A Study of Oscillation for Signal Stabilization of Nonlinear System

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Abstract: The phenomena of desynchronization, synchronization, and forced oscillation has been investigation using describing function theory for a two input and two output nonlinear system containing saturation-type nonlinearities and subjected to high-frequency deterministic signal for the purpose of limit cycle quenching. The analytical results have been compared with the results of digital simulation/Matlab-Simulink for a typical example varying the nonlinear element.

Key-Words: - signal stabilization, limit cycle, forced oscillation, dither, saturation.

Notation

B amplitude of high frequency external signal (dither)
C_1, C_2 amplitude of outputs of the two subsystem
G_1, G_2 transfer functions of linear elements
N_1, N_2 describing functions (DF) of nonlinear elements
N_{1i}, N_{2i} incremental input DFs of the nonlinear elements
N_{1eq} two sinusoidal input DF (DIDF) of nonlinear element N_1
X_1, X_2 amplitude of input to nonlinear elements
X_1', X_2' amplitude of input to nonlinear elements when the system exhibits forced oscillations
Y_1, Y_2 amplitude of output (fundamental) of nonlinear elements
U_1, U_2 inputs of the two subsystems
ω_f frequency of high frequency external signal
ω_B frequency of self oscillations (limit cycle)
Subscripts 1 and 2, correspond to quantities of two subsystems S_1 and S_2, respectively

1 Introduction

Recognition of nonlinear self-oscillations or limit cycles in multidimensional nonlinear systems as they are indeed has had a long history, and is closely related to the system stability [1-23]. Engineers are continually involved in the design of system simply to ensure that it meets the performance criterion, which strictly excludes the existence of limit cycles [1], [3], [4], [6] and [8]. One of the important and interesting methods of extinguishing such limit cycle is by the employment of high-frequency signal to the nonlinear system input. The high-frequency signal is usually called dither. The use of dither to turn limit cycles off is referred to as signal stabilization. In many cases the introduction of an extra signal is less expensive than actually replacing the nonlinear element [9]. It has been extensively investigated by Olden-burger and his students [9], among the first to discover this phenomenon experimentally and subsequently, to provide analytical justification. However, these are for single-input and single-output (SISO) systems with both deterministic and random inputs. Other notable works on signal stabilization of SISO systems can be seen from the wealth of literatures [4], [6], [9], [13-16]. There are, however, a large number of practical industrial problems with two- or higher-dimensional nonlinear control configurations [1], [4], [18], [19] and the analysis of signal stabilization there has a huge significance in its own right. Unfortunately, relatively small amount of work has been published on forced oscillation/signal stabilization of multidimensional systems and hence addressed here for a two-dimensional system subjected to a deterministic dither. The describing function (DF) method provides a convenient tool and by virtue of its inherent approximations leads to a significant reduction in the complexity of analysis [1-10], [13-17], [22], [23]. The dual input describing function (DIDF) is analogous to the conventional describing function as far as the manner of using is concerned.

Investigation of signal stabilization via describing function theory can be executed in two stage process [20]. First, by use of DIDF theory, the dither and the original nonlinear element are replaced with an equivalent nonlinear element, whose form implicitly accounts for the presence of dither, but which no longer explicitly displays the dither signal. Second, the resulting system is made the object of a DF analysis to reveal the presence or absence of limit cycle [5], [9], [13-17], [20]. The variation of amplitude and frequency of limit cycle with variation of forcing signal amplitude (phenomena of forced oscillations, synchronization and desynchronisation) has been analyzed. The exact magnitude of dither for which limit cycle is extinguished (i.e. synchronization) or induced (i.e. desynchronisation) has been found. The technique is derived from the basic concept of DIDF, incremental input describing function (IDF) and relationship between system variables. Apart from directness of application, the method outlined has the notable advantage that it brings out the influence of individual system (effect of interaction/coupling) on the forced oscillation parameter, and can be applied to a higher-dimensional system [10], [13]. This technique also forms the basis of computer algorithms for predicting limit cycle/forced oscillation [10]. This rather simple investigation scheme has been illustrated through examples and comparison of results with digital simulation without loss of generality [13], [21]. The system has also been simulated using Matlab 6.0 for forced oscillation investigation.

2 Signal stabilization

In this paper, we consider a two-dimensional nonlinear system configuration as represented in Fig. 1 with two inputs U_1 and U_2 and the two outputs C_1 and C_2 and saturation as the only nonlinearities present in both the subsystems S_1 and S_2. This nonlinearity is not uncommon. For example, frequently the valves used as actuators in process control applications give rise to nonlinearity as a result of actuator saturation, where the limit corresponds to a fully open or closed valve. Actuator saturator may lead to a large "overshoot" inducing a limit cycle [22]. This particular system has been used earlier by the authors for prediction of limit cycle parameters [9-13]. The characteristics of nonlinear elements used in the examples considered are shown in Fig. 2.

It is a general class of two-dimensional system developed by the author [11, 12] considering the coupling effect between subsystems and relationships between individual parameters of significance within the subsystem. The system claims to be more suitable for the analysis of limit cycle/signal stabilization.

The system shown in Fig. 1 exhibits a limit cycle in the autonomous state [8-13]. We now examine the possibility of quenching the limit cycle by injecting a high-frequency dither. The dither can be injected either at u_1 or u_2 or at both the inputs simultaneously. However, for the present investigation we confine attention to the case when the dither, $B\sin\omega_f t$, (ω_f is at least 10 times greater than limit cycle frequency [19]) is injected at $u1$ only while $u2$ is kept unexcited from external sources. When the dither amplitude at $u1$ is gradually enhanced, the system would exhibit forced oscillations. The signals at various points in the system would then be composed of signals of frequency (ω_f), signals of frequency of limit cycle (ω_S) and the combination frequencies, $k_1\omega_f \pm k_2\omega_S$ where k_1, along with k_2 assume various integer values. However, with increase of the dither amplitude *B,* the frequency of limit cycle (ω_S) would also gradually change [4], [9], [14]. For a certain amplitude of dither, *synchronization* would occur i.e., the limit cycle would vanish and the system would exhibit forced oscillations at the dither frequency of only [4], [9], [20].

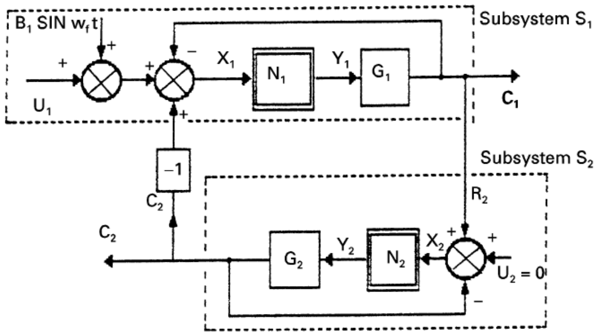


Fig.1: A general 2x2 nonlinear systems.

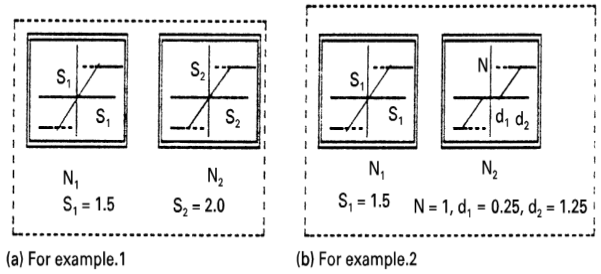


Fig.2: Characteristics of nonlinear elements used in Examples 1 and 2.

If subsequently the amplitude B is gradually re-duced, a point may be reached at which the limit cycle would reappear and the system would exhibit forced oscillations once again. This phenomenon has been termed as *desynchronisation* [10]. The analysis of such oscillation even in a relatively simple two-dimensional nonlinear system is

exceedingly complex. This paper presents analysis of these phenomena based on the following assumptions:
(a) The external signal is impressed on system only at u_1 (cf. Fig. 1).
(b) The linear elements composing various loops of the system possess low-pass characteristics (filter hypothesis) [1], [4], [9], [13-16], [20].

Because of the low-pass characteristics of the linear elements, the components of high-frequency signal at C_1 and C_2 would be very small. Hence, the component of the high-frequency signal at the input to the nonlinear element, N_1, would be equal to the magnitude of the dither at u_1.

It may be noted that just prior to desynchronisation, the system would be exhibiting a forced harmonic oscillation, and consequently, this phenomenon which is relatively easier to analyse and is, therefore, considered first.

2.1. Desynchronisation

Let the system in Fig. 1 be subjected to a dither, $B\sin\omega_f t$, and consider the situation for a reasonably large B, when the limit cycle has been quenched and, consequently, the system is exhibiting a harmonic oscillation at dither frequency (ω_f). Since the frequency of the dither is high, the magnitude of C_1 and C_2 can be assumed to be negligibly small. Hence, it follows that under these conditions, the inputs to the nonlinear elements N_1 and N_2 can be approximated, respectively, as (i) $B\sin\omega_f t$, and (ii) a vanishingly small signal. It has been shown in earlier works for SISO system [16-18], 20] that when B is gradually reduced the self-oscillations reappear at a point at which the forced oscillations become unstable and that this instability can be predicted by employing IDFs [4], [20]. Hence, in two-dimensional case also the limiting values of B at which the self-oscillations reappear can be obtained by replacing the nonlinear elements N_1 and N_2 by their IDFs, N_{1i} and N_{2i}, for vanishingly small signals superposed on the finite amplitude signals of frequency ω_f at their respective inputs. The linearised system is shown in Fig.3 and conditions for the stability limit can be obtained in a straightforward manner.

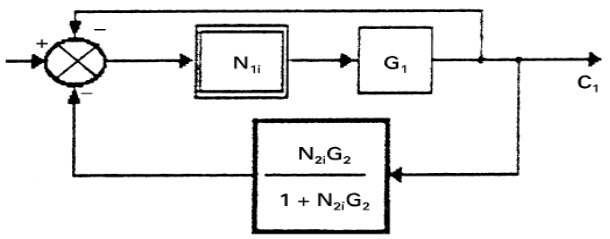


Fig.3: Equivalent linearisation for incremental signals for the system of Fig. 1.

The condition for self-oscillations to just reappear is thus obtained as

$$1 + N_{1i}G_1(j\omega)\left[1 + \frac{(N_{2i}G_2(j\omega))}{1 + (N_{2i}G_2(j\omega))}\right] = 0 \qquad (1)$$

As shown above, the magnitudes of the high-frequency signals at the inputs to the nonlinear elements N_1 and N_2 are approximated by $B\sin\omega_f t$ and zero, respectively. Hence, it follows that in Eq.(1), N_{1i} is the slope at the origin of the modified characteristic of N for an input $B\sin\omega_f t$, while N_{2i} is the slope at the origin of the characteristic of N_2 [1], [4], [23]. The following examples illustrate the procedure for determining the value of B for which desynchronisation would take place and self-oscillations would reappear.

Example 1. Considering the system of Fig. 1, where $G_1(s)=2/s(s+1)^2$, $G_2(s)=1/s(s+4)$ and the two nonlinear elements have ideal saturation characteristics as shown in Fig. 2a. Since the value of N_{2i} for small signals is equal to unity, Eq. (1) leads to

$$1 + G_1(j\omega)N_{1i}\left[1 + \frac{G_2(j\omega)}{1 + G_2(j\omega)}\right] = 0\,. \qquad (2)$$

Substituting $G1(j\omega)$ and $G2(j\omega)$ and separating in real and imaginary parts finally yields:

$$\omega^4 - 10\omega^2 + (8N_{1i} + 1) = 0$$

and

$$4N_{1i} + 6\omega^4 - (6 + 2N_{1i})\omega^2 = 0\,.$$

Simultaneous solution of the above equations yields: $N_{1i} = 0.51$ (critical).

The IDF for the saturation characteristic with given s_1 (= 1.5) is given by [1], [4], [17], [20]

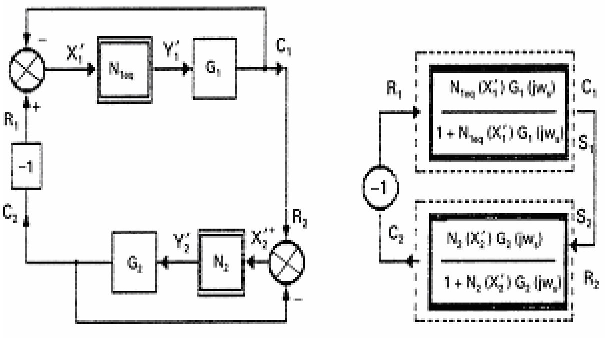
$$N_{1i} = \frac{2}{\Pi}\left(\sin^{-1}\frac{1.5}{B}\right).$$

Hence, the amplitude B of the dither that would make the IDF equal to the critical value of 0.51 is found to be B=2.09.

Example 2. Consider the system of Example 1 but the characteristics of nonlinear elements are as shown in Fig. 2b. The value of dither amplitude for desynchronisation is found to be B=1.5. It is important to note that the above analysis is based on the assumption that the amplitude of signal X_S (self-oscillation) is zero at all points in the system. However, once the amplitude of self-oscillation is different from zero, the signals at the various points would represent forced oscillations. Consequently, the frequency of self-oscillation that the system would eventually sustain after desynchronisation would be different from the one predicted above. So, the above analysis predicts only the critical amplitude B at which the process of desynchronisation sets in.

2.2. Forced oscillations

We now present an analysis of the forced oscillations in the system of Fig. 1 when it exhibits self-oscillations while being subjected to a high-frequency input $B\sin\omega_f t$ at u_1. As a consequence of assumed low-pass characteristics of linear elements, the input to the nonlinear element N_1 is composed of dither signal of frequency ω_f and self-oscillating signal of frequency ω_S, while the input to the nonlinear element N_2 is composed only of self-oscillating signal of frequency ω_S. Consequently, an analysis of the components of the frequency of self-oscillation, ω_S, can be visualized as the analysis of the system of Fig. 4a. The system of Fig. 4a is obtained by replacing the nonlinear element N_1 in Fig. 1 by its modified characteristics [3], [4], [9], [13-16], [20] determined by the component of the frequency ω_f at its input. In view of the low-pass characteristics of the linear elements and the high-frequency of dither, this component can be approximated as $B\sin\omega_f t$. The dither frequency should be much greater compared to self-oscillation frequency and the frequency ratio is considered irrational so that the DIDF will depend only on amplitude of two signals [1, 4, 18, 20].



a) b)

Fig.4: (a) Equivalent system for analysing forced oscillations of Fig. 1 with external input at U1. (b) Linearised equivalent for the system of Fig. 1 for analysing forced oscillation.

The component of frequency ω_f at the input to the non-linear element N_2 is negligibly small and therefore, the characteristics of the element N_2 in Fig. 4a would remain unaltered. The system of Fig. 4a can subsequently be analyzed for possible self-oscillations by employing the techniques developed in [4], [10]. If such an analysis shows the presence of self-oscillation for the system of Fig. 4a, then the system of Fig. 1 would exhibit forced oscillation of the frequencies ω_S and ω_f. A rigorous analysis of such a system is extremely complex. However, if the whole system is assumed to exhibit an oscillation predominantly at a single frequency and if the loops possess low-pass characteristics, then a simpler analysis, based on harmonic balance approach can be developed along the following lines.

The characteristic equation in frequency domain is obtained as

$$G_1(j\omega)N_{1eq}+G_2(j\omega)N_2+2G_1(j\omega)G_2(j\omega)N_{1eq}N_2=-1.0 \quad (3)$$

The three unknowns, X_1, X_2 and w require three independent equations for their evaluation. Separating the real and imaginary parts only two independent equations can be developed. The characteristic equation alone is not sufficient for analysis of self-oscillation in multidimensional systems. However, representing the system of Fig. 4a alternatively as in Fig. 4b, the following conditions must be fulfilled for ensuring harmonic balance.

(i) The phase condition

$$\theta_{c1}+\theta_{c2}=180^0 . \quad (4a)$$

where θ_c = loop angle of subsystems.

(ii) The gain condition:

$$(C_1/R_1)(C_2/R_2)=1. \quad (4b)$$

(iii) The amplitude ratio condition

$$\frac{X_1'}{X_2'}=\frac{\left|1+N_2G_2(j\omega)\right|}{\left|N_{1eq}G_1(j\omega)\right|} . \quad (4c)$$

It may be noted that N_{Ieq} and X_I' are related through the DF expression for the modified characteristic of the element N_I (DIDF), while N_2 and X_2' are related through the DF expression of the element N_2. Eq.(4) constitutes three equations for the solution of the three unknowns w, X_I', X_2'.

Example 3. Consider again the system of Example 1. Substituting $G_1(j\omega)$, $G_2(j\omega)$ in Eqs.(4a) and (4b) finally yields

$$N_2=\frac{\omega^2(11-3\omega^2)+\sqrt{\omega^2(11-3\omega^2)^2-8(1-\omega^2)^2(\omega^2+16)}}{4(1-\omega^2)} \quad (5)$$

and

$$N_{1eq}=\left(\frac{\omega^2-1}{8}\right)N_2+\frac{9\omega^2-\omega^4}{8}. \quad (6)$$

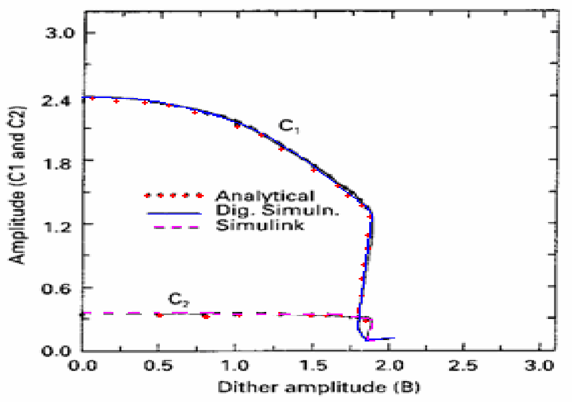
Again, substituting $G_1(j\omega)$, $G_2(j\omega)$ in Eq. (4) finally yields

$$\frac{X_1'}{X_2'}=\frac{\sqrt{\omega^2(\omega^2+16-4N_2)+2N_2^2}}{\omega\sqrt{(\omega^2+16)}}. \quad (7)$$

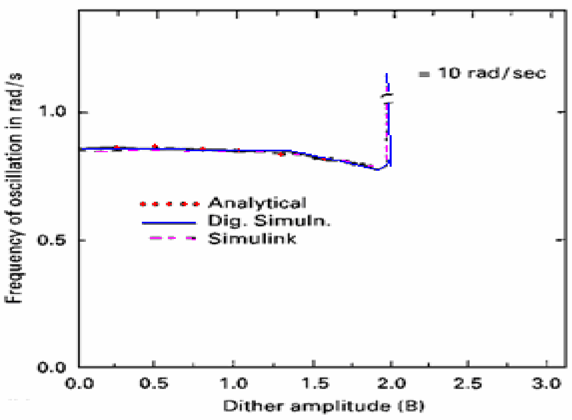
Furthermore, the relations between X_1' and N_{1eq} and, X_2' and N_2 are obtained from the given nonlinear characteristics as [2], [4]

$$N_{1eq}(X_1',B)=\frac{2}{\Pi X_1'}\int_{-\infty}^{\infty}\frac{\sin(S_1u)}{u^2}J_0(Bu)J_1(X_1'u)du \quad (8)$$

where J_0, and J_1 are Bessel's function of first kind of order 0 and 1, respectively.



a)



b)

Fig.5: (a) Variation of C_1 and C_2 with dither amplitude, illustration of limit cycle quenching (signal stabilisation), results of Example 3. (b) Variation of frequency with dither amplitude results of Example 3.

We also note that the input to nonlinear element N_2 can be approximated by a signal of frequency of self-oscillation alone. Therefore, the gain for the nonlinear element N_2 would be defined by its DF:

$$N_2 = \frac{2}{\Pi}\left[\sin^{-1}\frac{2}{X_2'} + \frac{2}{X_2'}\sqrt{\left(1-\left(\frac{2}{X_2'}\right)\right)}\right] \qquad (9)$$

The procedure for evaluation of the frequency of oscillations and other parameters is executed in the following sequential steps:

(a) certain value of ω is assumed;
(b) Eq. (5) yields a value of N_2;
(c) consequently Eq.(6) yields N_{1eq};
(d) subsequently Eq.(7) yields a value of X_1'/X_2';
(e) for the N_2 and N_{1eq} obtained in steps (b) and (c) above and for a particular value of B an alternative ratio X_1'/X_2' can be obtained from Eqs.(8) and (9);
(f) steps (a)-(e) are repeated for several assumed values of w, while keeping the value of B a fixed number.

The frequency for which the ratio X_1'/X_2' can be obtained by two alternative means are equal is the frequency of self-oscillation of the system. The other variables associated with self-oscillations can, subsequently be calculated. For example, if the frequency of oscillation is found out, N_{1eq} and N_2 can be determined from Eqs. (5) and (6). From these values X_1' and X_2' and hence C_1 and C_2 can be calculated. For various values of B, this procedure is repeated and the variations of C_1, C_2 and co for various B are depicted in Fig. 5 along with the results of digital simulation. The digital simulation technique used is similar to the Subramanian's work on SISO system [21] and also used by the authors' earlier work on limit cycle prediction for two-dimensional autonomous system [13]. The dither frequency chosen in the work is 10 rad/s. Fig.6 depicts the build up of subsystem output C_1 at B =1.0, which shows the periodic nature of oscillation. The forced oscillation has less settling time. The low-frequency demand signal is the excitation signal used to initiate the oscillation. The system was also simulated through MATLAB 6.0 for predicting the above phenomenon at various dither values. The simulation results are also shown in Fig.5. The analytical results have excellent agreement with simulation results.

Example 4. Consider the same system of Example 2. The results from simulation and analytical technique are compared in Fig.7. It can be seen that the simulation provides a good match with the frequency and amplitude of oscillation. Synchronization occurs at B=1.875.

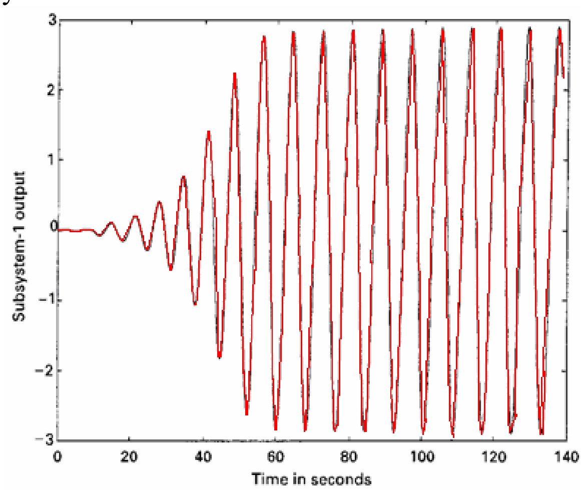
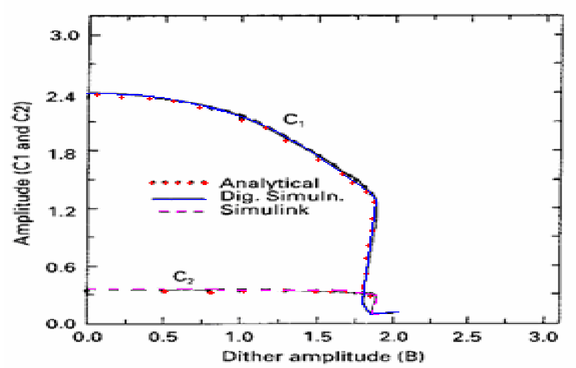


Fig.6: Build up of oscillation is subsystem-1.



a)

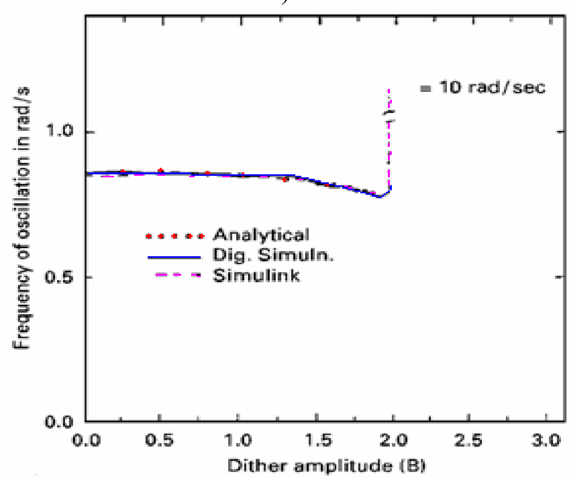


Fig.7: (a) Variation of C_1 and C_2 with dither amplitude, illustration of limit cycle quenching (signal stabilisation), results of Example 4.
(b) Variation of frequency with dither amplitude. Results of Example 4.

3 Conclusions

Comparison of analytical results with the results of digital simulation of the example considered, shows that the simplifying assumptions made in the analysis lead to results of acceptable accuracy. In addition, the method of analysis also aids the conceptual visualization of the mechanism leading to these interesting phenomena. However, the signal stabilization for the system comprising several interconnected subsystems exhibiting limit cycle at different frequencies are yet to be explored and this method of analysis may be appended by Neural Network model [15].

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