Controller Design using Bifurcation Map for Aircraft Spin Recovery

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Abstract: In this paper, we present a variable structure based sliding mode controller for recovery of an aircraft from spin. The spin recovery problem is formulated as a two point boundary value problem where the two points, different steady states of the aircraft, are obtained from a bifurcation analysis of the aircraft model. Using the bifurcation analysis results of the aircraft, the spin states of aircraft are identified. Once the aircraft enters into spin, the controller is activated to bring it back to a desired state which is a level flight trim solution also found from a bifurcation map of the aircraft model.

Keywords: Spin Recovery, Bifurcation Map, Sliding Mode Controller

1. INTRODUCTION

Spin is a complex, nonlinear, post-stall motion of aircraft in which an aircraft rotates about its center of gravity and an axis perpendicular to Earth descending vertically at high speed following a downward corkscrew path. An aircraft may enter into spin either voluntarily for testing air superiority in high-angle-of-attack flight regimes or unintentionally in a downburst like wind condition. It is one of the most dangerous nonlinear phenomena encountered by aircraft in high-angle-of-attack flight regimes. Spin leads to problems, such as, pilot spatial dis-orientation, uncontrolled aircraft motion, etc., resulting in fatal accidents and subsequent loss of aircraft. Spin tests have been recommended mandatory therefore for aircraft both in civil transport and combat categories [1].

Spin solutions for many high-angle-of-attack aircraft models have been computed using bifurcation analysis and continuation technique based methodology and reported in literature [2]. Bifurcation analysis results provide the onset points of bifurcations (at critical values of control surface defletions) that may inadvertently or voluntarily land an aircraft into spin. Standard control inputs using proper deflection of rudder to control yaw with simultaneous application of elevator to reduce the angle-of-attack of aircraft have been recommended for spin recovery [1]. Improper functioning of aerodynamic control surfaces during spin however often makes it difficult for the pilot to control the aircraft in this motion. Design of new generation aircraft with new configurations therefore cannot rely on the standard piloting strategies to recover the aircraft from spin. Instead, a uniform approach to design aircraft model based spin recovery strategies may be extremely useful. An introduction to spin related problems and needs to devise nonlinear controllers for spin recovery of aircraft have been reported in Ref. [2]. Using bifurcation analysis and continuation methods, in Ref. [2], authors first computed the steady states and bifurcations of an aircraft model in longitudinal dynamics as a function of the elevator deflection. Identifying the spin states of aircraft from the bifurcation analysis results and the state to which the aircraft was supposed to return to after spin recovery, they further developed an NDI (nonlinear dynamics inversion) technique based controller for aircraft recovery from spin. Robustness, simplicity and time optimality of controller are however three main issues very important for spin recovery controller design.

Variable structure based controllers are usually robust in the presence of system uncertainties and external disturbances and do not result in complicated control architecture. Use of variable structure based sliding mode controller for control of aircraft nonlinear maneuvers using discontinuous control deflections has been previously shown by Singh [3]. In this paper, the authors, using a sliding mode controller design technique in conjunction with the bifurcation analysis results for a nonlinear high-angle-of-attack research vehicle model of F-18 present novel stepwise procedures involved in controller design for aircraft recovery from spin. The sliding mode controller design for spin recovery is formulated as a two point boundary value problem where the boundary states are supplied from the bifurcation analysis results of the aircraft model.

The paper is organized as follows. In the next section, a brief description of bifurcation analysis and continuation methods relevant to this work is presented. In section 3,

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sliding mode control design technique is explained. Results and discussions are presented in section 4 and conclusions follow thereafter in section 5.

2. COMPUTATION OF AIRCRAFT STEADY STATES

The aircraft model considered here is the F-18 High-Angle-of-Attack Research Vehicle (HARV) which has highly nonlinear post-stall aerodynamics. The tabulated aerodynamic data for this aircraft is available in open literature in the range of angle-of-attack between -4 to $+90$ degrees. Besides, inertia coupling and kinematic coupling terms are also present in the full six-degree-of-freedom equations of aircraft motion [4]. In order to compute steady states of aircraft to identify spin states, AUTO2000 [5] continuation algorithm has been used in this work to solve simultaneous aircraft nonlinear equations of motion [4]

$$\dot{\underline{x}} \;=\; \underline{f}(\underline{x}, \underline{p}, \delta e) = 0. \tag{1}$$

as a function of the elevator deflection. In Eq. (1) and in what follows, $\underline{x} = [M,\ \alpha,\ \beta,\ p,\ q,\ r,\ \phi,\ \theta]$ is the vector represeting aircraft states, and $\underline{p} = [\eta,\ \delta a,\ \delta r]$ is the vector of aircraft control parameters. The vector $\underline{f}$ governs the aircraft dynamics [4]. The high-angle-of-attack bifurcation results as a function of the elevator deflection δe is shown in Fig. 1. The high-angle-of-attack flight regime for this aircraft shows existence of a stable oscillatory spin (SOS) solution branch and a branch of unstable steady spin (USS) solutions. Each of the solution on the SOS branch is a limit cycle attractor. A little exercise of simulating aircraft motion around these solution branches confirm the predictions made by using the bifurcation analysis results. One can further verify that simulation of aircraft equations of motion around one of the USS spin solutions eventually lands the aircraft into the stable oscillatory spin motion. The main objective of carrying out bifurcation analysis of the aircraft model in this paper is to compute states of aircraft in SOS to be used as input for the sliding mode controller design. One of the solutions on the SOS branch is used as the reference point '1' for sliding mode control design for spin recovery. The other reference point '2' is the state to which the aircraft is recovered to after applying the control commands computed using the sliding mode controller. The reference point '2' (a level flight trim) is computed by solving

$$\underline{f}(\underline{x}, \delta e, \eta(\delta e), \delta a(\delta e), \delta r(\delta e) \;=\; 0 \tag{2}$$

with changing elevator deflection δe as a continuation parameter. The schedules in the above Eq. (2), $\eta(\delta e)$, $\delta a(\delta e) = 0$ and $\delta r(\delta e) = 0$ are the ones required to keep the aircraft in level flight trims for different values of throttle and elevator deflections [6]. The results of this exercise are bifurcation diagrams for the aircraft in level flight trims shown in Fig. 2 (shown is only angle-of-attack vs elevator deflection). The reference point '2' is chosen as a stable level flight trim from the results shown in Fig. 2. Thus, the reference points for controller design are obtained from the bifurcation analysis results in the two cases as the followings:

Reference point '1' (Stable Oscillatory Spin State):

$$\begin{aligned}\underline{\mathrm{x}}_0 \;&=\; [V_0,\ \alpha_0,\ \beta_0,\ p_0,\ q_0,\ r_0,\ \phi_0,\ \theta_0]' \\ &=\; [196, 1.27, 0.03, -0.47, 0.047, -1.56, -0.03, -0.29]' \\ \underline{\mathrm{U}}_0 \;&=\; [\delta a_0,\ \delta e_0,\ \delta r_0,\ \eta_0]' \;=\; [0, -0.42, 0, 0.38]'\end{aligned}$$

Reference point '2' (Stable Level Flight Trim Condition):

$$\begin{aligned}\underline{\mathrm{x}}_0 \;&=\; [V_0,\ \alpha_0,\ \beta_0,\ p_0,\ q_0,\ r_0,\ \phi_0,\ \theta_0]' \\ &=\; [223, 0.3, 0, 0, 0, 0, 0, 0.3]' \\ \underline{\mathrm{U}}_0 \;&=\; [\delta a_0,\ \delta e_0,\ \delta r_0,\ \eta_0]' \;=\; [0, -0.01, 0, 0.54]'\end{aligned}$$

All the angles in the above are in radians and angular rates in radian per second. The velocity V is in feet per second.

3. PROBLEM STATEMENT AND SLIDING MODE CONTROL DESIGN TECHNIQUE

The main objective of this work is to compute control commands using Sliding Mode Control (SMC) technique which transfers the aircraft from an oscillatory spin state (reference point '1') to a steady level-flight trim state (reference point '2'). The sliding mode control design problem here is thus formulated as a two-point boundary value problem.

Sliding mode controller can be designed for a continuous nonlinear system of the general form [7],

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{U}, t), \tag{3}$$

where dim $\underline{x}$ = n and dim $\underline{U}$ = m. The above equation (3) for an aircraft can be written as:

$$\dot{\underline{x}} = A(\underline{x}, t) + B(\underline{x}, t)\underline{U}, \tag{4}$$

For the aircraft model used in this work, $\underline{x}$ and $\underline{U}$ are as defined in Eq. (1), thus dim A = 8 x 1 and dim B = 8 x 4. In this problem, A and B are not accurately known as the aerodynamic data available in the tabular form has to be interpolated and called and there is no linear relationship between the states and the inputs. In this work, we use available controls elevator, aileron and rudder for spin recovery, and therefore, we choose sliding functions of the form,

$$s = \begin{bmatrix} \dfrac{d}{dt}(\phi - \phi_d) + \lambda_1(\phi - \phi_d) \\ \dfrac{d}{dt}(\alpha - \alpha_d) + \lambda_2(\alpha - \alpha_d) \\ \dfrac{d}{dt}(\beta - \beta_d) + \lambda_3(\beta - \beta_d) \end{bmatrix}, \tag{5}$$

where λ_1, λ_2 and λ_3 are positive real numbers. The desired solution $s = 0$ is achieved using the criteria:

$$\begin{array}{llll} \text{if} & \mathrm{s}_i(x) > 0 & then & \dot{s}_i(x) < 0, \\ \text{if} & \mathrm{s}_i(x) = 0 & then & \dot{s}_i(x) = 0, \quad and \\ \text{if} & \mathrm{s}_i(x) < 0 & then & \dot{s}_i(x) > 0. \end{array}$$

The above equations and the control criteria can be put into a generalized form called 'Reaching Law':

$$\dot{s} = -Qsgn(s) - Kf(s), \tag{6}$$

where Q and K are diagonal matrices with positive elements and

$$sgn(s) = [sgn(s_1), sgn(s_2), ...sgn(s_m)]^T,$$
$$f(s) = [f(s_1), f(s_2), ...f(s_m)]^T.$$

m in the above is the number of sliding surfaces or the number of states to be commanded and the scalar functions f_i satisfy the condition

$$s_i f_i(s_i > 0) \text{ when } s_i \neq 0.$$

Various choices of Q and K specify different rates for s and yield different structures in the reaching law. In this case, $f(s)$ becomes

$$f(s) = \begin{bmatrix} \phi - \phi_d \\ \alpha - \alpha_d \\ \beta - \beta_d \end{bmatrix}. \tag{7}$$

The control law from reaching law in Eq. (6) can be derived as:

$$\dot{s} = \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial x}(A + Bu) = -Qsgn(s) - Kf(s). \tag{8}$$

By solving the above equation (8) for u (3×1), one gets expressions for u as:

$$u = -\left(\frac{\partial s}{\partial x}B\right)^{-1}\left[\frac{\partial s}{\partial x}A + Qsgn(s) + Kf(s)\right]. \tag{9}$$

The matrix $\frac{\partial s}{\partial x}$ is computed numerically. The control inputs computed are subjected to position and rate constraints on control surface deflections as listed in the Table 1.

Table 1. Control surface position and rate limits

Symbol	Position limits, deg	Rate limits, deg/s
δe	(-25,10)	± 40
δa	(-35,35)	± 100
δr	(-30,30)	± 82
η	(0,1)	None

4. RESULTS AND DISCUSSIONS

Time optimality and lesser complexity of the control commands are two important issues while designing any controller for aircraft spin recovery. Robustness of controller is another important issue. Sliding mode controller based on variable structure control technique for nonlinear systems is arguably the most robust of all the nonlinear controllers. In this work, using the bifurcation maps for the aircraft in longitudinal flight and in longitudinal level flight, a sliding mode controller is designed for aircraft spin recovery. A stable spin solution where aircraft may enter accidently is taken as reference point '1' (from Fig. 1) and another stable level flight condition where aircraft is supposed to recover to is taken as reference point '2' (from Fig. 2). A first order dynamics representing errors in aircraft three state variables is chosen for the controller design. Both the reference points known, errors between the actual aircraft states and the desired states are minimized in time so that aircraft follows a trajectory on what is known as sliding surface. Stability of the initial states and the desired states are ensured from the beginning which is as a result of the bifurcation analysis. Limits on the control surface deflections both in position and in rate are maintained during the computation of control commands. In the plots presented in Fig. 3, the high angle of attack portion of the plot is the stable spin motion and the low angle of attack portion is the steady level flight condition. In between transients represent recovery of aircraft from spin, for which, computed control commands are shown in Fig. 4. Successful implementation of sliding mode controller for spin recovery thus can been observed from Fig. 3. The discontinuous control deflections for the controller indicate chattering, which can be avoided by properly tuning the parameters of the controller.

5. CONCLUSIONS

In this work, a controller based on sliding mode control technique is designed for aircraft spin recovery. A prerequisite to control design based on sliding mode controller presented here is identification of aircraft steady states. A bifurcation analysis and continuation method is used to compute steady states of aircraft. Aircraft enters into spin through several bifurcations of steady states as the elevator deflection angle is increased in the negative direction. Two types of spin motion of aircraft are identified for the F-18/HARV model considered in this paper, viz. unstable steady and flat oscillatory. Numerical simulations of the aircraft further confirm existence of spin motion. One of the spin states is used in this work for controller design. The second state of the aircraft to which aircraft returns when the designed controller is applied is a level flight trim solution which is computed using an extended bifurcation analysis and continuation procedure. The sliding mode control commands for aircraft control deflections presented in this paper successfully recover the aircraft from spin.

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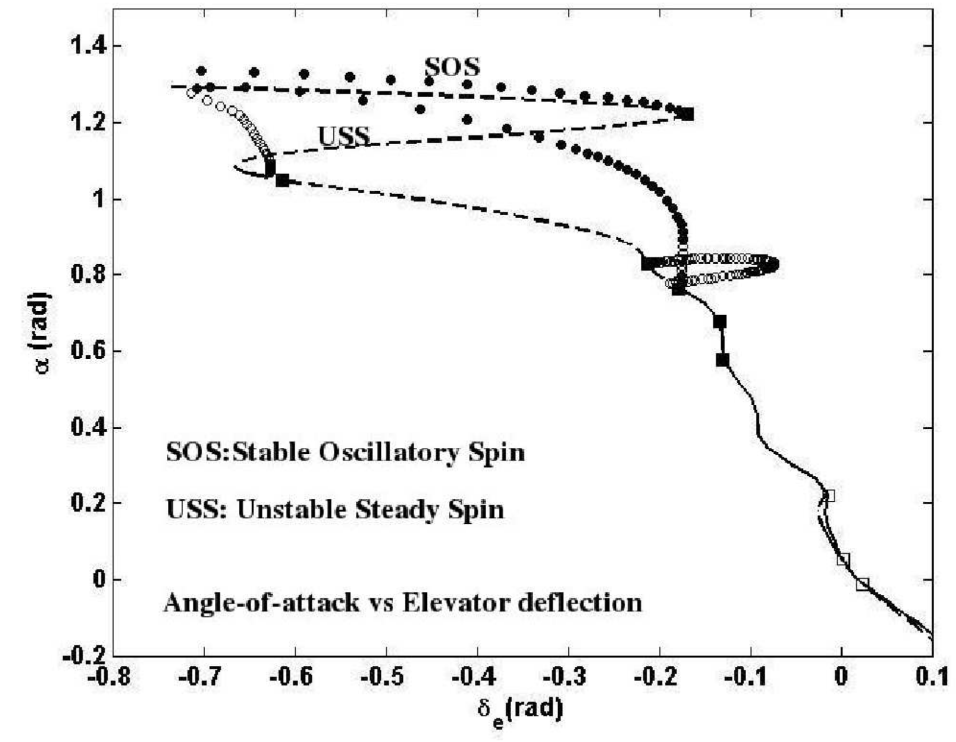

Fig. 1. Bifurcation diagram of angle-of-attack α versus the elevator deflection δe for $\eta = const.$, $\delta a = \delta r = 0$.

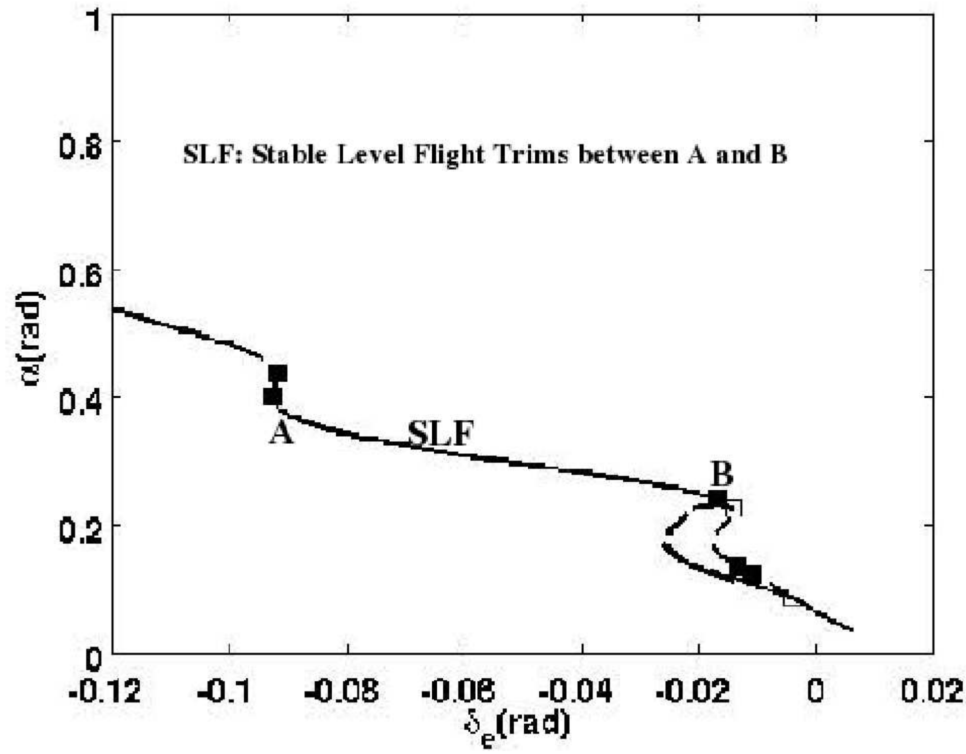

Fig. 2. Bifurcation diagram of angle-of-attack α versus the elevator deflection δe for $\eta = \eta(\delta e)$ and $\delta a = \delta r = 0$ in level flight conditions.

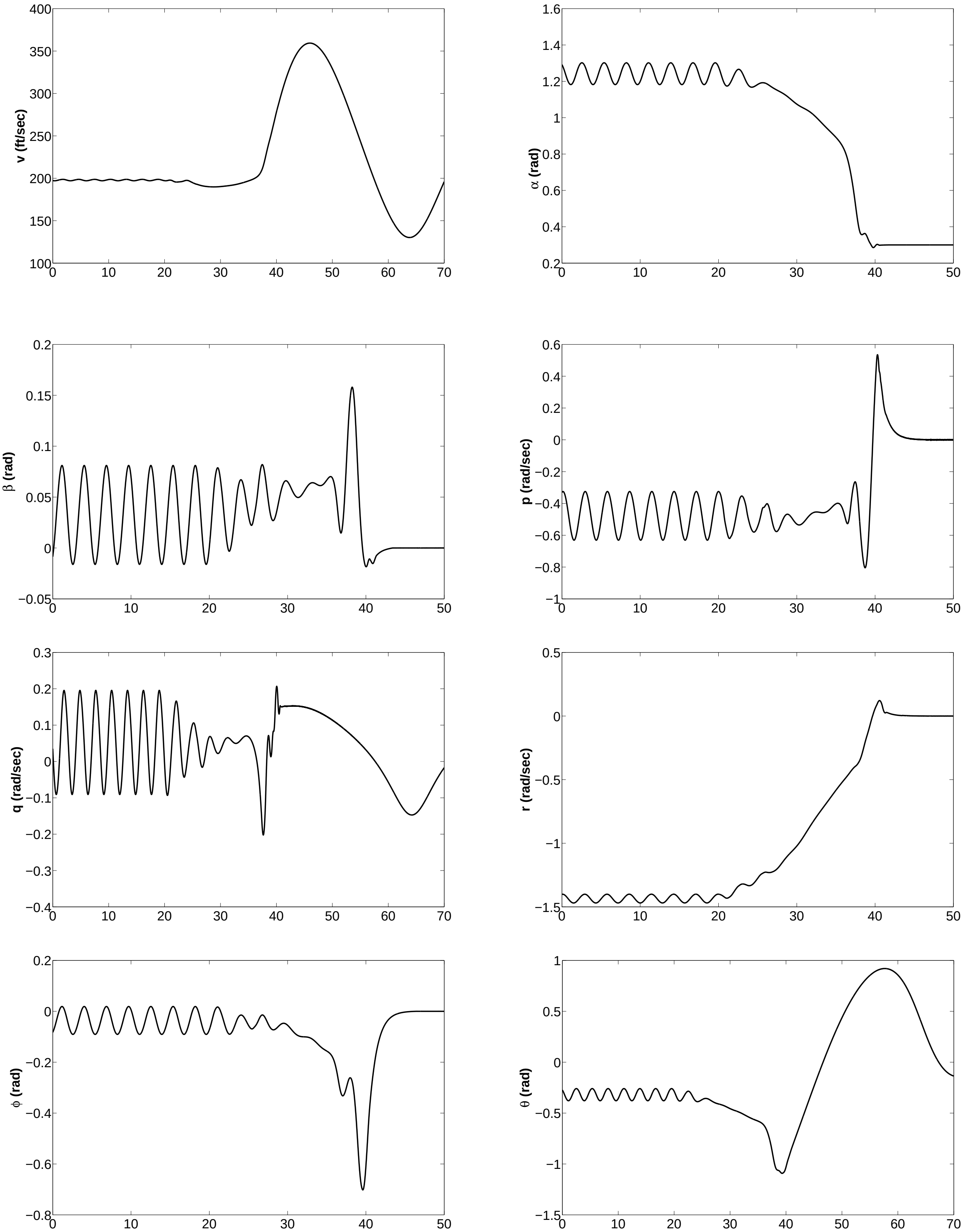

Fig. 3. Numerical simulation results showing aircraft spin recovery from spin to a level flight trim using Sliding Mode Controller (x-axis: time in seconds).

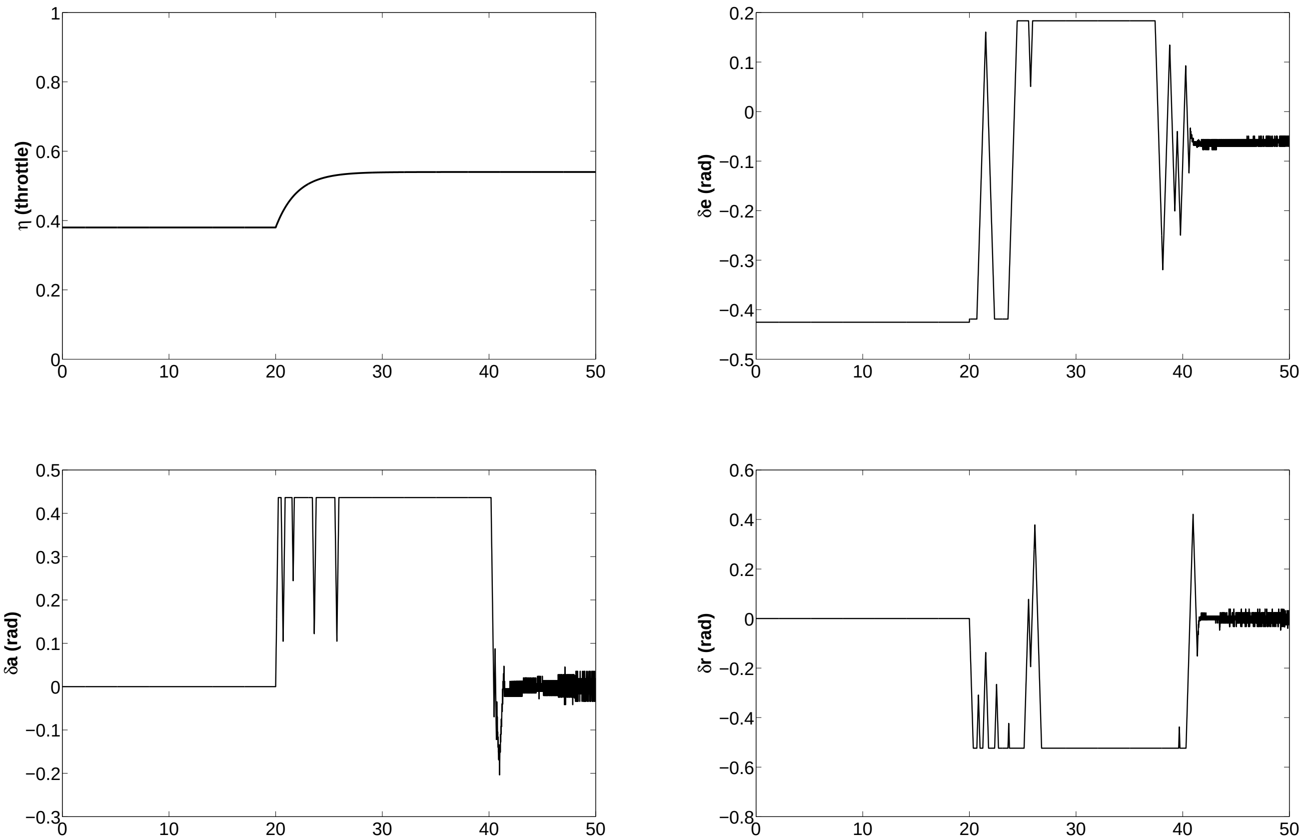

Fig. 4. Sliding mode control commands for spin recovery (x-axis: time in seconds).

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