Studies on the Mechanical Elastic Systems with nonlinear damping. 
Power and amplitude numerical analysis

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Abstract: This study is both a qualitative and a quantitative approach of the nonlinear vibrations of the mechanical elastic systems with polynomial damping. It gives the physical and mathematical modelling and numerical simulation of the dynamics of one degree of freedom mechanical systems and shows the diagrams of the curves of different characteristics (amplitude, displacement, power) function of the polynomial coefficients of damping. It also shows the variation of the same characteristics function of the frequency of the harmonic force of excitation. The numerical simulation has done with the data of real technological equipment: an industrial conveyor driven by inertial vibrator which has tested in site by the specialists of The Research Centre of Machines, Mechanic and Technological Equipments - MECMET.

Key-Words: vibrations, nonlinear damping, power, amplitude, numerical analysis.

1 Introduction
The usual dynamics approaches of vibrating technological equipment consider that the mechanical system (with 1DOF, 2DOF or more) has elastic and damping elements with linear behavior. But, there are a lot of situations, when the linear/linearized model of the vibrating systems cannot explain some resonance phenomena at the superior or inferior frequencies than the driving vibrator frequency or the necessity to supercharge the motor of the vibrator. In this case, a modeling of the system with nonlinear elasticity and/or damping can lead to some theoretical results more accurate.

This article is taking into consideration a nonlinear modeling for the damping, where the damping coefficient has a polynomial variation function of the velocity. The reason of this approach is done by experimental observation on an inertial vibrating conveyor used to transport raw/grain materials. For this kind of materials, the interaction forces between particles themselves or between particles and the eaves are very complex and often lavatory, that’s why a nonlinear damping can explain the observed phenomena of resonance at over or sub harmonic frequencies or the raise of necessary power for driving motor.

2 The model of the vibrating system
Figure 1 shows the model of a conveyor driven by inertial vibrator with two eccentric synchronized masses. The used notations are:
- \( M \) – the total mass of the conveyor
- \( 2m \) – the total unbalanced masses
- \( k \) – elasticity coefficient of the conveyor’s springs
- \( b \) – the dissipation coefficient (that include the damping of the eaves’ seat and the equivalent dissipation of the transported material)
- \( Z \) – the vibrating direction
- \( z \) – the displacement of the conveyor’s eaves
- \( z_m \) – displacement of unbalanced/eccentric masses
- \( \varphi \) – rotation angle of the eccentric masses
- \( \omega \) – rotation velocity of the eccentric masses

The data (measured and calculated) of the real inertial vibrating conveyor used to numerical calculus (also to numerical simulation) are:
- the total mass of the conveyor (vibrating mass):
The equivalent coefficient of elasticity (of the steel plate springs): 

\[ k = 3 \times 10^5 \text{ Nm}^{-1} \]

- the rotational speed of eccentric masses:

\[ n = 948 \text{ rpm} \]

- the frequency of inertial excitation:

\[ f = 15.8 \text{ Hz} \]

- the pulsation of one direction driving force:

\[ \omega = 99.27 \text{ rad/s} \]

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- the total static moment of the eccentric masses:

\[ 2mr = m_0r = 1.2583 \text{ Kgm} \]

- the amplitude of one direction inertial force:

\[ F_0 = 12400 \text{ N} \]

- the natural pulsation of the conveyor:

\[ p = 34.641 \text{ rad/s} \]

- the critical value of the damping coefficient:

\[ b_{cr} = 17320.5 \text{ Ns/m}^{-1} \]

- the value of the steady state amplitude of forced vibration: \( A_M = 5.033 \text{ mm} \)

### 3 The analyze of necessary power characteristics

According to [1], the equations of the steady state vibrations of the conveyor are done by

\[
\begin{align*}
M^2 + b^2 + k^2 & = 2mrz \cos \omega t \\
M_M & = 2mr(g - \dot{z}) \sin \omega t
\end{align*}
\]

where \( M_M \) is the necessary motor moment;

\( 2mr \) - total static moment of the unbalanced masses.

#### 3.1 The dissipated power – linear model

The forced steady state vibrations of the conveyor is done by the relation

\[
z f(t) = \frac{2mrz^2}{\sqrt{(c - M_\omega^2)^2 + b^2\omega^2}} \cos \varphi ,
\]

where \( \varphi = \omega t - \arctan \frac{b_0}{c - M_\omega^2} \)

Taking into consideration only the linear dissipation, the necessary average power to drive the vibrator is

\[
P_{\text{aver}} = \frac{(2mr)^2\omega_0^6}{M\left[p^2 - \omega^2\right]^2 + 4n^2\omega^2} ,
\]

where \( n = \frac{b}{2M} \) is the damping ratio

\[
p = \sqrt{\frac{k}{M}} - \text{ natural pulsation}
\]

Usually, the nominal RPM of inertial vibrators are \( \omega = (3...5)p \), that meaning the average power is proportional with the damping ratio \( n \) and with the square of the pulsation \( \omega \).

Analyzing the relation (4), it may say that for resonance (\( \omega \approx p \)), the average power is proportional with the \( n^{-1} \) and with \( \omega^4 \).

Figure 2 shows the curves of power characteristics for the linear model, function of the pulsation \( \omega \) and the damping coefficient \( b_0 \).

#### 3.2 The dissipated power – nonlinear model

To analyze the necessary power for the model with nonlinear damping it consider the same inertial vibrating conveyor from fig. 1 with the construction data from §2.

The nonlinear damping is described by the polynomial function

\[
b = b_0 + \sum_{i=1}^{\infty} b_i |z|^i ,
\]

where \( b_0 \) is the statically damping coefficient (which describe the linear dissipation); \( b_i \ i = 1, \infty \) - the dynamical damping coefficients (which describe the nonlinear dissipation)

Taking into consideration only the first two
dynamical damping coefficients (for the dissipations proportional on the square and on the cubic of the deformation velocity), the differential moving equation is nonlinear as follows:

\[ Mz'' + b_0z' + b_1z'^2 + b_2z'^3 + kz = 2mr_0^2 \cos \omega t \quad (6) \]

The equation (6) can be solved only using numerical methods. To solve the equation (6), the author of the article has integrated it numerical, using a programme on the Borland® Pascal 7.0 based on the algorithm Runge-Kutta IV. The values of the nonlinear dissipation coefficients taken into consideration are: \( b_0 = 0 \)–\( 60000 \) Ns/m; \( b_1 = 0 \)–\( 180000 \) Ns²/m²; \( b_2 = 0 \)–\( 600000 \) Ns³/m³.

Figure 3 shows the curves of power characteristics for the dissipation proportional with the square of the velocity \( |z'|^2 \).

Also, fig. 4 shows the curves of power characteristics for the dissipation proportional with the cube of the velocity \( |z'|^3 \).

Figure 5 shows the amplitude of motion for the linear model.

Figure 6 shows the maximal displacement for the nonlinear model.
the cubic of the velocity $|\dot{z}|^3$.

4 Conclusion

Analyzing the power characteristics of the linear model from fig. 2, it may take the next conclusions:
- due to small value of the pulsation (and as a result, a small value of the amplitude of forced vibration, also for the velocity), the dissipated power is very small for $\omega \in [0...20]$ rad/s;
- at resonance, the dissipated power is very high due to the greatest values for the amplitudes of the forced vibrations; in this case, as the damping coefficient $b_0$ is lower, the necessary power is higher
- after the resonance, the necessary power raises in function of the damping coefficient and the pulsation;
- if the level of necessary power is done by the amplitude in resonance regime, after the resonance the frequency of the forced vibration is the main factor to increase the power.

Analyzing the power characteristics of the nonlinear model from fig. 3 and from fig. 4, it can issue the next conclusions:
- for frequencies values domain before resonance frequencies of linear model, that meaning $\omega = 0...30$ rad / s, the dissipated power is very low in comparison with the power of linear model;
- for the frequencies nearby the amplitude resonance of the linear model, that meaning $\omega = 30...40$ rad / s, the dissipated power is smaller than the power of the linear model and this decrease of the power is more obvious how much more the damping coefficients $b_1$ and/or $b_2$ are bigger; it may say that the “power resonance” phenomenon doesn’t appear anymore for the nonlinear model; that can be explain by the severe decreasing of the displacement, velocity and the damping force. That’s why, the power analyze for the nonlinear model must be done taking into consideration also the amplitude ($A$) characteristic; see these characteristics in the fig. 5 for linear model and in the fig. 6 and fig. 7 for nonlinear model;
- the curves of power characteristics for the damping force proportional with the square of the velocity (see fig. 3) are more spaced out than the curves of power characteristics for the damping force proportional with the cube of the velocity (from fig. 3); that means the dissipated power is more sensitive to the variation of $b_1$ damping coefficients than to the variation of $b_2$.

The power analyze can be done for the nonlinear modeling of the damping forces other than polynomial; the approaches proposed by the author in this article are of principle and can be extended and for other kind of nonlinear models of mechanical vibrating systems.

References: