Evaluation of seepage problem under a concrete dam with finite volume method

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Abstract: - In most of countries, underground waters are the most important sources to provide drinking water. So it is necessary to make scheme and to do high protection of to achieve maximum beneficiary. Necessity of this management is going to be felt by developing these sources and human's interference. However, in the past overtopping phenomenon was the first reason of dam's destruction, but nowadays by increasing of spate design's period, the significant problem that researchers are interfere with, is seepage problem. The purpose of solving the underground problems is to procure height of water as a function of coordinate and time. In observation of practical industries, we can use the mathematical models that simulate the water flow in porous medias. Often, in the large areas, examinations are impossible and too expensive. So, with computer simulation, more areas are going to be studied. In this paper, mathematic model of moving water under the concrete dams in porous media is to discussed. In this case, by according to descritization methods of governed equations in porous medias such as finite difference and finite element, finite volume technique is selected. Presentation model shows the amount of water that is waste from under the concrete dam. Herein, to possibility of solving the problem with this method, the media is simplified. For solving the equations, unstructured mesh is used. The effect of under-relaxation coefficient to increase the rate of convergence is to be illustrated. This factor is calculated and is equal to 0.995 that shows 10 percent or improvement in rate of convergence. At the end, the results of amount of seepage by using 3 powerful seepage codes (seep/w, mseep and plaxis) that are based on finite element method which are compared with FVseep model that is based on finite volume method and shows 5.1 percent of improvement in accuracy.
Key-words: seepage, finite volume method, unstructured mesh

1 Introduction

At first, seepage phenomena that is one of the most significant motive of dam's demolition in recent years, is studied by experimental and analytical equations or by simulating in minor models at laboratories. In many cases, these methods have no economical justifications and also they are faced with problems such as no accordance of prototype with the main model. But after developing computers with the power of high calculations and presentation of methods that can solve differential equations, researchers are inclined to use numerical methods to examine this phenomenon like other scientific problems.

Numerical methods can be classified by the way of solving equations and types of meshes. Finite difference, finite element and finite volume are the principal numerical equations which are used to solve differential equations that each one has abilities and deficiencies. In this paper, combination of finite volume by using rectangular grids which are adjusted with simple geometries is present. In this model the main variables are concrete dams, seepage, rectangular mesh, finite volume method and etc. In this paper, after debating on the theory of water movement in porous medias, basis of numerical methods and circumstances of them in flow simulation under the concrete dams, are explained. After that, finite volume method is explained as a numerical method in 2-dimensional convection-diffusion situation. Then, governing equations of seepage is discreted by using finite volume technique. At the end, correction test and model's calibration is done by using 3 powerful seepage codes (Mseep, Plaxis and Seep/w) that are based on finite element method. Scientists and researchers were interested in seepage problems from the past. Henry Darcy was one of the persons who tendered its basis [1]. This phenomenon can be introduced by Darci's Law and continuity equations. Patankar has applied finite volume method to evaluate heat transfer in 1972. Between 1978 and 1979, he has used finite volume method to study the fluid mechanics and he got good results. In 1980, he has used this procedure with a high accuracy in fluid dynamics. This method has a high flexibility even in problems with unstructured meshes.

This system has been developed by White in 1986. At the same year, Anderson has done a vast study on usage of this method to solve differential equations and achieved valuable results. In 1988, Wheiser used finite element method to evaluate elliptic problems. At the same time, Samon presented a new method that is called quadratic convergence, to solve differential equations and compare it with classic methods and finally found out that this method is adequate with finite volume technique. In 1996, Baranger and his colleagues studied corresponding between finite volume method and finite element method and got worth results. In 2003, Eymard used finite volume method in solving Navier-Stokes equations. At the same time, Angerman and his assistants exhibited cell-centered style that works with finite volume method and according to achieved results we can assure that is a idealist way to discrete differential equations. In 2004, Bertezolai investigated that finite volume technique in solving convection diffusion equations with non-structured grids. Finite volume method analyzed by Domolo in 2005. The advantage of finite volume method is to use it in every grid and without any limitations.

2 Governing equations on seepage in porous Medias:

A public flow sources such as ϕ is contemplated. By emphasizing on conservation equations, the summation of ϕ rate increasing by pure flow rate ϕ in one element is equal to rate of increasing ϕ because of diffusion and source. This substance in mathematic is called convection equation. We consider this equation in (1) as below:

$$\frac{\partial(\rho\phi)}{\partial t} + div\,(\rho\phi\,\vec{u}) = div\,(\Gamma\, gard\ \phi) + S_\phi \quad (1)$$

where, div ($\rho\phi\vec{u}$) is the convection term and $(\Gamma \mathrm{grad}\phi)$ is the diffusion term (Γ is the diffusion coefficient) [].

This equation is the first step on calculations in finite volume method. If we integrate from this equation in 3 dimensions, convection equations will be gained.

$$\int_{c\forall} \frac{\partial(\rho\phi)}{\partial t} d\forall + \int_{c\forall} div\,(\rho\phi\,\vec{u})d\forall = \int_{c\forall} div\,(\Gamma\, grad\ \phi)d\forall + \int S_\phi\, d\forall \quad (2)$$

By using Gauss Divergence Theory, we can suppose that convection and diffusion term can be explained as integral equations on boundary condition surfaces:

$$\frac{\partial}{\partial t}\int_{c\forall} \rho\phi\ d\forall + \int_{A} n.(\rho\phi\ \vec{u})dA = \int_{A} n.(\Gamma\,\mathrm{grad}\ \phi)dA + \int_{c\forall} S_\phi d\forall \quad (3)$$

In steady state problems, because of no mutation toward time, we can compress equation (3) and gain below equation:

$$\int_A n.(\rho\phi\ \vec{u})dA = \int_A n.(\Gamma\ grad\ \ \phi)dA + \int_{C\forall} S_\phi dA \quad (4)$$

In turbulent flows, we should integrate from equation (2) between time t to $t+\Delta t$:

$$\int_{\Delta t}\frac{\partial}{\partial t}\left(\int_{c\forall}\rho\phi d\forall\right)dt + \int_{\Delta t}\int_{A} n.(\rho\phi\vec{u})dAdt = \int_{\Delta t}\int_{A} n.(\Gamma\mathrm{grad}\ \phi)dAdt + \int_{\Delta t}\int_{c\forall} S_\phi\ d\forall dt \quad (5)$$

2.1. Finite volume method in diffusion:

Diffusion equation can be obtained by omitting the unsteady sources and convection term:

$$div(\Gamma grad\,\phi) + S_\phi = 0 \quad (6)$$

By integrating on equation (6), the final equation has been prepared:

$$\int_{C\forall} div(\Gamma grad\ \phi)d\forall + \int_{C\forall} S_\phi d\forall = \int_A n.(\Gamma grad\ \phi)dA + \int_{C\forall} S_\phi d\forall = 0 \quad (7)$$

Now we can descrited the above equation in 2-D media.

2.2. Usage of finite volume method in 2-D diffusion problems:

Governing equation on 2-D diffusion is illustrated as equation (8):

$$\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right) + S = \cdot$$

In figure (1) we can observe part of a 2-D grid that is used for descriting equations.

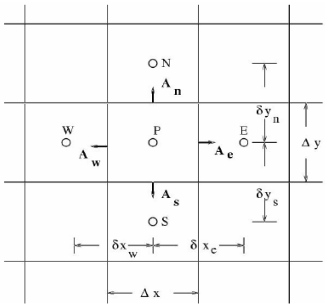


Figure (1).2-D grids

In 2-D grids, in addition to points which are situated on right (E) and left (W), two more points denominated N and S are positioned on up and down. Now, we can use the (8) equation as (9):

$$\int_{\Delta\forall}\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)\partial x\partial y+\int_{\Delta\forall}\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)\partial x\partial y+\int_{\Delta\forall}S_{\phi}\partial\forall=0 \quad (9)$$

According to $A_e = A_w = \Delta y$ and $A_n = A_s = \Delta x$, equation (9) can be corrected as follows:

$$\left[\Gamma_e A_e\left(\frac{\partial\phi}{\partial x}\right)_e-\Gamma_w A_w\left(\frac{\partial\phi}{\partial x}\right)_w\right]+\left[\Gamma_n A_n\left(\frac{\partial\phi}{\partial y}\right)_n-\Gamma_s A_s\left(\frac{\partial\phi}{\partial y}\right)_s\right]+\bar{s}\Delta\forall=0 \quad (10)$$

As is mentioned before, this equation shows the equilibrium of ϕ in a control volume. Therefore, we can calculate the transmission flow in a control volume as below equations:

Transmission flow from left

$$\Gamma_w A_w\left.\frac{\partial\phi}{\partial x}\right|_w=\Gamma_w A_w\frac{(\phi_p-\phi_w)}{\delta x_{wp}} \quad (11)$$

Transmission flow from right

$$\Gamma_e A_e\left.\frac{\partial\phi}{\partial x}\right|_e=\Gamma_e A_e\frac{(\phi_e-\phi_p)}{\delta x_{pE}} \quad (12)$$

Transmission flow from top

$$\Gamma_n A_n\left.\frac{\partial\phi}{\partial y}\right|_n=\Gamma_n A_n\frac{(\phi_N-\phi_P)}{\delta y_{PN}} \quad (13)$$

Transmission flow from below

$$\Gamma_s A_s\left.\frac{\partial\phi}{\partial y}\right|_S=\Gamma_S A_S\frac{(\phi_P-\phi_S)}{\delta y_{sp}} \quad (14)$$

By substituting these equations in (10) equation, we can have equation (15):

$$\Gamma_e A_e\frac{(\phi_E-\phi_P)}{\delta x_{PE}}-\Gamma_W A_W\frac{(\phi_P-\phi_W)}{\delta x_{wp}}+\Gamma_n A_n\frac{(\phi_N-\phi_P)}{\delta y_{PN}}-\Gamma_S A_S\frac{(\phi_P-\phi_S)}{\delta y_{sp}}+\bar{S}\Delta\forall=0$$

(15)

If we consider the source term as a linear equation, equation (15) can be illustrated like equation (16) as follows:

$$\left(\frac{\Gamma wA_w}{\partial x_{wp}}+\frac{\Gamma_e A_E}{\partial x_{wp}}+\frac{\Gamma_s A_S}{\partial Y_{PN}}+\frac{\Gamma_n A_n}{\partial y_{pN}}-S_P\right)\phi_P= \quad (16)$$

$$\left(\frac{\Gamma_W A_W}{\partial x_{wp}}\right)\phi_w+\left(\frac{\Gamma_e A_e}{\partial x_{pE}}\right)\phi_E\left(\frac{\Gamma_S A_S}{\partial Y_{SP}}\right)+\phi_S\left(\frac{\Gamma_n A_n}{\partial y_{PN}}\right)\phi_n+S_u$$

$$a_p\phi_p=a_W\phi_W+a_E\phi_E+a_s\phi_s+a_n\phi_n+S_u$$

This equation is a final form of Laplace equation and we should program our code in Matlab software. This code is called "FVseep" that contains sub-programs that each one calculates particular things. These sub-programs are managed by a main m-file and after preparing unstructured meshes, flow lines and potential lines and also the amount of wasting water are calculated.

2.3. Presentation of FVseep Model:

Fvseep model has a main routine and four sub-routines. The main program is managed sub-routines and controlled them. At first, by calling sub-programs, Laplace equation has solved in porous media. This program has got some abilities like soil situation definition, soil porosity and hydraulic conductivity in two directions.

2.3.1. Structure of sub-routines:

Sub-programs have below roles in FVseep model:

- Geometry Definition
- Calculations of control volume
- Calculating governing equations
- Preparing graphical solutions

This model, divides the porous media by rectangular meshes. In this case, we have 504 elements that intersect the media into smaller pieces. An appropriate grid in numerical methods is a grid that can increase the rate of solutions in calculation's procedure and causes the best accuracy with minimum iterations. So, type and the geometry of grids have a high effect on results in numerical methods. In this paper, we use special grids that are adapted with our geometry. For the first time, Copler has used unstructured meshes in 2-D Medias. Unstructured grids have different dimensions in length. Seven nodes are illustrated in figure (2) to evaluate average velocities by FVseep model. The coordinates of the nodes are given in table (1).

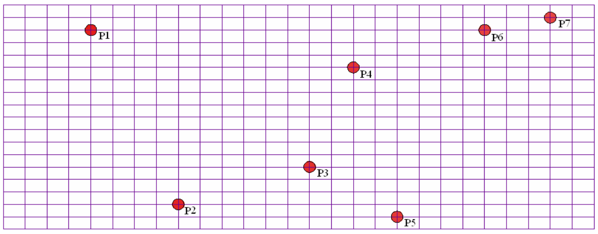


Figure (2).positions of nodes

Y(M)	X(M)	NODE NUMBER
5.33	2.22	P1
0.67	4.44	P2
1.67	7.78	P3
4.33	8.89	P4
0.33	10.6	P5
4.33	12.2	P6
5.67	14.44	P7

Table (1).node's coordinate

Here, comparison of the results between FVseep model and three finite element programs are given in table (2).

In figure (3) the amount of velocity in two different directions is illustrated at point P4 by using FVseep model. This model has the ability to show these values for favorite nodes.

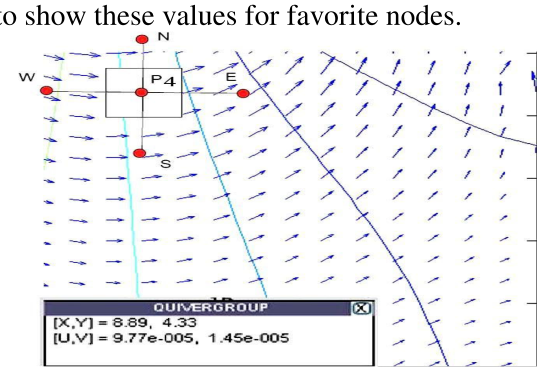


Figure (3). Vector of velocity in 2 directions

According to the above explanation, we have compared the value of velocities in mentioned nodes by finite element method and the results are given in table (2).

FVSEEP	MSEEP	شمارهٔ گره
$\times 10^{-5}\frac{m}{s}$	$\times 10^{-5}\frac{m}{s}$	
5.0640	4.9300	P1
5.1145	4.7440	P2
8.8090	8.5163	P3
9.8770	9.7956	P4
4.6777	4.6278	P5
6.8589	6.7634	P6
4.5063	4.3226	P7

Table (2). Velocity results by finite element and finite volume method

In the same manner that is mentioned in table (2), the maximum difference between finite volume method and finite element method is observed in node P2 and its value is 7.2 percent that shows a desirable conformity of results between two descriting methods. To increase safety factor, we have compared these consequences by using statistic science that can assure the integrity of conclusions. There is a graphical method in statistics which is called Q-Q graph to evaluate quadrants and shows appropriate comparison. In this manner, one line is automatically pached into nodes. If the nodes are gathered around this line, we can get higher independency of these results and we can rely on our model.

In this paper, to prepare this graph, we use Bloom's method that is resulting a linear line as is shown in figure (1).

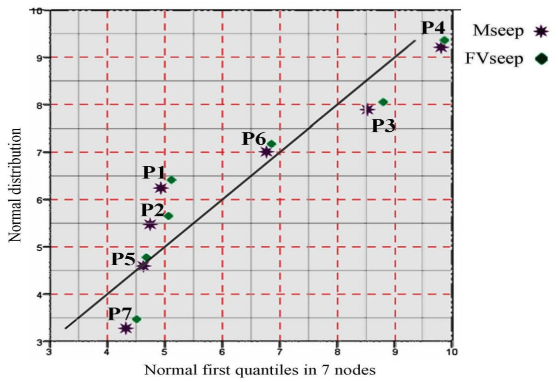
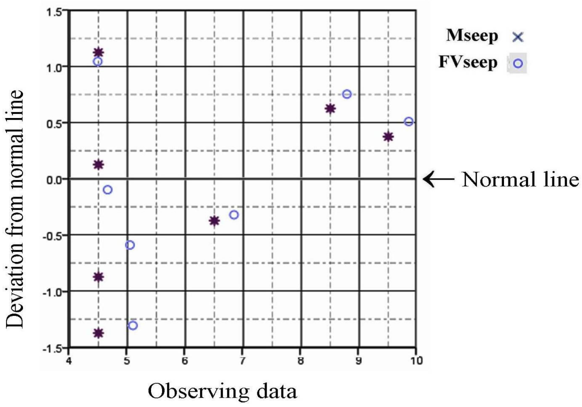


Figure (1). Q-Q plot

As it is shown in figure (1), the value of velocities that are calculated by finite element and finite volume method have an appropriate distribution around line $y=x$. This subject shows high independency between results of finite element method and finite volume method.

In figure (2), deviation of velocity values are illustrated. As it is shown, similar nodes approximately have the same distance from normal line.



Figure(2).deviation from normal line

So, we can trust the results of FVseep model with high safety factor. In continue, we precisely analyzed the amount of seepage under a small concrete dam by using 3 powerful software that are based on finite element method and then compared their results by FVseep model.

Figure (4) shows geometry of this case study. This figure shows a small concrete dam. Potential head on upstream is 10 m and on down stream is 1 m. hydraulic conductivity coefficient in 2 directions is $10^{-4}\,\dfrac{m}{s}$. This area is supposed to be porous media, rectangular and has a 15 m in length and 6 m in height.

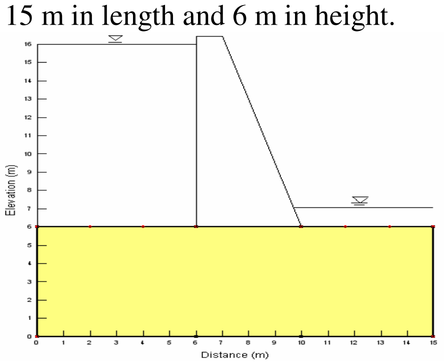
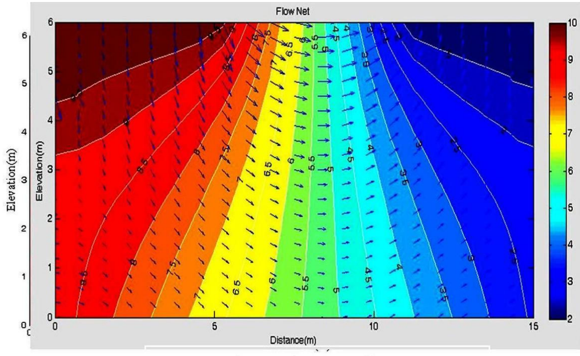
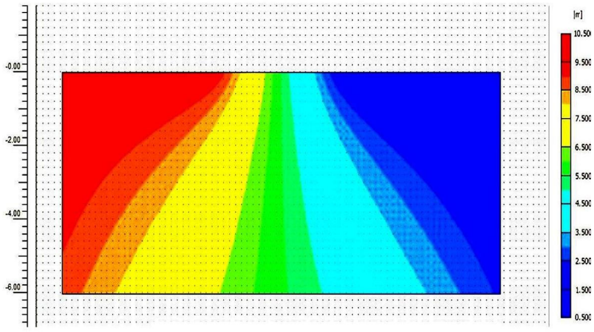


Figure (4).Geometry of porous media

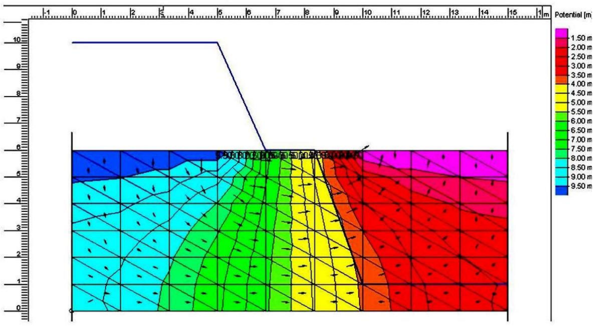
The amount of error in all soft wares is 0.5×10^{-6} and the maximum of iterations is 1000. In figure (5) are shown flow lines which are solved by finite element method and finite volume method.



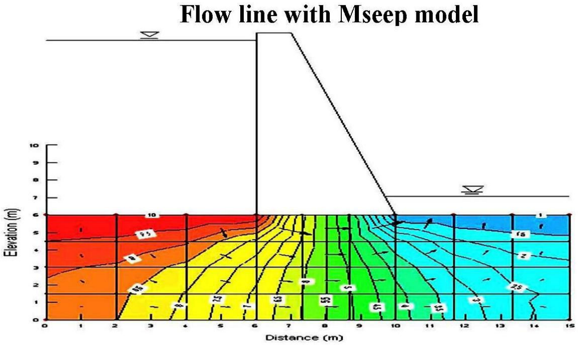
Flow line with FVSeep model



Flow line with Plaxis model



Flow line with Mseep model



Flow line with Seep/w model

Figure (5).finite element and volume flow lines

As it is seen, they have approximately similar graphical results. In table (3) velocity and amount of discharge results are given. By according to these values, difference between the results is about 5.1 percent that is appropriate with numerical methods.

DISCHARGE $\frac{m^3}{s}\times10^{-5}$	ELEMENT NUMBER	TYPE OF MESH	SOFTWARE
25.9722	504	unstructured	FVseep
26.3680	207	unstructured	Mseep
28.2182	216	unstructured	Plaxis
27.3920	36	unstructured	Seep/w

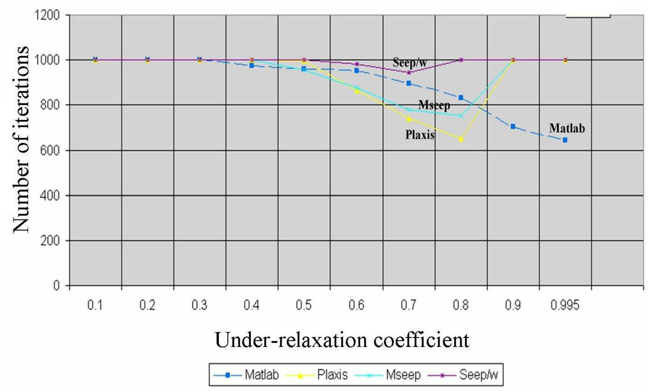
Table (3).amount of discharges by 2 method

In continue, for improving and increasing the rate of convergence, we have studied the effect of under relaxation coefficient and we are going to see what happened in number of iterations.
Under-Relaxation method:
When we use iterative methods for obtaining solutions or resolving nonlinearities, it is normally necessary to control the rate of each variable that are changing during iterations. For example when we have a strong non-linearity in a temperature source term and our initial guess is far from the solution, we may get large oscillations in the temperature computed during the iterations, making it difficult for the iteration to proceed. In such cases, we often employ under-relaxation. This method was presented by Yang and Franckel in 1950. We can write equation () to explain under-relaxations method:

$$\frac{a_p}{\alpha}\phi_p = \sum_{nb} a_{nb}\phi_{nb} + b + \frac{1-\alpha}{\alpha}a_p\phi_p^*$$

Now by using this coefficient and enter it to our code and also in 3 finite element program, allows the solution to move quickly to convergence, but may be more prone to divergence in prime iterations.
Results show us that amount of iterations by using this coefficient has improved about 10 percent that should be a good result.
We should consider that this value is not unique and in each problem by according to many conditions can be gain just by .

Graph (3) shows circumstances of the process of divergence in each program. This graph shows changes of under-relaxation coefficient against number of iterations.



Graph (3).divergence process

We can conclude two results, first one is the number of iterations until getting convergence negotiate descend procedure. The second one by increasing under-relaxation coefficient, we see intensive in the primary iterations.
We should consider that if we don't use under-relaxation coefficient in FVSeep code, convergence is going to be gain after 702 iteration, but by applying this value, iterations going to be less and convergence is going to be get after 643 iterate, that shows 10 percent improvement in convergence rate.
The value of under-relaxation coefficient is about 0.995. By according to adjacency of this number to 1, we can find out that the result is appropriate.

Conclusions:

- While using iterative methods to solve seepage problem in FVSeep model, we can find out this method is more appropriate than straight methods. Because by increasing the geometry of problem in 2 or 3 dimensions, we are facing with the memory of computers is going to be less. In this case iterative methods are more comfortable to solve matrix.
- By using under-relaxation coefficient, that are produced in first steps of iteration are neutralized.
- We are assured that under-relaxation is only a change in the path to solution, and not in the discretization itself. Thus, using or

deny using of this method have no effect in results and just rate of convergence is going to increase.

- The optimum value of α depends strongly on the nature of the system of equations we are solving, on how strong the non-linearities are, on grid size and so on. A value close to unity allows the solution to move quickly towards convergence, but may be more prone to divergence.

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