

The use of topological quantities to detect hierarchical properties in financial markets: the Financial sector in NYSE

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Abstract: In previous studies we have analyzed dynamical filtered graphs (namely the Minimum Spanning Trees and Planar Maximally Filtered Graphs) constructed from correlation matrices of daily return time series for the 300 most capitalized stocks of the New York Stock Exchange, and we have introduced centrality/peripherality synthetic indices which allowed to detect the stocks' relative positions in the network. In this paper we investigate the central role that the Financial sector is playing in the system hierarchy. We find that some Financial subsectors are particularly well clustered while others are more heterogeneous. Although most financial stocks are located in the central regions of the network, there are some notorious companies which have different behaviors and interact with more peripheral regions. We find that the stocks' relative positions are deeply related to their main economic activities.

Key-Words: Econophysics; Financial Data Correlations; Graph Theory; Networks; Minimum Spanning Tree; Planar Maximally Filtered Graph; Complex Systems

1 Introduction

In recent papers [1, 2, 3, 4, 5], we have studied some topological statistical properties of the New York Stock Exchange (NYSE) market from a network theory point of view, for the period 2001-2003 and for the 300 most capitalized companies. We have represented the market as a graph of interactions where the intensity of the relation that binds two stocks together depends on the linear correlation of their returns time series. Adopting a dynamical perspective we have computed the dynamical correlation matrices for log-price daily returns $Y_s(t)$ (where $s = 1, \dots, 300$ and $t = 1, 2, \dots, T - \Delta t + 1$ being $T = 748$ days) over a moving time window of size $\Delta t = 1, 2, 3, 4, 6, 12$ months:

$$c_{i,j}(t, \Delta t) = \frac{\langle Y_i Y_j \rangle_{(t, \Delta t)} - \langle Y_i \rangle_{(t, \Delta t)} \langle Y_j \rangle_{(t, \Delta t)}}{\sqrt{\left(\langle Y_i^2 \rangle_{(t, \Delta t)} - \langle Y_i \rangle_{(t, \Delta t)}^2 \right) \left(\langle Y_j^2 \rangle_{(t, \Delta t)} - \langle Y_j \rangle_{(t, \Delta t)}^2 \right)}} \quad (1)$$

where $\langle f \rangle_{(t, \Delta t)} = \frac{1}{\Delta t} \sum_{\tau=0}^{\Delta t-1} f(t+\tau)$ is the time average of a given series $f(\tau)$ from time t to time $t + \Delta t - 1$.

We make use of Minimum Spanning Trees (MST) [6] and Planar Maximally Filtered Graphs

(PMFG) [1, 7, 8] in order to extract the most relevant and meaningful information and neglect redundancies. At each time t , the networks lose some edges and gain some new ones, and the overall structure of the system fluctuates stochastically.

We have observed that the structure of such networks is rather complex: there is a central core of nodes very well connected among themselves and to which a set of peripheral cores is connected. Stocks residing in the central core have usually very high connectivity but it happens to observe as well low connectivity stocks which are also located in the central core in virtue of their connection with few very well connected stocks. Analogously, stocks residing in the peripheral cores may have high connectivity with respect to other stocks belonging to the same core or may be isolated in the extreme periphery of the system, far away from both the center and its "suburbs".

In order to detect smoothly each stocks' relative position and to be able to visualize it graphically, we have elaborated two centrality/peripherality synthetic indexes. Namely, the first index measures highly connected central nodes and it is the ranking average of degree + betweenness - eccentricity - closeness from MST and PMFG generated dynamically over the moving window. The second index measures highly connected but eccentric nodes and it is computed as the

ranking average of degree + betweenness + eccentricity + closeness from MST and PMFG generated dynamically over the moving window. A plot reporting the two indexes for the 300 most capitalized stocks in the NYSE analyzed between the period 2001 to 2003 is shown in Fig.1. We have found that many sectors of the market are well localized in specific regions. We can distinguish stocks well connected and central; stocks well connected but at the same time peripheral; stocks poorly connected but central; stocks poorly connected and peripheral. We have observed that this differentiation is in agreement with the independent classification of stocks in economic sectors and that these results are robust over time and this differentiation becomes stronger when the window size increases. In Fig.1 the stocks belonging to the Financial sector are highlighted with large symbols. Not surprising we observed that the Financial sector plays a central role in the network with its stocks mostly gathered in the region associated with high centrality. However, we have also observed some exceptions and peculiarities which we will further investigate in this paper.

2 Sectors and subsectors

According to the Forbes Magazine classification of economic activities [9], each sector can be subdivided into different subsectors. Table 1 shows those of the Financial sector.

Table 1: Name of Financial subsectors and corresponding number of stocks. The total number of stocks for the Financial sector is 53.

| Subsector | Number of Stocks |
|---------------------------------|------------------|
| Consumer Financial Services | 3 |
| Insurance Accidental & Health | 7 |
| Insurance Life | 2 |
| Insurance Miscellaneous | 2 |
| Insurance Properties & Casualty | 11 |
| Investment Services | 8 |
| Money Center Banks | 3 |
| Regional Banks | 14 |
| S&Ls/Savings Banks | 1 |
| N\A | 2 |

Let us now look at the different positioning in Fig.1 of the stocks belonging to these subsectors. Generally speaking, we can observe that some sub-

sectors such as Investment Services, Money Center Banks and Insurance Life that are located in central position of the MST and PMFG and therefore are located towards the right corner of Fig.1. On the other hand, there are other subsectors such as Regional Banks, Insurance Accidental & Health and Insurance Properties & Casualty that contain also stocks located in other areas showing mixed positioning.

In particular the subsector *Investment Services* is the most central in the system and is made of the world's most important financial institutions of the time including important stocks like Merrill Lynch (MER), Bear Stearns (BSC), Lehman Brothers (LEH) and Mellon Financial (MEL). They are represented by the six pointed stars in Fig.1 and they all lay on a line by the upper right corner. The financial crisis which struck the world in 2008, has brought many of them to bankruptcy, shaking the world economy to its very foundations.

The small subsector of *Money Center Banks*, constituted of only three stocks represented by down pointing triangles, include the Bank of America (BAC), the Bank of New York (BK) and Wells Fargo (WFC) which are among the largest financial companies in the world. They are all located in the central region of the network and are well gathered among themselves, constituting a homogeneous subsector. We can note that The Bank of New York (BK) is positioned very close to Mellon Financial (MEL). Remarkably, in July 2007 the two firms merged. Bank of America is very close to Merrill Lynch which it acquired in January 2009.

The two circles represent the *Insurance Life* subsector, Jefferson-Pilot (JP) and Lincoln National (LNC), which are very close to each other and are situated among the most central stocks. In March 2006, Lincoln National acquired Jefferson-Pilot.

A populated subsector, *Regional Banks*, constituted of 14 stocks and represented by diamonds, is also largely included in the central region. Only three stocks (City National Corporation; Hibernia Corporation; Provident Financial Bancorp) lie relatively far, among the scarcely connected stocks located in the center of the network.

Subsector *Insurance Accidental & Health* behaves in a peculiar way. Four out of seven of its stocks (squares) are positioned in the upper left side of the plot, showing to be central but characterized by low connectivity; two of them, Wellpoint (WLP) and Humana (HUM) are positioned in the lower left side instead, showing the opposite feature, peripherality and high connectivity; only one of them, Torchmark (TMK), is positioned near the right corner, displaying centrality. The positions of Wellpoint and Humana, the only two financial stocks in that area of the fig-

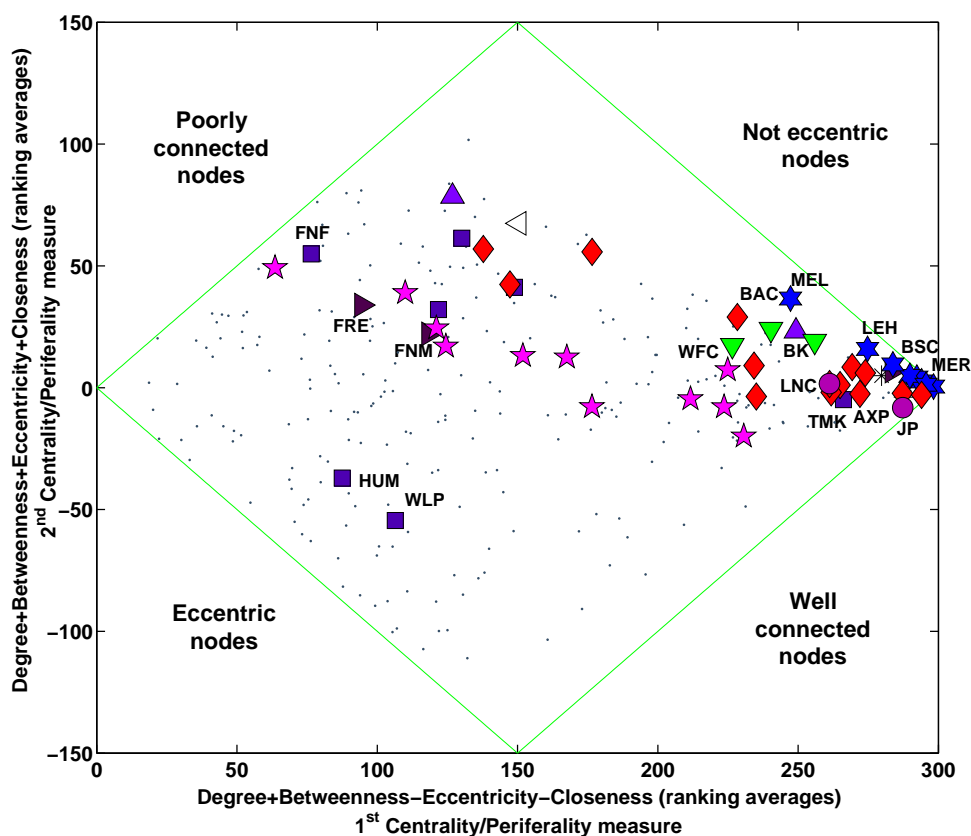


Figure 1: Average rankings from MST and PMFG ordered by Degree, Betweenness, (\pm)Eccentricity and (\pm)Closeness. Large symbols are associated to stocks in the Financial sector and differentiate among subsectors. Small dots represent all other stocks in the NYSE 300.

ure, are not entirely unexplained: in the same region, stocks belonging to *Healthcare* (not shown in the figure) are also positioned.

Another unusual but explicable positioning regards two of the companies that have most drawn the attention in the context of the 2008 financial crisis, the Federal National Mortgage Association (Fannie Mae) and the Federal Home Loan Mortgage Corporation (Freddie Mac) - also known as Fannie & Freddie, two government sponsored enterprises placed into conservatorship by the US Treasury in September 2008, nationalization that constituted a capital event in the course of the ongoing subprime mortgage crisis. Fannie & Freddie, together with the American Express Company (AXP), were classified as *Consumer Financial Services*, represented in Fig.1 with right pointing triangles. While, not surprisingly, the American Express Company, one of the most important companies in the credit card business, is adjacent to the stocks belonging to *Investment Services*, in the most central region of the map, Fannie & Freddie are located in a peripheral region and very close to one another. In

order to highlight the possible causes of such anomalous positions, we need to consider not only the ambiguous institutional and legal status, but also the peculiar activity of the two companies which operate in the mortgage market through actions aimed at ensuring that funds are available to the institutions that lend money to home-buyers. In the upper subplot of Fig.2, we show the same map as in Fig.1 highlighting some subsectors that are by nature connected to this activity, these being above all: *Services-Real Estate Operations*, represented by the nine hexagons; *Consumer Non Cyclical-Personal & Household Products*, represented by eight squares; *Capital Good-Construction Services*, represented by two pentagons. Other subsectors which are not directly connected to Fannie & Freddie economic activity but which seem to follow a similar structural pattern are *Consumer Non Cyclical-Food Processing*, eleven downward-pointing triangles, *Services-Retail Grocery*, five upward-pointing triangles, *Services-Retail Specialty*, five diamonds.

The bottom subplots of Fig.2 represent the trend of the first centrality/periferality measure over time

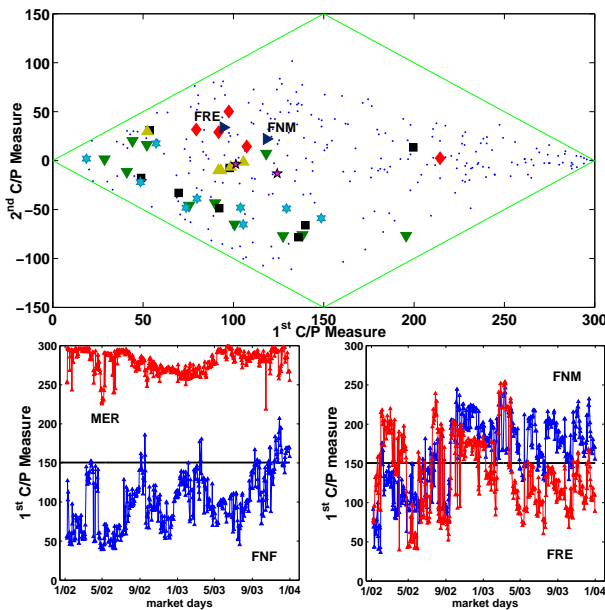


Figure 2: (top) Positioning of Fannie & Freddie with respect to other stocks from sectors different from the Financial sector. (bottom) Examples of dynamical behavior of the first centrality/periferality measure for $\Delta t = 12$.

for four stocks when $\Delta t = 12$ months. We see that Merrill Lynch (MER), which is among the most central stocks of the network, shows a trend that is systematically above the global average, always close to the maximum possible value of 300; Fidelity National Financial (FNF) which, like Fannie & Freddie, is involved in the real estate but belongs to the *Insurance Properties & Casualty* subsector, is one of the most peripheral stocks of the financial sector and of the whole network, it is located in a region of the map which is surrounded by stocks operating in the Real Estate market, the trend of its centrality measure is systematically below the average, although occasional fluctuations above the average rarely occur; Freddie & Fannie, which are very close in Fig.1, show very similar patterns for the centrality measure over time, their trends are superimposed and practically indistinguishable. These curves also show larger fluctuations than that of MER as function of time.

3 Conclusion

We have developed a quantitative method that allows us to measure the centrality/periferality properties of NYSE stocks from networks generated by filtering correlations among stocks. We have demonstrated that this method is able to catch the proximity of

stocks which are known to operate in similar economic activities. Important sectors and subsectors result near and very well clustered when displayed in a simple and intuitive graphical representation with the two centrality/periferality measures reported on the Cartesian axes (Fig.1).

This tool might be used to make hypotheses regarding the probability of merging between stocks (as in BK-MEL and JP-LNC cases) or to help clarify the different causes of market events: for example the peculiar positioning of Fannie & Freddie may suggest that while their fragility might have been predictable due to their anomalous institutional status, this is not so for companies such as Merrill Lynch, Lehman Brothers or Bear Stearns whose crises seem to have been generated by causes of a more systemic and dangerous nature which threaten to touch the very marrow of the financial system.

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