Dynamic Analysis Of An Off-Road Vehicle Frame

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Abstract: The paper presents computational methods applied for the study of an off road vehicle frame. The structure is analyzed first using finite element method. Internal energy consumed by the plastic hinges is determined and then the joint stiffness is computed. Also a multibody model is developed and analyzed. Joint data are exchanged between the numerical models and the results are compared.

Keywords: Vehicle frame finite elements, multibody method, plastic hinge characterization

1 INTRODUCTION
Past and modern construction solutions for off road vehicles includes a frame that is used as a support for the vehicle body, powertrain, axles and other components and it also has an important role in consuming the impact energy. Typically it may contribute in consuming the vehicle initial kinetic energy, by deformation, about 20-30% of the total impact energy. Therefore the configuration of the frame must comply with its functional purpose but it must be also an impact efficient structure.

The configuration must account the number, position and plastic hinges mechanism.

Plastic hinge is a collapse model of the structures and this process must be very carefully analyzed and understood in order to obtain the maximum of performances from a structure that may have a role in global vehicle structural crashworthiness.

Analytical techniques are available and applicable for a wide variety of models [1,2,3,11].

2 VEHICLE FRAME MODEL
The frame of an off road vehicle will be analyzed using a both finite element method and a multibody model. Figure 1 present the CAD model of the entire frame structure.

The objective is to study the function of the frontal structure of the frame during a full frontal impact.

Therefore from the assembly only the first section of the frame will be analyzed using numerical methods.

Fig. 2. Front side frame structure. Numerical model.

3 MECHANICAL SYSTEM
Mechanical systems may be represented as a collection or rigid or flexible bodies joined together by kinematic joints and force elements [9].

Multibody systems consisting in $n_b$ interconnected bodies are modelled using a set of $n_c$ independent coordinates connected through $l$ kinematic constrains. The number of system degrees of freedom is defined to be the number of total degrees of freedom minus the number of independent constraint equations.

3.1 Generalized coordinates and kinematical constraints
The configuration of a multibody model is identified by a number of variables called coordinates or generalized coordinates that must define completely
the location and orientation of each body belonging to the system.

For 2D systems each separate body belonging to the system can be completely defined using three independent coordinates: two coordinates describing the location of the origin of the body local coordinate frame and one coordinate describing the orientation of the local coordinate frame with respect to the global or fixed coordinate frame.

For an arbitrary point \( P \) on the body \( i \) the position with respect to the global coordinate frame \((XYZ)\) is:

\[
r^i_P = R^i + A^i \cdot \bar{u}^i
\]

where \( R^i \) is the position of the local coordinate frame \((xyz)\) attached to body \( i \); \( A^i \) is the transformation matrix from local coordinate system to the global coordinate system and \( \bar{u}^i \) is a vector defining the position of point \( P \) with respect to the local coordinate frame.

The vector of generalized coordinates will be denoted with \( \mathbf{q}^i \) and for planar analysis the vector is defined as:

\[
\mathbf{q}^i = [x^i \ y^i \ \theta^i]^T
\]

where \( x^i \) and \( y^i \) are the coordinates of the local reference frame and \( \theta^i \) is the rotation of the local reference frame about \( Z \) axis.

Figure 3 presents a planar revolute joint. The two bodied share the common point \( P \).

With respect to the global frame the coordinates of point \( P \) belonging to both bodies are the same. Thus one may write that:

\[
r^j_P = r^i_P
\]

where \( i \) denotes the base body while \( j \) denotes the follower body in the system.

Using equation 1 and with respect to equation 2 the following identity may be stated [9]:

\[
R^i + A^i \cdot \bar{u}^i = R^j + A^j \cdot \bar{u}^j
\]

The set of equations defined between the bodied in the system may be defined in condensed form as:

\[
\Phi(q_1, q_2, \ldots, q_n, t) = \Phi(q, t) = 0
\]

Therefore one may write that:

\[
\Phi(q) = 0
\]

### 3.2 Dynamic equations

The constraint equations are written as:

\[
\Phi(q) = 0
\]

while the velocity constraints are

\[
C \cdot \dot{q} = 0
\]

Using equation \( \Phi = \dot{C} \cdot \dot{q} + C \cdot \ddot{q} \) and the systems constraints \( \Phi(q) = 0 \) the following will be obtained:

\[
0 = \dot{C} \cdot \dot{q} + C \cdot \ddot{q}
\]

Equation of motion for a dynamical system is:

\[
M \cdot \ddot{q} - Q + D^T \cdot \lambda = 0
\]

Where: \( M \) is the inertia matrix, \( Q \) is the sum \( Q = Q_e + Q_s \) of gyroscopic terms \((Q_e)\) and the externally applied forces and moments \((Q_s)\) and \( \lambda \) is the vector of Lagrange multipliers.

The equations of motion of the constrained multibody system may be written as one matrix equation:

\[
\begin{pmatrix}
M & C^T \\
C & 0
\end{pmatrix}
\begin{bmatrix}
\dot{q} \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
-Q \\
-\dot{C} \cdot \dot{q}
\end{bmatrix}
\]

### 3.3 Numerical model

The mechanical system to be analysed has a number of 6 kinematical joints (fig. 4).

Body 1 is related to the ground by a translational constraint. There are two constrained degrees of freedom, namely the vertical displacement and body rotation.

Referring to the next section (4) the ground has associated the global coordinate frame.

The mechanical system may be analysed and the total number of constrained equations identified. As there are 6 kinematical joint and in this case each one of these joins constrains a number of two degrees of freedom the total number of constraint equations is 12.
• for body 1 of the system the coordinates are defined as follows:

\[
\begin{align*}
  x_1 &= x_1^0 \\
  y_1 &= y_1^0 = 0
\end{align*}
\]  
(10)

• for the second body the coordinates are defined as follows:

\[
\begin{align*}
  x_2 &= x_2 + l_2 \cdot \cos \theta_2 \\
  y_2 &= y_1 + l_2 \cdot \sin \theta_2
\end{align*}
\]  
(11)

• for the third body the coordinates are defined as follows:

\[
\begin{align*}
  x_3 &= x_2 + l_2 \cdot \cos \theta_3 + l_3 \cdot \cos \theta_3 \\
  y_3 &= y_2 + l_2 \cdot \sin \theta_2 + l_3 \cdot \sin \theta_3
\end{align*}
\]  
(12)

• for the forth body the coordinates are defined as follows:

\[
\begin{align*}
  x_4 &= x_3 + l_3 \cdot \cos \theta_4 + l_4 \cdot \cos \theta_4 \\
  y_4 &= y_3 + l_3 \cdot \sin \theta_3 + l_4 \cdot \sin \theta_4
\end{align*}
\]  
(13)

• for the fifth body of the system the coordinates are defined as follows:

\[
\begin{align*}
  x_5 &= x_4 + l_4 \cdot \cos \theta_5 \\
  y_5 &= y_4 + l_4 \cdot \sin \theta_4
\end{align*}
\]  
(14)

The first time derivative of the geometrical constrain equations will define the velocities of the bodies of the system and the definition of kinematic constrain will be completed. Therefore the first body the following constrains are defined:

\[
\begin{align*}
  \dot{x}_1 &= 0 \\
  \dot{y}_1 &= 0
\end{align*}
\]  
(15)

or with For the second body the equations are:

\[
\begin{bmatrix}
  x_1 \\
  y_1 \\
  \theta_1 \\
  \dot{x}_1 \\
  \dot{y}_1 \\
  \dot{\theta}_1
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\]

The system of 12 constraints equations are obtained from the numerical analysis of the model using finite element method. The joint resistant torque is defined as

\[
M_{ij} = k_{ij} \left( \theta_i - \theta_j^0 \right)
\]

(23)
where \( k_{ij} \) is the measured linear or nonlinear joint stiffness in Nm/rad; \( \theta_{i,j} \) are the rotations of two adjacent sections measured in radians while \( \theta^{0}_{i} \) are the initial rotation of the sections.

\[
\begin{align*}
M_{23} &= k_{23} \cdot (\Delta \theta_2 - \Delta \theta_3) \\
M_{34} &= k_{34} \cdot (\Delta \theta_3 - \Delta \theta_4)
\end{align*}
\]  

(24)

3.4 Initial conditions

In order to obtain the correct and required motion of the mechanical system correct initial conditions must be defined \([4, 8]\). Initial conditions include the correction of the system’s geometry and also the correction of translational and rotational velocities. The correction of geometrical positions and velocities must be performed in accordance with the constraint equations defined. Otherwise inconsistent conditions will be set that will lead to a violation of both geometrical and kinematic constraints.

Geometrical constrains applied to this model will lead to the following equalities:

\[
\begin{align*}
x^{0}_1 + L_2 \cdot \cos \theta^0_1 + L_1 \cdot \cos \theta^0_2 + L_4 \cdot \cos \theta^0_4 &= L_0 \\
L_2 \cdot \sin \theta^0_1 + L_1 \cdot \sin \theta^0_2 + L_4 \cdot \sin \theta^0_4 &= y^0_3
\end{align*}
\]  

(25)

Considering the current time the following equations must be applied:

\[
\begin{align*}
x_1 + L_2 \cdot \cos \theta_1 + L_1 \cdot \cos \theta_2 + L_4 \cdot \cos \theta_4 &= L_0 - L_4 \cdot \cos \theta_4 \\
L_2 \cdot \sin \theta_1 + L_1 \cdot \sin \theta_2 + L_4 \cdot \sin \theta_4 &= y_3 - L_4 \cdot \sin \theta_4
\end{align*}
\]  

(26)

In order to define the actual positions of the bodies Newton Raphson method is used to determine the correct positions of bodied 4 and 5. The nonlinear geometrical constrain functions are defined as follows:

\[
\begin{align*}
f_x(\theta_1, \theta_2) &= L_1 \cdot \cos \theta_2 - (x_1 + L_2 \cdot \cos \theta_1 + L_1 \cdot \cos \theta_2 + L_4 \cdot \cos \theta_4) = 0 \\
f_y(\theta_1, \theta_2) &= L_1 \cdot \sin \theta_2 - y_3 - (L_2 \cdot \sin \theta_1 + L_1 \cdot \sin \theta_2 + L_4 \cdot \sin \theta_4) = 0
\end{align*}
\]  

(27)

The system’s Jacobian is defined as follows:

\[
J = \begin{bmatrix}
\frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} \\
\frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2}
\end{bmatrix}
\]  

(28)

Or in an explicit form:

\[
J = \begin{bmatrix}
- L_1 \cdot \sin \theta_1 & 0 \\
L_1 \cdot \cos \theta_1 & -1
\end{bmatrix}
\]  

(29)

The solution of the nonlinear system of equations is obtained iteratively as follows:

\[
x = x^{0} - J^{-1} \cdot f(x^{0})
\]  

(30)

Starting from the initial configurations:

| Body 1 | 0 0 0° |
| Body 2 | 0.5 0 0° |
| Body 3 | 1.226 0.105 −25° |
| Body 4 | 1.699 0.167 −10° |
| Body 5 | 1.945 0.124 0° |

Using the Newton Raphson method to find the correct and initial position of the bodies, the following sets of coordinates is obtained:

| Body 4 | 1.679 0.105 −25° |
| Body 5 | 1.906 0.014 0° |

Figure 6 presents both initial “out of position” configuration and the final corrected configuration.

![Figure 6. Multibody model. Correction of the sets of coordinates.](image)

While for the geometrical configuration of the system is obtained solving a nonlinear equations system the initial velocities of the bodies are obtained solving the following system:

\[
\begin{bmatrix}
C_d & C_i \\
0 & I
\end{bmatrix}
\begin{bmatrix}
v_d \\
v_i
\end{bmatrix} = \begin{bmatrix} 0 \\
v_i^0 \end{bmatrix}
\]  

(31)

where the Jacobian matrix \( C_{di} \) is partitioned as:

\[
C_{di} = \begin{bmatrix} C_d & C_i \end{bmatrix}
\]  

(32)

or as presented in an explicit form:

\[
\begin{bmatrix}
v_d \\
v_i
\end{bmatrix} = \begin{bmatrix} C_d & C_i \end{bmatrix}^{-1} \begin{bmatrix} 0 \\
v_i^0 \end{bmatrix}
\]  

(33)

Starting from the vector of initial velocities where the values for the translational Ox velocity of body 1 and angular velocities of bodied 2 and 3, parameters that were set as independent coordinates the vector of all translational and angular velocities of the system’s bodies are well defined.
4 RESULTS AND DISCUSSION

The vehicle structure will be tested under a constant applied force condition. First a numerical simulation using finite element method is performed and the results are analyzed. The total simulation time is of 0.05 s and 0.04 s for multibody and a second run using the finite element model.

As the joint will be represented as a nonlinear torsion spring the stiffness may be computed using equation (34):

\[ U = \frac{1}{2} k_\theta \cdot \theta^2 \]  
(34)

Where:

- \( U \) is the computed internal energy;
- \( k_\theta \) is the joint stiffness;
- \( \theta \) is the bending angle measured from the initial configuration \( \theta^0 \).

Analytical techniques can be used to describe the plastic hinge considering the section dimensions and the material model [3]. The plastic moment is calculated and then applied in equations that may define the reaction torque [6].

The joint stiffness dependence is used for the multibody model of the frame.

The results of the simulation using finite element method are presented in figure 10, while figure 11 presents the solution using the multibody model.

It may be noticed that there is a good agreement between the finite element model and the multibody model obtained.

Regarding the joints stiffness, the values used for the multibody model can be verified by analysing model’s results as the translational velocity of the first body.

If the joint response for the multibody model is the one required then the results (finite elements model and multibody) should match. Otherwise if a higher stiffness is used the body will increase its...
velocity slower. Figure 12 presents the velocity of the first body compared with the results obtained from the finite elements simulation.

![Figure 12](image1)

Fig.12. Results analysis.
Multibody vs. finite elements model

Figure 13 presents the velocity history of the first body when a higher value for the joints stiffness is used.

![Figure 13](image2)

Fig.13. Results analysis.
Multibody vs. finite elements model

Analytical model to study the plastic hinges are available in the literature [3]. By defining the requirements of such a joint / plastic hinge the geometrical dimensions can be defined.

5 CONCLUSIONS

Numerical method are necessary tools for structural performance improvement.

Finite element method is a very detailed, and structures can be analyzed in a very detailed manner. The main shortcoming of the method is that it requires specialized software to run the simulations. On the other hand multibody methods can be used very easy because is based on analytical structure description is it does not require sophisticated solver.

The paper presents both finite element and multibody method applied for the study of a vehicle frame.

Although some of the input data for the multibody model are based on the finite element simulations the plastic hinges frame sections can be also analyzed using physical testing.

Further work will contain optimization methods [7] applied to the structure and impact simulations [11] with initial velocity conditions and internal impact/contact forces [1,2,10].

References