A Feature Extraction Method of Image Based on Dimensional Transformation and SVD Algorithm

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Abstract: - In this paper, a new matrix based on dimensional transformation is used to present a image, and then a set of efficient image features are extracted by using SVD algorithm. Through Theory analysis and experiment results, the method has well stability to have broad application and high efficient recognition rate.

Key-Words: - Dimensional transformation; SVD algorithm; Feature extraction;

1 Introduction
Feature of the image itself is a reflection of the characteristics. Feature of the image is extracted to reflect on these images in real or abstract mathematical representation. As a result, image extraction is completed to identify the feature of the premise. At present, of the main types for the image to identify the feature are:
① intuitive features, such as the edge of the image, contour, texture and so on;
② gray demographic features, such as histograms, and so on;
③ transform coefficient features, such as Fourier transform, the Walsh transform, Haar transform, and so on. In addition, the features of algebra are often used to solve the problem of image recognition. SVD algebra as an effective feature extraction methods has been widely applied, because of their excellent properties in data compression, signal processing and pattern recognition and many other fields. For example, a researcher at the SVD algorithm based on a good face image feature extraction and recognition. Described in its image recognition only under certain constraints within the framework of an effective (such as cameras and face the relative location, direction and attitude can not be a significant change), because this image feature extraction method does not have the image of the translation, rotation and size of the non-sensitive, and thus do not have a wide range of applications[1][2].

2 A Matrix based on dimensional transformation which is used to present a image
In general, an image is indicated by matrix when extracting algebra feature, that is, the original image of \( m \times n \) is directly indicated in the form of \( A_{m \times n} \) matrix. We think this method does not have good stability, so this paper brings up a new image matrix method. As shown in Figure 1: Suppose \( o \) is the object center of mass, \( o \) is a circumscribed circle of object with a radius of \( R \), \( \alpha A_1 \) is a horizontal axis. The matrix \( A_{mn} = (a_y) \) which indicates the image sample can be generated by following steps: ① Choosing \( n \) pixels averagely on \( oA_1 \) and putting its gray value of the matrix \( A_{mn} \) as the first line \((a_{11}, a_{12}, a_{13}, \ldots, a_{1n})\); ② The \( \alpha A_2 \) axis can be acquired through rotating \( 360^\circ/m \) degrees anti-clockwise around \( o \), and the rotation starts from \( \alpha A_1 \); ③ Choosing \( n \) pixels averagely on \( oA_2 \) and putting its gray value of the matrix \( A_{mn} \) as the next line; ④ Repeating the above steps[3][4].

Figure 1 image matrix
In the course of choosing points at the same interval, the chosen points may fall between the two...
actual pixels. For example: If \( R / n \cdot j = p + q \), \( p \) is a non-negative integer, and \( 0 < q < 1 \), so the point corresponding to \( A_{ij} \) falls on the \( O A_i \) axis between \( P \) th and \((P+1)\)th pixel. At this time \( A_{ij} \) can be taken the following formula:

\[
a_{ij} = q \times g(p) + (1-q) \times g(p+1)
\]

\( g(p) \) expresses the gray value of \( P \) th pixel, and \( g(p+1) \) expresses the gray value of \( P+1 \) th pixel. Obviously, the matrix has a similar energy through above transforming by the same type of image samples.

3 SVD decomposition and image feature extraction

SVD singular values decomposition can transform any real matrix into the form of diagonal matrix. In the image processing, the following reasons are the main theoretical background about the application of SVD: ① The stability of the image singular values are better. Namely, though the image is imposed on the small disturbance, the singular values of image does not have any major changes; ② The singular values of image are shown rather than the intrinsic properties of the visual features. From the perspective of linear algebra, a gray image can be seen as a non-negative matrix, if an image is expressed as \( A \) and defined as \( A \in R^{m \times n} \) (for the sake of convenience, only the square matrix to discuss), \( R \) is defined as the real domain. So the definition of SVD decomposition about matrix \( A \) is the following formula:

\[
A=US\bar{V}=(\begin{array}{c}
\sqrt{\lambda_1} \\
0 \\
\sqrt{\lambda_2} \\
0 \\
\sqrt{\lambda_3} \\
\end{array}) (\begin{array}{c}
y_1 \ y_2 \ldots \ y_n \\
0 \\
0 \\
0 \\
0 \\
\end{array})
\]

Both \( U \in R^{m \times n} \) and \( V \in R^{n \times n} \) are real matrix, \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \), \( \sqrt{\lambda_i} \) is singular value of \( A \) which can be determined uniquely by formula (2), and \( \sqrt{\lambda_i} = \) the square root of \( AA^T \) eigenvalue.

For any real matrix \( A \), the singular value decomposition is unique in \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \) restrictions. According to the singular value decomposition of the image matrix is unique, we can use the singular value eigenvector of image matrix to describe the two-dimensional gray image. Singular vectors through dimensional transformation have several important characteristics as image features:

(1) dimensional transformation of image and immutability of displacement

A method based on dimensional transformation to present matrix for image of similar objects has good stability. Specially, it should meet the following condition for image movement and dimensional transformation:

\[
\| A-B \|_F < \varepsilon
\]

\( A \) is a matrix of dimensional transformation to original image, and \( B \) is a dimensional matrix through movement or zoom transformation to original image. In theory, it can be proved that the singular values decomposition of matrix is stable. Namely, when the matrix occurs small change, but SV eigenvectors do not have significant change. The following definition illustrates this point.

Definition: if matrix \( A, B \in R^{m \times n} \), and \( B-A = \Delta \), their corresponding singular values are \( \delta_1 \geq \delta_2 \geq \ldots \geq \delta_m, \tau_1 \geq \tau_2 \geq \ldots \geq \tau_n \), so the following formula is tenable to any invariable norm on \( R^{m \times n} \):

\[
\| diag(\delta_1, \ldots, \delta_m - \tau_1, \ldots, \tau_n)\| \leq \| B-A \|_F = \| \Delta \|
\]

Especially, if norm \( \varepsilon \), so formula (4) will become to formula (5):

\[
\sqrt{\sum_{i=1}^{m} (\tau_i - \delta_i)^2} \leq \| B-A \|_F
\]

To image movement and dimensional transformation, formula (6) can be acquired from formula (3) and (5).

\[
\sqrt{\sum_{i=1}^{m} (\tau_i - \delta_i)^2} \leq \varepsilon
\]

On-show: the new matrix method that adopted in this paper is used to present an image can assure extracted SV eigenvectors no sensitivity on moving in the displace and changing in the scaling.

(2) immutability of rotation transformation for image

According to the new matrix method based on dimensional transformation is used to present a image, any rotation transformation is equivalent to corresponding row replacement for the matrix of image. According to matrix theory, i and j rows of switching matrix \( \Delta \) are equivalent to the left side of
the matrix which are multiplied by such a following formula:

\[ E_y = E - (e_i - e_j)(e_i - e_j)^T \]  

(7)

In it, \( e_i, e_j \) are expressed in \( i \) th and \( j \) th of identity matrix \( E \) respectively.

Suppose \( A \) expresses original image, and \( E_y.A \) expresses the image after rotation transformation, so \((E_y.A)(E_y.A)^T \) of feature equation is:

\[ |(E_y.A)(E_y.A)^T - \lambda E| = 0 \]  

(8)

As \( E_y = E_y^T = E_{ji}^T \), formula (8) can be written:

\[ |E_y.A.A^T.E_y^T - \lambda E| = |AA^T - \lambda E| = 0 \]  

(9)

By this token, original image \( A \) and the image \( E_y.A \) after rotation transformation have the same eigenvector, namely have immutability of rotation transformation. We know, extracted features should have immutability in algebra and geometry for extracted methods of image recognition. Analysis above indicate SV features have these immutability for the matrix based on dimensional transformation to present an image\(^{[11]}\).

### 4. Experiment result

In order to identify the validity of feature extraction and recognized method, this paper designed an experiment about recognizing a plane model and human face. Sample five times under different sunshine condition, five black and white images can be acquired in the gray area in between 0 to 255 with spatial resolution of 256 × 256. By adjusting the height of the cameras, video cameras and aircraft model with the face of the relative position, Make five gray images exist rotation(refer to the plane rotation), movement or the scale differences. If \( I \) is sample image, \( \Omega = \{I_{ji}^{(i)}\}_{j=1}^{N_i} \) is the training sample collection of the \( i \) th image, and \( i = 1, 2, \ldots, M \), so the extraction and recognition course of image feature is\(^{[10]}\):

1. Work out the dimensional transformation \( A_{ji}^{(i)} \) corresponding to image \( I_{ji}^{(i)} \), in here \( A_{ji}^{(i)} \) is 32×32.
2. Work out the average image \( A^{(i)} \) of the \( i \) th plane image, namely:

\[ A^{(i)} = \frac{1}{N_i} \sum_{j=1}^{N_i} A_{ji}^{(i)} \]

3. SVD decompose \( A_{ji}^{(i)} \) to acquire the singular value eigenvector \( e = (\lambda_{1j}^{(i)}, \lambda_{2j}^{(i)}, \ldots, o, \ldots, o)_{1:32} \) of \( i \) th image.
4. According the nearest neighbor criteria to classify and recognise. Namely, if \( Y \) is a recognized image, which matrix of dimensional transformation is \( A_y \), and singular value eigenvector of \( A_y \) is \( e = (\lambda_{1j}^{(i)}, \lambda_{2j}^{(i)}, \ldots, o, \ldots, o)_{1:32} \), so the criteria of classification and identification is:

If

\[ \min_i \left\{ \sum_{j=1}^{32} (A_{ji}^{(i)} - A_{ij}^{(y)})^2 \right\} = \sum_{j=1}^{32} (A_{ji}^{(y)} - A_{ij}^{(j)})^2, r \in \{1, 2, \ldots, M\} \]

So \( Y \in \Omega \)

![Figure 2 recognition of different objects](image_url)

The experiment above shows: Because of adopting the matrix method based on dimensional transformation to present image, the matrix has a similar energy under Frobenius norm, so the feature extraction of algebra based on dimensional transformation of matrix, especially SVD decomposition and the singular value eigenvector have better stability, but they are Insensitive to image movement, rotation and scale change. Otherwise,
feature extraction and recognition method are also insensitive in this paper to aperture focus of vidicon, as well as image blur and image noise by sunshine condition. Thus it can assure the method has better applied area and higher recognition.

References: