Digital Phase-Locked Loop and its Realization

Tsai-Sheng Kao[1], Sheng-Chih Chen[2], Yuan-Chang Chang[1], Sheng-Yun Hou[1], and Chang-Jung Juan[1]
[1]Department of Electronic Engineering,
Hwa-Hsia Institute of Technology.
No. 111 Gong Jhuan Road, Chung Ho,
Taipei County 23568, Taiwan, R.O.C.
TsaiShengKao@gmail.com
[2]Graduate Program of Digital Contens and Technologies,
ChengChi University.
No. 64, Sec. 2, ZhiNan Rd., Wenshan District,
Taipei City 11605, Taiwan, R.O.C.

Abstract: The realization of a digital phase-locked loop (DPLL) requires to choose a suitable phase detector and to design an appropriate loop filter; these tasks are commonly nontrivial in most applications. In this paper, the DPLL system is first formulated as a state estimation problem; then an extended Kalman filter (EKF) is applied to realize this DPLL for estimating the sampling phase. Therefore, the phase detector and loop filter are simply realized by the EKF. The proposed DPLL has a simple structure and low realization complexity. Computer simulations for a conventional DPLL system are given to compare with those for the proposed timing recovery system. Simulation results indicate that the proposed realization can estimate the input phase rapidly without causing a large jittering.

Key–Words: Digital phase-locked loop (DPLL), state estimation, extended Kalman filter (EKF)

1 Introduction

Phase-locked loop (PLL) which constitutes a basic building block for many synchronizers like carrier recovery or timing recovery is essential in most digital communication systems. Owing to the continued advancement in VLSI, all digital phase-locked loop (DPLL) has been under extensive investigation for several years [1, 2]. To realize a DPLL system, however, the selection of a phase detector [3, 4, 5] is crucial and the design of a loop filter is nontrivial.

A DPLL is, in general, a nonlinear system due to the nonlinear behavior of the phase detector. Unfortunately, few studies have been published on modeling a phase detector. Hence, the loop filter design often ignores the dynamics of the phase detector, causing the performance of the DPLL less reliable. The conventional loop filter design involves in selecting the order of the loop filter and determining its loop gains such that the performance of a DPLL satisfies fast phase acquisition and small phase jitter. However, the two characteristics of conventional DPLL systems with fixed loop gains are contradictory since fast phase acquisition requires wide loop bandwidth and small phase jitter requires narrow loop bandwidth [6, 7]. Moreover, the determination of the loop gains is difficult using the transfer function approach, especially when the order of the loop filter is high. A Kalman filter (KF) was realized as a loop filter to fulfill the above characteristics together with time-variant loop gains [8, 9, 10, 11], and these Kalman gains were shown to be equivalent to the time-variant loop gains of a DPLL. The performance of this DPLL bit synchronizer is significantly improved by using these time-variant loop gains in place of the fixed gains of a conventional DPLL.

Although Driessen [8] used a KF to realize the loop filter of a DPLL, this realization did not take the phase detector into account and the timing information was assumed to be known in advance. In this paper, we use an extended Kalman filter (EKF) to realize the loop filter as well as the phase detector of a DPLL, and the loop gains are easily obtained via the extended Kalman filtering techniques. The proposed system has a simple structure and low realization complexity.

The rest of the paper is organized as follows. In section II, the channel model is described and the function of a DPLL is briefly reviewed. In section III, we formulate the DPLL system as a state estimation problem and apply an EKF to realize this DPLL. In section IV, phase domain models of both a conventional DPLL and the proposed EKF-based DPLL are described. In section V, simulations are shown to verify the proposed DPLL. Finally, conclusions are given in section V.

2 Channel Model and DPLL System Overview

The baseband model of a synchronous data transmission system is shown in Fig. 1. The information sequence $\{a_k\}$ is independently chosen from the set of $\{1, -1\}$ with equal probability, and the data bit a_k is transmitted through a transmission channel at time instant $t_k = (k - \epsilon_k)T$, where T is the bit interval and ϵ_k is the input phase, normalized with respect to T. Owing to the channel imperfections, an equalizer is commonly included to eliminate the intersymbol interference. Thus, the overall impulse response $h(t)$ encompasses the transmission channel and an equalizer; the output of the equalizer can then be described as:

$$r(t) = \sum_{i=-\infty}^{\infty} a_i h(t - (i - \epsilon_i)T) + n(t) \qquad (1)$$

where $n(t)$ is the filtered noise.

Assume ϵ_k is slowly time-variant, and write the sampling data of (1) at time instant $\hat{t}_k = (k - \hat{\epsilon}_{k|k-1})T$ as:

$$r_k = \sum_{i=-L}^{L} a_{k-i} h_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T) + n_k \qquad (2)$$

where $r_k = r((k - \hat{\epsilon}_{k|k-1})T)$, $n_k = n((k - \hat{\epsilon}_{k|k-1})T)$, and L is chosen such that the term, $h_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T) = h(iT + (\epsilon_k - \hat{\epsilon}_{k|k-1})T)$, is negligible for $|i| > L$. In a compact form, (2) is rewritten as:

$$\begin{aligned} r_k &= \boldsymbol{a}_k^T \boldsymbol{h}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) + n_k \\ &= y_k + n_k \end{aligned} \qquad (3)$$

where the data vector $\boldsymbol{a}_k = [a_{k+L} \cdots a_k \cdots a_{k-L}]^T$, the channel parameter vector $\boldsymbol{h}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) = [h_{-L}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h_0((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h_L((\epsilon_k - \hat{\epsilon}_{k|k-1})T)]^T$, and the superscript T denotes the transpose. Thus, y_k denotes the noise-free data of r_k. The DPLL processes the measurement data r_k to adjust the sampling time $\hat{t}_k$ such that the timing error $e_k = t_k - \hat{t}_k = (\epsilon_k - \hat{\epsilon}_{k|k-1})T$ is approaching zero, where $\hat{\epsilon}_{k|k-1}$ is the predicted estimate of ϵ_k. Specifically, the information sequence $\{a_k\}$ is either known as a prior in the training mode or replaced by its estimate $\{\hat{a}_k\}$ in the tracking mode, and this class of DPLLs is classified as the data-aided (DA) structure.

3 Extended Kalman Filter for a DPLL System

In this section, we formulate the DPLL system as a state estimation problem, and an EKF is applied to realize this DPLL for estimating the sampling phase due to the nonlinear relationship between the input phase and the noise-free data y_k. To establish the model [8], define the state vector $\boldsymbol{x}_k = [\epsilon_k \ \dot{\epsilon}_k]^T$, where $\dot{\epsilon}_k$ is the timing offset; write the process equation as:

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{v}_k \qquad (4)$$

where the state transition matrix $\boldsymbol{\Phi} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\boldsymbol{v}_k = \begin{bmatrix} u_k \\ w_k \end{bmatrix}$ representing a zero mean phase jitter u_k and zero mean timing offset jitter w_k. Since the noise-free data y_k of the measurement equation (3) is a nonlinear function of the state vector $\boldsymbol{x}_k = [\epsilon_k \ \dot{\epsilon}_k]^T$, it is linearized about the predicted estimate $\hat{\boldsymbol{x}}_{k|k-1}$ of $\boldsymbol{x}_k$ as:

$$y_k = \boldsymbol{a}_k^T \boldsymbol{h}(0) + \boldsymbol{H}_k(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1}) \qquad (5)$$

The vector $\boldsymbol{h}(0)$ contains the samples of the overall channel impulse response without a phase error, i.e., $\boldsymbol{h}(0) = [h_{-L}(0) \cdots h_0(0) \cdots h_L(0)]^T$. Notably, the transpose of the Jacobian matrix plays the role of the measurement matrix in the regular Kalman filtering and is given by

$$\begin{aligned} \boldsymbol{H}_k &= \begin{bmatrix} \frac{\partial y_k}{\partial \epsilon_k} & \frac{\partial y_k}{\partial \dot{\epsilon}_k} \end{bmatrix}_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_{k|k-1}} \\ &= \begin{bmatrix} \boldsymbol{a}_k^T \boldsymbol{h}'(0) & 0 \end{bmatrix}_{\epsilon_k = \hat{\epsilon}_{k|k-1}} \end{aligned} \qquad (6)$$

where $\boldsymbol{h}'(0) = [h'_{-L}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h'_0((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h'_L((\epsilon_k - \hat{\epsilon}_{k|k-1})T)]^T_{\epsilon_k = \hat{\epsilon}_{k|k-1}}$ and $h'_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T) = \frac{\partial h_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T)}{\partial \epsilon_k}$ for $i = -L, \ldots, 0, \ldots, L$. Thus, the EKF for a DPLL system can be described as:

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{v}_k \\ z_k = \boldsymbol{H}_k(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1}) + n_k \end{cases} \qquad (7)$$

where the data $z_k = r_k - \boldsymbol{a}_k^T \boldsymbol{h}(0)$.

Furthermore, assume $\boldsymbol{v}_k$ and n_k are white Gaussian noise with zero mean and their corvariance matrices are given by

$$E[\boldsymbol{v}_k \boldsymbol{v}_i^T] = \begin{cases} \boldsymbol{Q}_k, & i = k \\ 0, & i \neq k \end{cases} \qquad (8)$$

and

$$E[n_k n_i] = \begin{cases} R_k, & i = k \\ 0, & i \neq k \end{cases} \qquad (9)$$

where $E[.]$ denotes the expectation operation. Denote $\boldsymbol{P}_k = E[(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)^T]$ and $\boldsymbol{P}_{k+1|k} =$

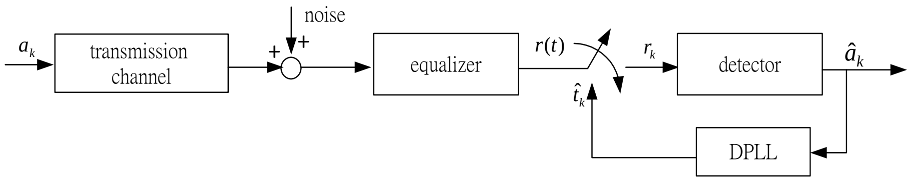

Figure 1: Model for the synchronous data transmission system

$E[(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})^T]$. The extended Kalman filter algorithm for estimating the phase and timing offset of a DPLL for $k \geq 0$ is described in the following with the initial state vector $\hat{\boldsymbol{x}}_{0|-1}$ and the corvarinace matrix $\boldsymbol{P}_{0|-1}$ [12, 13]. The extended Kalman gain equation is

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T + R_k)^{-1} \quad (10)$$

The estimation equation is

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k(r_k - \boldsymbol{a}_k^T\boldsymbol{h}(0)) \quad (11)$$

The error covariance equation is

$$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k|k-1} \quad (12)$$

The prediction equations are

$$\hat{\boldsymbol{x}}_{k+1|k} = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_k \quad (13)$$

and

$$\boldsymbol{P}_{k+1|k} = \boldsymbol{\Phi}\boldsymbol{P}_k\boldsymbol{\Phi}^T + \boldsymbol{Q}_k \quad (14)$$

The computational steps using equations (6) and (10-14) are depicted in Fig. 2.

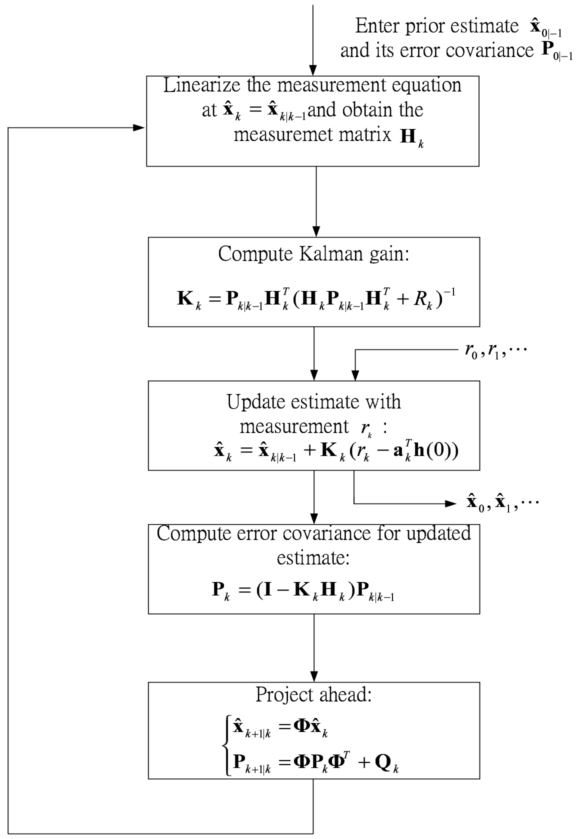

Figure 2: Extended Kalman filter algorithm for the DPLL

4 Phase Domain Analyses of a DPLL

In this section, the proposed EKF approach is further shown to resemble a 2nd-order DPLL with time-variant loop gains. Before doing this, we briefly describe a conventional 2nd-order DPLL and then compare it with the proposed EKF-based DPLL.

The phase domain model of a conventional 2nd-order DPLL is depicted in Fig. 3, which consisting of a phase detector, a loop filter and a numerical-controlled oscillator (NCO). Although the phase detector is, in general, a nonlinear device, it is often mathematically linearized as a constant gain. That is,

$$\tau_k = K_{pd}(\epsilon_k - \hat{\epsilon}_{k|k-1}) \quad (15)$$

where τ_k denotes the phase detector output and K_{pd} is called the phase detector gain. The conventional design approach has to choose a suitable phase detector and determine the fixed constants K_p and K_i of the loop filter [14] such that the estimated phase follows the input phase in some performance criteria. However, these tasks are usually difficult and time-consuming because the nonlinear dynamics of the phase detector has been ignored. In this study, an EKF is used to completely describe the DPLL with time-variant loop gains and these design parameters can be easily obtained by the Kalman filtering techniques.

The phase domain model of the EKF-based DPLL is derived as follows. By substituting (11) into (13),

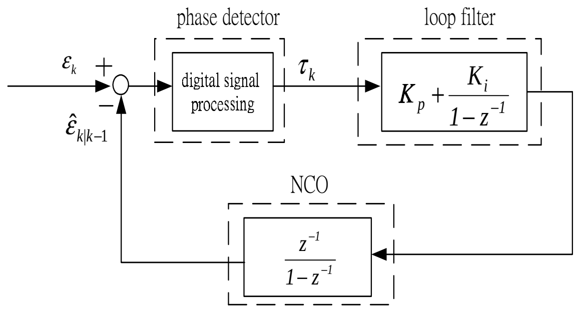

Figure 3: Phase domain model for a conventional 2nd-order DPLL

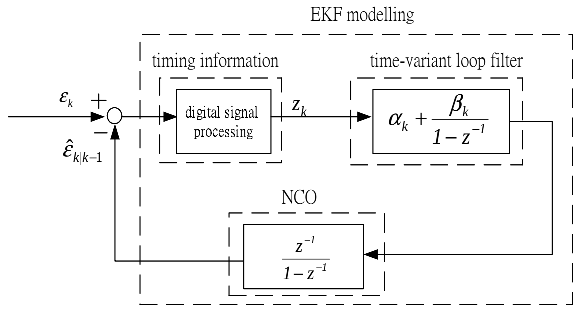

Figure 4: Phase domain model for an EKF-based DPLL

the state estimation equation is given by

$$\hat{\boldsymbol{x}}_{k+1|k} = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{\Phi}\boldsymbol{K}_k(r_k - \boldsymbol{a}_k^T\boldsymbol{h}(0)) \quad (16)$$

We further define $\boldsymbol{K}_k = [\alpha_k \ \beta_k]^T$, and then rewrite (16) in the following. For $k = 0$,

$$\hat{\epsilon}_{1|0} = \hat{\epsilon}_{0|-1} + \hat{\dot{\epsilon}}_{0|-1} + \alpha_0 z_0 + \beta_0 z_0 \quad (17)$$

and

$$\hat{\dot{\epsilon}}_{1|0} = \hat{\dot{\epsilon}}_{0|-1} + \beta_0 z_0 \quad (18)$$

For $k = 1$ and using (18), write

$$\begin{aligned} \hat{\epsilon}_{2|1} &= \hat{\epsilon}_{1|0} + \hat{\dot{\epsilon}}_{1|0} + \alpha_1 z_1 + \beta_1 z_1 \\ &= \hat{\epsilon}_{1|0} + \hat{\dot{\epsilon}}_{0|-1} + \beta_0 z_0 + \alpha_1 z_1 + \beta_1 z_1 \\ &= \hat{\epsilon}_{1|0} + \alpha_1 z_1 + \Sigma_{i=0}^{1}\beta_i z_i \quad (19) \end{aligned}$$

Finally, the estimate of $\hat{\epsilon}_{k|k-1}$ is obtained recursively by

$$\hat{\epsilon}_{k|k-1} = \hat{\epsilon}_{k-1|k-2} + \alpha_{k-1}z_{k-1} + \Sigma_{i=0}^{k-1}\beta_i z_i \quad (20)$$

The phase estimate equation (20) is updated by inputting the phase information z_k through a filter with time-variant loop gains, α_k and β_k. An EKF that completely models the DPLL and governs the phase update equation (20) is depicted in Fig. 4.

5 Computer Simulations

In this section, computer simulations for the conventional DPLL system are given to compare with those for the proposed timing recovery system. The overall channel impulse response is assumed to be a raise-cosine function with the roll-off factor $\alpha = 0$ and $L = 1$. The signal-to-noise ratio (SNR) is defined as $10 \log E[\frac{(r_k - n_k)^2}{n_k^2}]$; in this example, SNR is set to be 20 dB. First, assume a constant phase delay between the transmission time instant and the sampling time instant, i.e., $\epsilon_k = 0.2$, and $\dot{\epsilon}_k = 0$ for $k > 0$. For the conventional DPLL design, Mueller and Müller's phase detector [4] is adopted, that is, $\tau_k = r_k a_{k-1} - r_{k-1} a_k$ and $K_{pd} = -2$. After several simulation trails, set $K_p = -2.75 * 10^{-2}$ and $K_i = -3.88 * 10^{-5}$ for the loop filter. The result is depicted in Fig. 5 (a), and the estimated phase converges for $k > 200$ with a slightly large jittering. To have a smooth response, set $K_p = -9.3 * 10^{-3}$ and $K_i = -4.93 * 10^{-5}$ and Fig. 5 (b) shows the simulation result. The estimated phase converges slowly for $k > 400$. Notably, both fast phase acquisition and small jitter cannot be simultaneously met for a conventional DPLL with fixed loop gains.

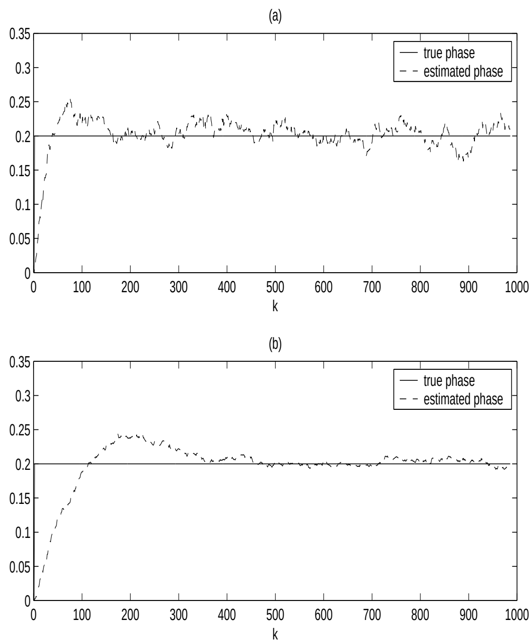

Figure 5: The responses of the conventional DPLL system: (a) fast acquisition, and (b) small jitter

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For the EKF-based DPLL, set the covariance matrices $\boldsymbol{Q}_k = 10^{-10}\boldsymbol{I}$ and $R_k = 0.01$, where $\boldsymbol{I}$ is the identity matrix. The initial settings $\hat{\boldsymbol{x}}_{0|-1}$ and $\boldsymbol{P}_{0|-1}$ are zeros and $0.1\boldsymbol{I}$, respectively. The estimated phase $\hat{\epsilon}_k$ and timing offset $\hat{\dot{\epsilon}}_k$ are respectively plotted in Fig. 6(a) and (b). Note that the proposed DPLL rapidly estimates the input phase and timing offset for $k \geq 60$ with a small jittering.

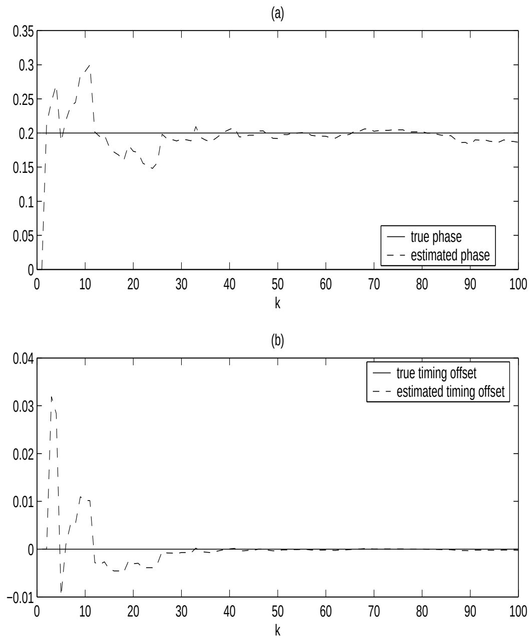

Figure 6: (a) The estimated phase $\hat{\epsilon}_k$, and (b) the estimated timing offset $\hat{\dot{\epsilon}}_k$

In the second simulation, the proposed DPLL is further demonstrated to track the input phase with a nonzero timing offset; in this case, $\epsilon_k = \epsilon_{k-1} + \dot{\epsilon}_{k-1}$, and $\dot{\epsilon}_k = 0.002$ for $k > 0$. Figure 7(a) and (b) plot the estimated phase $\hat{\epsilon}_k$ and timing offset $\hat{\dot{\epsilon}}_k$, and show that the estimated ones closely follow the true ones for $k \geq 70$.

6 Conclusions

The EKF has been successfully applied for realizing a DPLL, which completely describes the phase detector and loop filter. Thus, the time-variant loop gains can be obtained by the extended Kalman filtering techniques. In addition, the proposed timing recovery system has been compared with a conventional DPLL system. Simulation results indicate that the proposed realization can estimate the input phase rapidly without causing a large jittering.

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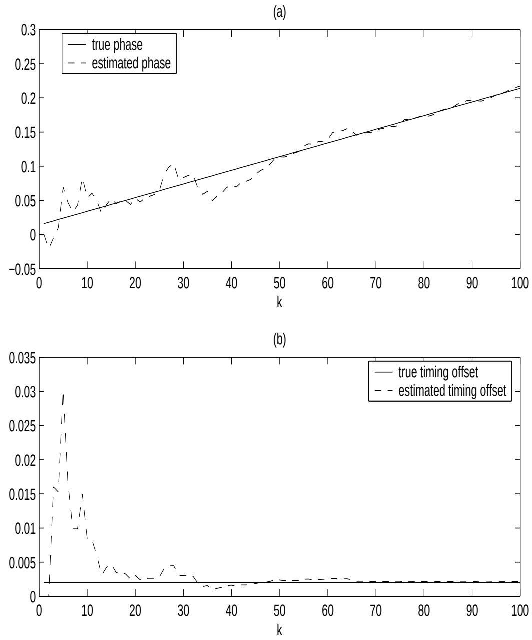

Figure 7: (a) The estimated phase $\hat{\epsilon}_k$, and (b) the estimated timing offset $\hat{\dot{\epsilon}}_k$

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