and quadrature subbranches. Traditional hybrid selection/maximal-ratio combining is then applied over $2N$ subbranches. $M$-ary pulse amplitude modulation, including coherent binary phase-shift keying (BPSK), with quadrature subbranch hybrid selection/maximal-ratio combining is investigated. The SER performance of the generalized receiver under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes are investigated and compared with the conventional hybrid selection/maximal-ratio combining receiver. The obtained results show that the generalized receiver with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes outperforms the traditional hybrid selection/maximal-ratio combining receiver.

Key Words: - Generalized detector, Diversity combining, Symbol error rate, Fading channel, Hybrid selection/maximal-ratio combining.

1 Introduction

In this paper we investigate the generalized receiver, which is constructed based on the generalized approach to signal processing in noise [1]–[5], under quadrature subbranch hybrid selection/maximal-ratio combining for 1-D modulations in Rayleigh fading and compare its symbol error rate (SER) performance with that of the traditional hybrid selection/maximal-ratio combining scheme discussed in [6]. It is well known that the hybrid selection/maximal-ratio combining receiver selects the $L$ strongest signals from $N$ available diversity branches and coherently combines them [7]–[13]. In traditional hybrid selection/maximal-ratio combining scheme, the strongest $L$ signals are selected according to signal-envelope amplitude [7]–[13]. However, some receiver implementations recover directly the in-phase (I) and quadrature (Q) components of the received branch signals. Furthermore, optimal maximum likelihood estimation of the phase of a diversity branch signal is implemented by first estimating the in-phase and quadrature branch signal components and obtaining the signal phase as a derived quantity [14] and [15]. Other channel-estimation procedures also operate by first estimating the in-phase and quadrature branch signal components [16]–[18]. Thus, rather than $N$ available signals, there are $2N$ available quadrature branch signal components for combining. In general, the largest $2L$ of these $2N$ quadrature branch signal components will not be the same as the $2L$ quadrature branch signal components of the $L$ branch signals having the largest signal envelopes. In this paper, we investigate how much improvement in performance can be achieved by using the generalized receiver with modified hybrid selection/maximal-ratio combining, namely, with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes, instead of the conventional hybrid selection/maximal-ratio combining scheme for 1-D signal modulations in Rayleigh fading. At the generalized receiver, the $N$ diversity branches are split into $2N$ in-phase and quadrature subbranches. Then the generalized receiver with hybrid selection/maximal-ratio combining scheme [19] is applied to these $2N$ subbranches. Obtained results show that the better performance is achieved by this quadrature subbranch hybrid selection/maximal-ratio combining scheme in comparison with the traditional hybrid selection/maximal-ratio combining scheme for the same va-
value of average signal-to-noise ratio (SNR) per diversity branch.

2 System Model

We assume that there are \( N \) diversity branches experiencing slow and flat Rayleigh fading, and all of the fading processes are independent and identically distributed. During analysis we consider only the hypothesis \( H_1 \) "a yes" signal in the input stochastic process. Then the equivalent received baseband signal for the \( k \)-th diversity branch takes the following form:

\[
x_k(t) = h_k a(t) + n_k(t), \quad k = 1, \ldots, N,
\]

where \( a(t) \) is a 1-D baseband transmitted signal that without loss of generality, is assumed to be real, \( h_k \) is the channel gain for the \( k \)-th branch subjected to Rayleigh fading, and \( n_k(t) \) is a zero-mean white complex Gaussian noise process with two-sided power spectral density \( \frac{N_0}{2} \) with the dimension \( \frac{W}{Hz} \).

At the generalized receiver front end, for each diversity branch, the received signal is split into its in-phase and quadrature signal components. Then, the conventional hybrid selection/maximal-ratio combining scheme is applied over all of these quadrature branches, as shown in Fig.1. We can present as

\[
g_k = \begin{cases} h_{kI}, & k = 1, \ldots, N \\ h_{k-NQ}, & k = N + 1, \ldots, 2N \end{cases}
\]

and \( v_k = \begin{cases} n_{kI}(t), & k = 1, \ldots, N \\ n_{(k-N)Q}(t), & k = N + 1, \ldots, 2N \end{cases} \)

the in-phase signal component and quadrature signal component of the received signal are given by

\[
x_{kI}(t) = h_{kI} a(t) + n_{kI}(t),
\]

\[
x_{kQ}(t) = h_{kQ} a(t) + n_{kQ}(t).
\]

Since \( h_k \) \((k = 1, \ldots, N)\) are subjected to independent and identically distributed Rayleigh fading, \( h_{kI} \) and \( h_{kQ} \) are independent zero-mean Gaussian random variables with the same variance [20]

\[
D_h = \frac{1}{2} M \{ |h_k|^2 \}.
\]

Further, the in-phase \( n_{kI}(t) \) and quadrature \( n_{kQ}(t) \) noise components are also independent zero-mean Gaussian random processes, each with two-sided power spectral density \( \frac{N_0}{2} \) with the dimension \( \frac{W}{Hz} \) [14]. Due to the independence of in-phase \( h_{kI} \) and quadrature \( h_{kQ} \) channel gain components and in-phase \( n_{kI}(t) \) and quadrature \( n_{kQ}(t) \) noise components, the \( 2N \) quadrature branch received signal components conditioned on the transmitted signal are independent and identically distributed. We can reorganize the in-phase and quadrature components of the channel gains \( h_k \) and Gaussian noise \( n_k(k = 1, \ldots, N) \) as \( g_k \) and \( v_k \), given, respectively, by

\[
g_k = h_{kI}, \quad k = 1, \ldots, N
\]

\[
v_k = \begin{cases} n_{kI}(t), & k = 1, \ldots, N \\ n_{(k-N)Q}(t), & k = N + 1, \ldots, 2N \end{cases}
\]

Figure 1. Block diagram of the generalized receiver under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes.

The signal at the output of the generalized receiver with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes takes the following form:

\[
Z_{QBHS/MRC}^k(t) = s^2(t) \sum_{k=1}^{2N} c_k^2 g_k^2 + \sum_{k=1}^{2N} c_k^2 g_k^2 [v_k^2(t) - v_k(t)]
\]

where \( v_k^2(t) - v_k(t) \) is the background noise forming at the output of the generalized detector for the \( k \)-th branch;

\[
v_k = \begin{cases} n_{kI}(t), & k = 1, \ldots, N \\ n_{(k-N)Q}(t), & k = N + 1, \ldots, 2N \end{cases}
\]

\( n_k(t) \) is the reference zero-mean white complex Gaussian noise process with two-sided power spectral density \( \frac{N_0}{2} \) introduced according...
to the generalized approach to signal processing in noise [1]–[5]; $c_k \in \{0,1\}$ and $2L$ of $c_k$ equal to 1.

## 3 Performance Analysis

### 3.1 Symbol Error Rate Expression

Let $q_k$ denote the instantaneous signal-to-noise ratio per symbol of the $k$-th quadrature branch ($k=1,\ldots, 2N$) at the output of the generalized receiver under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes. In line with [2], this instantaneous signal-to-noise ratio (SNR) $q_k$ can be defined as

$$q_k = \frac{E_b}{4\sigma_n^2} s_k^2,$$

where $E_b$ is the average symbol energy of the transmitted signal $a(t)$. Assume that we choose $2L (1 \leq L \leq N)$ quadrature branches out of the $2N$ branches. Then, the SNR per symbol at the output of the generalized receiver under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes may be presented as

$$q_{QBHS/MRC} = \frac{2}{2L} \sum_{k=1}^{2L} q_{(k)},$$

where $q_{(k)}$ are the ordered instantaneous SNRs $q_k$ and satisfy the following condition

$$q_{(1)} \geq q_{(2)} \geq \cdots \geq q_{(2N)}.$$  

When $L=N$, we obtain the maximal-ratio combiner, as expected.

Using the moment generating function method discussed in [11] and [21], the SER of an $M$-ary pulse amplitude modulation (PAM) system conditioned on $q_{QBHS/MRC}$ is given by

$$P_b(q_{QBHS/MRC}) = \frac{2(M-1)}{M \pi} \int_0^{0.5\pi} \exp\left(-\frac{g_{M-PAM}}{\sin^2 \theta} q_{QBHS/MRC}\right)d\theta,$$

where

$$g_{M-PAM} = \frac{3}{M^2 - 1}.$$  

Averaging (14) over $q_{QBHS/MRC}$, the SER of the $M$-ary pulse amplitude modulation system is determined in the following form:

$$P_b = \frac{2(M-1)}{M \pi} \int_0^{0.5\pi} \exp\left(-\frac{g_{M-PAM}}{\sin^2 \theta} \right)d\theta,$$

where

$$\phi_q(s) = M_q \{\exp(sq)\}$$

is the moment generating function of random variable $q$, $M_q \{\cdot\}$ is the mathematical expectation of the moment generating function with respect to SNR per symbol.

When $M=2$, the average bit error rate of a coherent binary phase-shift keying (BPSK) system using the quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes can be determined in the following form:

$$P_b = \frac{1}{\pi} \int_0^{0.5\pi} \phi_q(q_{QBHS/MRC}) \left(-\frac{1}{\sin^2 \theta}\right)d\theta.$$

### 3.2 Moment Generating Function of $q_{QBHS/MRC}$

Since all of the $2N$ quadrature branches are independent and identically distributed, the moment generating function of $q_{QBHS/MRC}$ takes the following form [13]:

$$\phi_{q_{QBHS/MRC}}(s) = 2L \left(\frac{2N}{2L}\right)\int_0^{0.5\pi} \exp(sq)f(q)[\phi(s,q)]^{2L-1}[F(q)]^{2(N-L)}dq,$$

where $f(q)$ and $F(q)$ are, respectively, the probability density function and the cumulative distribution function of $q$, the SNR per symbol, for each quadrature branch, and

$$\phi(s,q) = \int_0^q \exp(x)f(x)dx$$

is the marginal moment generating function of the SNR per symbol of a single quadrature branch.

Since $g_k$ and $g_{k+N}$ ($k=1,\ldots, N$) follow a zero-mean Gaussian distribution with the variance $D_h$ given by (6), one can show that $g_k$ and $q_{k+N}$ follow the Gamma distribution with probability density function given by [20]
where

\[
\overline{q} = \frac{2E_s D_h}{4\sigma_h^2}
\]

is the average SNR per symbol for each diversity branch. Then the marginal moment generating function of the SNR per symbol of a single quadrature branch can be determined in the following form:

\[
\phi(s, q) = \frac{1}{\sqrt{1 - s\overline{q}}} \text{erfc} \left( \sqrt{\frac{1 - s\overline{q}}{\overline{q}}} \right)
\]

and the cumulative distribution function of \( q \) becomes

\[
F(q) = 1 - \phi(0, q) = 1 - \text{erfc} \left( \sqrt{\frac{q}{\overline{q}}} \right),
\]

where

\[
\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2) dt
\]

is the error function.

### 4 Simulation Results

In this section we discuss some examples of the performance of the generalized detector under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes and compare with the conventional hybrid selection/maximal-ratio combining receiver. The average SER of coherent BPSK and 8-PAM signals under processing by the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes as a function of average SNR per symbol per diversity branch for various values of \( 2L \) with \( 2N = 8 \) is shown in Fig.2. It is seen that the performance of the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes as a function of average SNR per symbol per diversity branch for various values of \( 2N \) with \( 2L = 4 \) is presented in Fig.3. We note the substantial benefits of increasing the number of diversity branches \( N \) for fixed \( L \). Comparison with the traditional hybrid selection/maximal-ratio combining receiver is made. Advantage of using the generalized detector is evident. Average SER of coherent BPSK and 8-PAM signals under processing by the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes as a function of average SNR per symbol per diversity branch for various values of \( 2N \) with \( 2L = 4 \) is shown in Fig.2. It is seen that the performance of the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes as a function of average SNR per symbol per diversity branch for various values of \( 2N \) with \( 2L = 4 \) is shown in Fig.4. To achieve the same value of average SNR per bit per diversity branch, we should choose \( 2L \) quadrature
Figure 3. Average SER of coherent BPSK and 8-PAM for the generalized detector under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes versus the average SNR per symbol per diversity for various values of $2N$ with $L = 4$.

Figure 4. Comparison of the average BER of coherent BPSK and 8-PAM for the generalized detector under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes for various values of $2L$ with $N = 8$.

The generalized receiver designs that process the quadrature signal components will require $2L$ receiver chains for either the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes or the generalized detector with traditional hybrid selection/maximal-ratio combining scheme. Such receiver designs will use only little additional power, as the generalized receiver chains consume much more power than the comparators. On the other hand, the generalized receiver designs that implement co-phasing of the branch signals without splitting the branch signals into the quadrature components will require $L$ receiver chains for the generalized detector with traditional hybrid selection/maximal-ratio combining scheme and $2L$ receiver chains for the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes, with corresponding hardware and power consumption increases.
5 Conclusions

In this paper, the performance of the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes for 1-D signal modulations in Rayleigh fading was investigated. The symbol error rate of M-ary pulse amplitude modulation, including coherent BPSK modulation, was derived. Results show that the generalized detector with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes performs substantially better than the generalized detector with traditional hybrid selection/maximal-ratio combining scheme, particularly when $L$ is smaller than one half $N$, and much better than the traditional hybrid selection/maximal-ratio combining receiver.

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References: