Fractal Dimension and Lacunarity of Psoriatic Lesions — A Colour Approach —

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Abstract: - Fractal measures like fractal dimension and lacunarity are widely-used for the analysis of textures, or any images with self-similar content. However, all the existing approaches are defined for one dimensional signals or binary images with extension to grey-scale images. We propose a colour approach, derived from the existing probabilistic algorithm for the computation of the fractal dimension and lacunarity. We used this approach for the evaluation of the skin lesions, in case of psoriasis. We show that there is correlation between the severity of a psoriatic lesion and the fractal dimension, when we compare two lesions of the same person. Lacunarity can be used as a complementary tool to interpret the results and distinguish between two lesions with the same fractal dimension. We discuss the results and then draw the conclusions.

Key-Words: - fractal dimension, lacunarity, colour images, texture, psoriasis, skin lesions

1 Introduction

The fractal geometry introduced by B. Mandelbrot in 1983 is used to describe self-similar sets called fractals and natural objects that are impossible to describe by using the classical geometry [1]. Fractal dimension is a measure that characterizes the irregularity and the complexity of a fractal set, indicating how much space is filled, while the lacunarity is a mass distribution function indicating how the space is occupied [2]. These two fractal properties are successfully used to discriminate between different structures exhibiting a fractal-like appearance [3, 4, 5], for classification and segmentation, due to their invariance to scale, rotation or translation. The fractal geometry proved to be of a great interest for the digital image processing and analysis in an extremely wide area of applications, especially in medicine [6, 4, 7].

From the different expressions directly linked to the theoretical one of the Minkowski-Bouligand [8], the box-counting approach is one the most popular due to the simplest algorithmic formulation, compared to the Hausdorff dimension. The box-counting fractal dimension is $D=-\log N_\delta/\log\delta$, where N_δ is the number of boxes of size δ needed to cover the fractal set.

The first practical approach initiated by Mandelbrot, was soon followed by the elegant probability measure of Voss [9, 10]. On a parallel research path, Allain and Cloitre [11] and Plotnick [12] developed their approach as a version of the basic box-counting algorithm. All the other approaches for the computation of the fractal dimension, like δ-parallel body method1 [13] or fuzzy [14] are more complex from a point of view of implementation and more difficult to extend to a colour multi-dimensional space. On the other hand, despite the large number of algorithmic approaches for the computation of the fractal dimension and lacunarity, only few of them offer the theoretical background that links them to the Hausdorff dimension.

However, such tools were developed long time ago for gray-scale small-size images and they are used "as they are" despite the fact that the acquisition techniques evolved (therefore the spatial resolution and quantification have changed) and the world of images became coloured. In addition, in medicine and biology, the colour is an important characteristic, very useful–for instance–for the identification of a certain tissue or the evaluation for of the severity of a skin lesion. The very few existing approaches for the computation of fractal measures for colour images are restricted to a marginal colour analysis and transform a grey scale problem in false colour [7]. We propose to extend the existing probabilistic algorithm by Voss [9] to colour domain [15] and therefore to define and compute the fractal dimension and the lacunarity for colour images.

2 The Colour Approach

The probabilistic algorithm defined by Voss [9] upon the proposal from Mandelbrot [1] considers the image as a set of points S in a Euclidian space of dimension E. The special arrangement of the set is characterized by the probability matrix $P(m,\delta)$, the probability of having m points inside a cube of size δ (called box), centered in an arbitrary point of S. The matrix is normalized so that

$$\sum_{m=1}^{N} P(m,\delta) = 1 \qquad (1)$$

where N is the number of pixels included in a box of size δ. Given the total number of points in the image is M, the number of boxes that contain m points is $(M/m)P(m, \delta)$.

The total number of boxes needed to cover the image is:

$$\langle N(\delta) \rangle = \sum_{m=1}^{N} \frac{M}{m} P(m,\delta) = M \sum_{m=1}^{N} \frac{1}{m} P(m,\delta) \qquad (2)$$

A grey-level image is a discrete surface $z = f(x,y)$ where z is the luminance in every (x,y) point of the space. A colour image is a hyper-surface in a colour space, like RGB for instance: $f(x,y) = (r,g,b)$. Therefore in the case of colour images we deal with a 5-dimensional Euclidian hyper-space and each pixel can be seen as a 5-dimensional vector. The classical algorithm of Voss uses boxes of variable size δ centered in the each pixel of the image and counts the number of pixels that fall inside that box. We generalize this by counting the pixels $F = f(x,y, r,g,b)$ for which the Minkowski infinity norm distance to the center of the hypercube is smaller than δ. Practically, for a certain square of size δ in the (x,y) plane, we count the number of pixels that fall inside a 3-dimensional RGB cube of size δ, centered in the current pixel. The theoretical development and validation on synthetic colour fractal images can be found in [15].

Even from the very beginning, when Mandelbrot introduced the fractal geometry, he was aware by the fact that the fractal dimension itself is not sufficient to fully capture the complexity of non deterministic objects. He defined and used the lacunarity as a complementary metric. Later on, Voss expressed it based on the probabilities $P(m, \delta)$ and using the first and second order moments of the measure distribution (3). Following the previous consideration, the lacunarity can be therefore computed for colour images as well.

$$\Lambda(\delta) = \frac{M^2(\delta) - (M(\delta))^2}{(M(\delta))^2} \qquad (3)$$

$$M(\delta) = \sum_{m=1}^{N} mP(m,\delta) \qquad (4)$$

$$M^2(\delta) = \sum_{m=1}^{N} m^2 P(m,\delta) \qquad (5)$$

The lacunarity is linked to the topological organisation of objects in an image, being a scale-dependent measure of spatial heterogeneity. Images with small lacunarity are more homogeneous with respect to the size distribution and spatial arrangement of gaps. On the other hand, images with larger lacunarity are more heterogeneous. In addition, lacunarity must be taken into consideration after inspecting the fractal dimension: in a similar manner with the Hue-Saturation couple in colour image analysis, the lacunarity becomes of greater importance when complexity, i.e. the fractal dimension, increases.

3 Fractals and Psoriatic Lesions

Human skin has a particular colour and texture that varies from individual to individual. Skin colour is determined by the content of the melanic pigment and its distribution in melanosomes, the cutaneous vascularization, the quantity of reduced hemoglobin and oxyhemoglobin in the superficial derma, the quantity of carotene and other coloring substances, as well as the thickness and distribution of the keratin layer. Skin surface is defined by thin ditches that intersect, forming a rhomboidal network characteristic for the normal healthy skin; by pores and by hair, of different dimensions, thickness and density [16].

However, the surface structure of a skin lesion reflects both the structure (anatomical elements like ditches, hair etc.) and the physio-pathological modifications, and implicitly the morpho-pathological modifications whose expression corresponds to the redness, scaliness and thickness of a psoriatic lesion. The redness is due to intense blood vessel dilatation, the scaliness due to the increased cell turn-over of the derma, while thickness is due to the inflammatory phenomena. All these elements lead to a non-uniform, fractal-like aspect of the skin lesion. The goal of a treatment is to block the physio-pathological mechanisms of the disease, in order to reestablish the normal functions and anatomy of the skin. The consequence is the reduction of the "defects" of the skin and the rendering of the original healthy aspect of the skin surface. In terms of fractal analysis, this means reducing the complexity of the structure - i.e. the colour digital image - corresponding to the skin sample and therefore reducing the fractal dimension or lacunarity. The tools we propose can be used not only to quantify the severity of a skin lesion, but also to evaluate the effectiveness of any treatment through the objective assessment of the therapeutical consequences regarding the skin aspect.

4 Results

For a uniform colour image, the fractal dimension is 2.0, indicating zero complexity. In the case of healthy skin, the colour fractal dimension is usually less than 3.0, being comprised between 2.5 and 2.8, depending on the complexity of the texture of the skin. Such values express a great complexity in the grey-scale domain, but as the value is less than 3.0, they are close to a complex surface in the colour domain.

Psoriasis lesions are of particular interest from the point of view of colour fractal analysis: the erythema, the scaliness and the thickness determine random variations

in the shape and size of ditches, as well in the local contrast of the image. The consequence is a large spread of information in the 5-dimensional RGB colour space (see Figure 1).

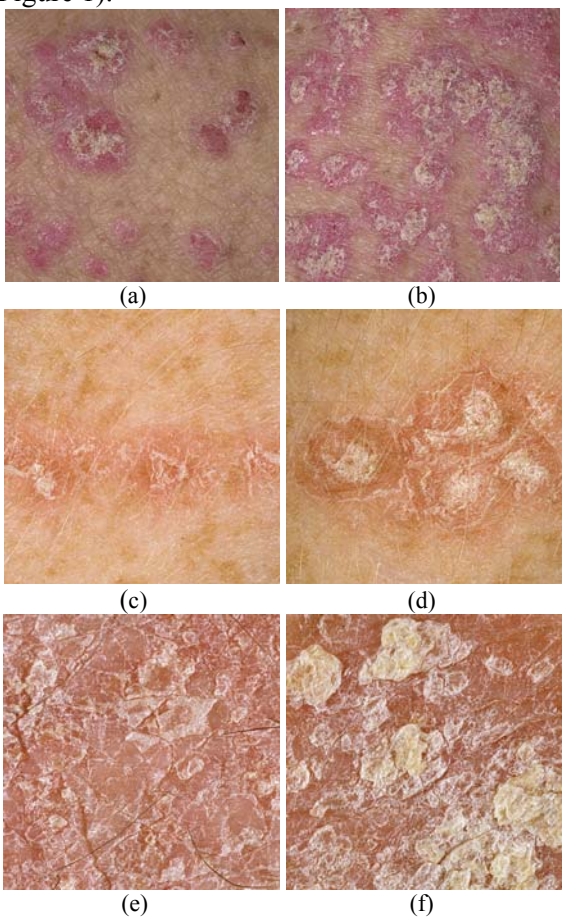

Fig. 1. Six psoriasis lesions (courtesy of Dermnet Skin Disease Image Atlas, http://www.dermnet.com).

As shown in Table 1, the fractal dimension computed for images of psoriatic lesions have a higher complexity than healthy skin images. As visually perceived, the texture complexity is due to the addition of some new characteristics of lesions on the skin texture, and this increased complexity is well captured by the fractal dimension. Due to the colour formalization of our approach, the probability measure is more sensitive to important local contrast compared to the natural skin colour. In these images, such contrasts are produced by the alternation between redness and scaliness of the lesions.

To validate our colour fractal dimension and lacunarity as objective measures, we compare them against the scores evaluated by the dermatological specialist for three characteristics of the psoriatic lesions: the erythema (redness), the scaliness and the induration (thickness) (see Table 1) The three scores are on a severity scale from 0 to 4, in accordance with the PASI score widely-used to evaluate the severity of psoriasis lesions [17]. The last row represents the global severity, by taking into account all three characteristics of the lesions. We compared the six images by pairs: (a,b), (c,d) and (e,f). As we can see, the relationship between the severity–either global or for each of the three characteristics–of the lesions and the fractal dimension is direct.

image	a	b	c	d	e	f
f.dim.	3.16	3.33	3.64	3.78	3.88	3.99
erythema	3	3	1	2	2	3
scaliness	1	2	1	2	2	3
thickness	3	3	0	2	1	1
Severity	2	3	1	2	1	2

Table 1. The fractal dimension, the erythema, the scaliness, the thickness and the global severity.

The corresponding lacunarity curves are depicted in Figure 2.

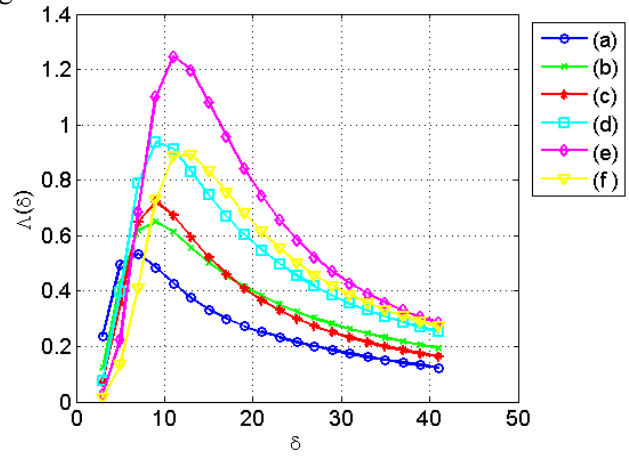

Fig. 2. Lacunarity curves for the images in Figure 1.

One can see that the curve for image (b) is placed higher than the curve for (a) indicating a more lacunar and heterogeneous image. The same statement is valid for (d) and (c). The complexity revealed by the lacunarity curves is therefore in accordance with the fractal dimension and the severity from Table 1, a less lacunar image being close to the healthy skin. However, a situation that looks peculiar is the fact that the curve for image (e) is on top of all curves, indicating the most lacunar image. This is valid for the size boxes in our experiments (between 10 and 30 pixels or RGB difference) where despite the fact that the first order moment is larger for the image (f), the second order moment is larger as well. For larger boxes—larger than the size of the white objects in image (f)—the situation has the tendency to invert. Note that for the images (e) and (f) the level of detail is increased compared to the others, therefore the contrasts are important especially for very small objects. Also note the similarity and the inversion for curves (b) and (c) - for small boxes image (c) appears to be more lacunar, while for larger boxes image (b) is more lacunar—the colour "mountains" and

"valleys" are very similar for both images, but the complexity revealed by the fractal dimension is different.

5 Conclusions and Future Work

In this paper we show how our colour extension of the classical probabilistic algorithm widely used to compute the fractal dimension and lacunarity can be used for the analysis of colour digital images of psoriatic lesions. We demonstrated that our mathematical tool is able to reveal the relative complexity of colour images of skin lesions and how the results can be related to the severity of the lesions. Therefore, the fractal dimension can be used as a global score to objectively assess the severity of the lesions. We are aware that just one single value—the fractal dimension—and a distribution—the lacunarity—cannot solve the issue of the correct diagnostic. However, our purpose was to develop a complementary approach to existing ones, based on fractal features. Because fractal features are well correlated to the perceived complexity by the human vision system, they are of great interest as objective tools in a diagnostic set. In addition, we want to determine the lighting and distance conditions for image acquisition, in order to work on calibrated images, that will allow for a more pertinent comparison and fractal analysis.

The perceived complexity of a colour image representing a psoriatic lesion may be smaller than the one indicated by our colour fractal dimension, due to the choice of colour space. We chose to use the RGB colour space because it perfectly suits the probabilistic approach, and the extension from cubes to hype-cubes was natural and intuitive. In addition, like in any colour approach, it is not possible to find the right colour space for all the decisions. However, this colour expression of the fractal measure is not limited to only one colour space. Our plan is to consider a formalism which embeds a measure and complementary information to separate the different aspects of the problem. In particular, by linear geometric transformation of the RGB space, it is possible to express directly the erythema without changing the formulation. In this way, the colour formulation remains generic, while the transformation of the colour space adapts to the purpose.

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