A Seven Degrees of Freedom (7 DOF) Human Body – Vehicle Structure Model for Impact Applications

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Abstract: This paper presents an analytical model of the human body for impact applications. A seven degrees of freedom model is developed and validated starting from a simple restrained model the human body response during the frontal impact is analysed.

Keywords: human body, d’Alembert – Lagrange equations of motion, finite element model.

1 Introduction
Vehicle safety is one of the most demanding subjects in the field of mechanical engineering. Early stages of systems’ development rely on the use of mathematical models of the human body and their implementation and application using specialised software for numerical simulation.

There is a number of application of these models for defining and improving the restrain system [2, 4, 6, 9, 10, 11, 12], improving the passenger’s protection in rear end collisions, evaluation of the structural performances of the vehicle and the passengers’ trauma level [5, 10], study of head/brain trauma, human body response to directional accelerations to the optimization of coupled systems like seat – passenger system.

Finite element method is a very powerful one and the mechanical systems are evaluated within an acceptable level of certainty. Analytical models are in this case a much faster way to define the requirements for the passenger’s protection system.

The number of elements involved in these models is significantly lower. The control of each element is more accurate. Although the detail level is not high, the structure definition and response can provide valuable information for further development of the simulation environment and safety restrain systems definition and applications.

2 Mechanical Model
2.1 Model overview
This paper presents an analytical model of the human body for impact applications. A seven degrees of freedom model is developed and validated.

As there is about the definition of a computational model of the human body during the frontal impact, the vehicle structure is represented by its mass (Θ) and the appropriate stiffness. Vehicle stiffness is used to obtain the deceleration profile in order to evaluate the human body response.

Regarding the human body, there are six main segments taken into account: pelvis (1), torso (2), head (3), hand (4), upper leg (5) and lower leg (6) (figure 1). Body masses, inertia [8] and dimensions are presented in table 1.

The human body – vehicle structure is presented in figure 1. Figure 1a is an overview of the system while figure 2b adds dimensional information, coordinates and some of the force elements (vehicle stiffness and passenger – vehicle interface). The passenger’s restrain system is represented only by the pelvic seat belt. This will allow larger displacements for the passenger’s body. Although these displacements are not realistic, the restrain system configuration was defined this way in order to analyze the motion of the passenger during the vehicle’s frontal impact.

<table>
<thead>
<tr>
<th>Body</th>
<th>Mass [kg]</th>
<th>Inertia [kg·m²]</th>
<th>Dimensions [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>-</td>
<td>x₀ = 0</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>0.14</td>
<td>x₁ = 0</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>0.30</td>
<td>θ₂ = −20°; l₂ = 0.285; L₂ = 0.570; l₂₄ = 0.550;</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.04</td>
<td>L₃ = 0.173; L₄ = 0.346</td>
</tr>
<tr>
<td>5</td>
<td>5.5</td>
<td>0.025</td>
<td>θ₄ = 100°; l₄ = 0.240; L₄ = 0.480</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>0.07</td>
<td>θ₅ = 80°; l₅ = 0.240; L₅ = 0.480</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.08</td>
<td>θ₆ = 125°; l₆ = 0.240; L₆ = 0.480</td>
</tr>
</tbody>
</table>

Table 1. Body masses, inertia and dimensions
According to the model definition (figure 1) the generalized coordinate are:

\[
\mathbf{q} = [x_0, x_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]
\]  
(1)

These general coordinates refers to the displacement (deformation) of the vehicle structure \( x_0 \), displacement of the human body inside the passengers’ compartment \( x_1 \), angular rotation of the torso \( \theta_2 \), angular rotation of the head \( \theta_3 \), angular rotation of the hand \( \theta_4 \), angular rotation of the upper leg \( \theta_5 \) and angular rotation of the lower leg \( \theta_6 \).

Figure 2 presents the definition of the global coordinate system \( (XOY) \) and local coordinate systems \( (x_i y_j) \) attached to model’s bodies. The local coordinate systems are applied to the gravity centre of each body defined in the mechanical systems.

With respect to the global coordinate system the coordinates of the gravity centre of the bodies are:

\[
\begin{align*}
\mathbf{x}_0 &= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \\
\mathbf{x}_1 &= \begin{bmatrix} x_0 + x_1 \\ y_0 + y_1 \end{bmatrix} \\
\mathbf{x}_2 &= \begin{bmatrix} x_0 + x_1 + l_2 \cdot \sin \theta_2 \\ y_0 + y_1 + l_2 \cdot \cos \theta_2 \end{bmatrix} \\
\mathbf{x}_3 &= \begin{bmatrix} x_0 + x_1 + l_2 \cdot \sin \theta_2 + l_3 \cdot \sin \theta_3 \\ y_0 + y_1 + l_2 \cdot \cos \theta_2 + l_3 \cdot \cos \theta_3 \end{bmatrix} \\
\mathbf{x}_4 &= \begin{bmatrix} x_0 + x_1 + l_2 \cdot \sin \theta_2 + l_3 \cdot \sin \theta_3 + l_4 \cdot \sin \theta_4 \\ y_0 + y_1 + l_2 \cdot \cos \theta_2 + l_3 \cdot \cos \theta_3 + l_4 \cdot \cos \theta_4 \end{bmatrix} \\
\mathbf{x}_5 &= \begin{bmatrix} x_0 + x_1 + l_2 \cdot \sin \theta_2 + l_3 \cdot \sin \theta_3 + l_4 \cdot \sin \theta_4 + l_5 \cdot \sin \theta_5 \\ y_0 + y_1 + l_2 \cdot \cos \theta_2 + l_3 \cdot \cos \theta_3 + l_4 \cdot \cos \theta_4 + l_5 \cdot \cos \theta_5 \end{bmatrix} \\
\mathbf{x}_6 &= \begin{bmatrix} x_0 + x_1 + l_2 \cdot \sin \theta_2 + l_3 \cdot \sin \theta_3 + l_4 \cdot \sin \theta_4 + l_5 \cdot \sin \theta_5 + l_6 \cdot \sin \theta_6 \\ y_0 + y_1 + l_2 \cdot \cos \theta_2 + l_3 \cdot \cos \theta_3 + l_4 \cdot \cos \theta_4 + l_5 \cdot \cos \theta_5 + l_6 \cdot \cos \theta_6 \end{bmatrix}
\end{align*}
\]  
(2)

The velocity and acceleration of the gravity centres of the bodies can be easily determined using equations (2)-(8).

The motion of the vehicle in constrained in \( OY \) direction, therefore \( y_0 = 0 \). The same constraint is applied for the human body pelvis \( (y_1 = 0) \).

### 2.2 Equations of motion

In order to derive the equations of motion the general theorems of mechanics are used (figure 3).

For this scope, force and moment of inertia were added to each body. The free body equations are:

- **Body** 0
  \[
  m_0 \cdot \ddot{x}_0 = -k_0 \cdot x_0 + k_{10} \cdot (x_1 - x_0)
  \]  
(9)

- **Body** 1
  \[
  m_1 \cdot \ddot{x}_1 = -H_2 - H_5 - k_{10} \cdot (x_1 - x_0)
  \]  
(10)
Body 2
\[ m_2 \ddot{x}_2 + H_2 + H_1 - H_4 \]
\[ m_2 \ddot{y}_2 + g = + V_2 - V_3 - V_4 \]
\[ J_2 \ddot{\theta}_2 = - H_2 \cdot l_2 \cdot \cos \theta_2 - H_1 \cdot (L_2 - l_2) \cdot \cos \theta_2 \]
\[ - H_4 \cdot (L_2 - l_2) \cdot \cos \theta_2 + V_2 \cdot l_2 \cdot \sin \theta_2 \]
\[ + V_4 \cdot (L_2 - l_2) \cdot \sin \theta_2 + V_4 \cdot (l_2 - l_2) \cdot \sin \theta_2 \]
\[ + M_{12} - M_{23} + M_{42} \]

Body 3
\[ m_3 \ddot{x}_3 = H_3 \]
\[ m_3 \ddot{y}_3 + g = V_3 \]
\[ J_3 \ddot{\theta}_3 = - H_3 \cdot l_3 \cdot \cos \theta_3 + V_3 \cdot l_3 \cdot \sin \theta_3 - M_{32} \]

Body 4
\[ m_4 \ddot{x}_4 = H_4 \]
\[ m_4 \ddot{y}_4 + g = V_4 \]
\[ J_4 \ddot{\theta}_4 = - H_4 \cdot l_4 \cdot \cos \theta_4 + V_4 \cdot l_4 \cdot \sin \theta_4 - M_{42} \]

Body 5
\[ m_5 \ddot{x}_5 = H_5 - H_6 \]
\[ m_5 \ddot{y}_5 + g = V_5 - V_6 \]
\[ J_5 \ddot{\theta}_5 = - H_5 \cdot l_5 \cdot \cos \theta_5 - H_6 \cdot (L_5 - l_5) \cdot \cos \theta_5 \]
\[ + V_5 \cdot l_5 \cdot \sin \theta_5 + V_6 \cdot (L_5 - l_5) \cdot \sin \theta_5 \]
\[ + M_{53} - M_{65} \]

Body 6
\[ m_6 \ddot{x}_6 = H_6 \]
\[ m_6 \ddot{y}_6 + g = V_6 \]
\[ J_6 \ddot{\theta}_6 = - H_6 \cdot l_6 \cdot \cos \theta_6 + V_6 \cdot l_6 \cdot \sin \theta_6 - M_{65} \]

The interface forces (vehicle deformation force and vehicle–passenger) are defined by (figure 4a):
\[ F_{ij} = k_{ij} \cdot \left( (x_i - x_j^0) - (x_i - x_j^0) \right) \]

Since joint stiffness was added too [1, 8], the resistant torque is defined as (figure 4b):
\[ M_{ij} = k_{ij} \cdot \left( (\theta_i - \theta_i^0) - (\theta_j - \theta_j^0) \right) \]

Substituting equations containing the velocities and accelerations, and solving equations (9)-(15) with respect to the reaction forces $H_2, H_3, H_4, H_5, H_6$ and $V_2, V_3, V_4, V_5, V_6$ Lagrange’s second order equations are obtained. In condensed form equations of motion may be written as:
\[ \mathbf{A} \cdot \ddot{\mathbf{q}} = \mathbf{B} \]

The $\mathbf{A}$ matrix coefficients are:
\[ A_{1,1} = m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6 \]
\[ A_{1,2} = A_{1,3} = A_{1,4} = A_{1,5} = A_{1,6} = A_{1,7} = 0 \]
\[ A_{2,1} = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 \]
\[ A_{2,2} = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 \]
\[ A_{2,3} = (m_2 \cdot l_1 + m_3 \cdot L_2 + m_4 \cdot l_24 \cdot \cos \theta_2) \]
\[ A_{2,4} = m_3 \cdot l_1 \cdot \cos \theta_3 \]
\[ A_{2,5} = m_4 \cdot l_1 \cdot \cos \theta_4 \]
\[ A_{2,6} = 0; A_{2,7} = 0 \]
\[ A_{3,1} = (m_2 \cdot l_2 + m_3 \cdot L_2 + m_4 \cdot l_24 \cdot \cos \theta_2) \]
\[ A_{3,2} = (m_3 \cdot l_1 + m_3 \cdot L_2 + m_4 \cdot l_24 \cdot \cos \theta_2) \]
\[ A_{3,3} = m_2 \cdot l_2^2 + m_3 \cdot l_1^2 + m_4 \cdot l_2^4 \]
\[ A_{3,4} = m_3 \cdot l_1 \cdot L_2 \cdot \cos \theta_2 \]
\[ A_{3,5} = m_4 \cdot l_4 \cdot l_24 \cdot \cos \theta_2 \]
\[ A_{3,6} = 0; A_{3,7} = 0 \]
\[ A_{4,1} = m_3 \cdot l_1 \cdot \cos \theta_3 \]
\[ A_{4,2} = m_3 \cdot l_1 \cdot \cos \theta_3 \]
\[ A_{4,3} = m_3 \cdot l_1 \cdot L_2 \cdot \cos \theta_2 \]
\[ A_{4,4} = m_4 \cdot l_2^2 + J_4 \]
\[ A_{4,5} = m_4 \cdot l_2^2 + J_4 \]
\[ A_{4,6} = 0 \]
\[ A_{5,1} = m_4 \cdot l_4 \cdot \cos \theta_4 \]
\[ A_{5,2} = 0 \]
\[ A_{5,3} = m_4 \cdot l_4 \cdot l_24 \cdot \cos \theta_2 \]
\[ A_{5,4} = m_4 \cdot l_4 \cdot \cos \theta_4 \]
\[ A_{5,5} = m_4 \cdot l_2^4 + J_4 \]
\[ A_{5,6} = 0; A_{5,7} = 0 \]
\[ A_{6,1} = (m_5 \cdot l_5 + m_6 \cdot L_3) \cdot \cos \theta_5 \]
\[ A_{6,2} = 0 \]
\[ A_{6,3} = 0 \]
\[ A_{6,4} = (m_5 \cdot l_5 + m_6 \cdot L_3) \cdot \cos \theta_5 \]
\[ A_{6,5} = 0 \]
\[ A_{7,1} = m_5 \cdot l_5 \cdot \cos \theta_5 \]
\[ A_{7,2} = 0; A_{7,3} = 0 \]
\[ A_{7,4} = m_5 \cdot l_5 \cdot \cos \theta_5 \]
\[ A_{7,5} = 0; A_{7,6} = m_6 \cdot l_5 \cdot L_5 \cdot \cos(\theta_5 - \theta_6) \]
\[ A_{7,7} = m_6 \cdot l_5 \cdot \cos \theta_6 \]

Table 2 presents the stiffness definitions of the mechanical system

<table>
<thead>
<tr>
<th>Definition</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle stiffness</td>
<td>$k_0$</td>
<td>5e5 [Nm]</td>
</tr>
<tr>
<td>Passenger’s restraint system</td>
<td>$k_{01}$</td>
<td>1e5 [Nm]</td>
</tr>
</tbody>
</table>

The vector $\mathbf{B}$, including the generalized forces, is defined as follows:
finite elements passenger with a restrain system similar to the one used in both analytical and multibody models was performed.

4 Validation of the analytical-multibody models.
Using the numerical procedures to solve equations (35) the motion of the passenger during a frontal impact is analysed. Some visualization procedures were used in order to improve the results’ graphical representation.

The simulation time is of 100 miliseconds. Figure 6 presents the displacements of both multibody and finite elements model at different time steps.

3 Finite element model.
A numerical model of the passenger, defined using finite elements was used in order to evaluate the performances of the analytical and multibody 7 DOF models [38].

The finite elements model was implemented in Ls-Dyna 3D software [27]. The dummy is a Hybrid 50% rigid model (figure 5). An analysis using the
Figure 7 presents the accelerations measured at the gravity centres of the head and pelvis as resulted by running simulations using the multibody model and the finite elements model (table 3), as these are necessary value to evaluate the injury criteria and trauma level.

The calculated error between the values obtained using the multibody model and the ones obtained from the finite elements model are between 1.2% for the head section and 7.8% for the pelvis section. A good agreement between the resulting values from the simulation model is obtained.

Table 3. Average accelerations

<table>
<thead>
<tr>
<th>Value</th>
<th>Section</th>
<th>Model</th>
<th>Measured</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average acceleration</td>
<td>Head</td>
<td>Analytic</td>
<td>244</td>
<td>3.17%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FE</td>
<td>252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pelvis</td>
<td>Analytic</td>
<td>193</td>
<td>8.29%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FE</td>
<td>177</td>
<td></td>
</tr>
</tbody>
</table>

5 Impact model

Springs and dampers are the basic energy absorbing components in a mechanical system. Previously the barrier force \( F_0 \) was defined as a displacement proportional force using the average vehicle stiffness coefficient (table 2) obtained from experiment or numerical simulations with full scale vehicle models (equation 36).

\[
F_0 = k_0 \cdot x_0
\]  

The same linear force model was used for the definition of the passenger’s restrain system (table 2).

\[
F_{10} = k_{10} \cdot x_1
\]  

In order to consider damping effects for the vehicle – wall force and for the passenger – vehicle interface force Kelvin model is used.

Therefore the forces are:

\[
F_0 = k_0 \cdot x_0 + c_0 \cdot \dot{x}_0
\]  

For vehicle impact applications the interface forces are characterized by damping factors with a value of about 0.2 [3, 7].

The maximum acceleration of the passenger’s head recorded at the centre of the gravity will be computed using the model with 7 degrees of freedom when for both vehicle wall force and passenger – vehicle interface force the damping factor in ranging from 0 to 0.25.

Figure 8 presents the maximum acceleration recorded at the gravity centre of the passenger’s head. The simulation time of the impact event is of 100 milliseconds.

\[
F_{10} = k_{10} \cdot x_1 + c_{10} \cdot \dot{x}_0
\]  

The equation of motions for a generally defined system is:

\[
m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = 0
\]  

or:

\[
\ddot{x} + 2 \cdot \zeta \cdot \omega \cdot \dot{x} + \omega^2 \cdot x = 0,
\]  

where:

\[
\omega = \sqrt{\frac{k}{m}}
\]

is the undamped natural frequency, and

\[
\zeta = \frac{c}{c_0}
\]

is the damping factor.

With respect to the damping factor the system may be:

- underdamped if \( 0 < \zeta < 1 \);
- critically damped if \( \zeta = 1 \);
- overdamped \( \zeta > 1 \);

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Figure 8 presents the maximum acceleration recorded at the gravity centre of the passenger’s head. The simulation time of the impact event is of 100 milliseconds.
acceleration to be not the maximum value that may occur during the impact event. Therefore a second plot is required that contains the time location of the maximum acceleration (figure 9).

Fig. 9. Time location of the maximum acceleration of the passenger’s head

Figures 8 and 9 will point that in order to obtain a maximum acceleration of the passenger’s head within the 100 milliseconds interval the vehicle’s structure force must have a damping factor between 0 and 0.05 while for the restrain system must be between 0 and 0.1.

6 Conclusions

A planar model of the human body is developed and validated using finite elements model. The motion is defined using d’Alembert – Lagrange equations. The sets of equations are solved using numerical procedures like Runge – Kutta. Some procedures for results visualization are used. Although the degrees of freedom of the model refer the gravity centres of the bodies the actual contour of the body overlays the skeleton representation.

The model may be adapted easily to various sets of anthropometric dimensions and masses. The position of the gravity centres can be adjusted independently with respect to the length of the human body segment. The open structure of the model allows further development of the model. There is a good agreement between the results obtained using the multibody model and the finite elements model. It is not the purpose of a simple 7 degree of freedom model to replace the more detailed representation of the human body using finite elements, but it may be a valuable tool for early validation of structure performances in terms of passenger’s protection and evaluation of trauma’s level.

It may be a tool for developing procedures to improve safety restrain system or to provide a tool to evaluate the human body response to different crash pulses.

References


