Elasticity Evaluation of Carbon and Aramid Fibre-Reinforced Laminates

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Abstract: - The paper presents the elasticity evaluation of some advanced symmetric composite laminates based on epoxy resin reinforced alternatively with HM carbon, HS carbon and kevlar49 fibres. The laminates taken into account into this evaluation have following plies sequences: [0/30/-30/60]S, [0/45/90]2S, [90/452/0]S and are subjected to off-axis loading systems. The elastic constants as well as the tensile-shear interaction have been determined. In order to obtain equal stiffness in all loading systems, a composite laminate have to present balanced angle plies.

Key-Words: - Elasticity, Carbon, Aramid, Laminate, Loading system, Fibres.

1 Introduction
The elasticity evaluation of advanced composite laminates is for a great importance in designing composite structures both for aerospace, automotive and medical techniques. It is well known that composite laminates with aligned reinforcement are very stiff along the fibres, but also very weak transverse to the fibres direction. This fact is more obvious in the case of advanced composite laminates reinforced with anisotropic carbon or aramid fibres. Getting equal stiffness of laminates is a demand.

The solution to obtain equal stiffness of laminates subjected in all directions within a plane is presented by various authors by stacking and bonding together plies with different fibres orientations [1-5].

A composite laminate subjected to off-axis loading system presents tensile-shear interactions in its plies. Tensile-shear interactions lead to distortions and local micro-structural damage and failure, so in order to obtain equal stiffness in all off-axis loading systems, a composite laminate have to present balanced angle plies [6-12].

Tensile-shear interaction in a fibre-reinforced composite laminate occurs only if the off-axis loading system does not coincide with the main axes of a single lamina or if the laminate is not balanced.

2 Theoretical Background
A composite laminate (fig. 1) formed by a number of unidirectional reinforced laminae subjected regarding to the loading scheme presented in fig. 2 is considered.

The elasticity law for a unidirectional lamina can be written as follows:

\[
\begin{bmatrix}
\sigma_{xx}^K \\
\sigma_{yy}^K \\
\tau_{xy}^K
\end{bmatrix} = \begin{bmatrix}
r_{11}^K & r_{12}^K & r_{13}^K \\
r_{12}^K & r_{22}^K & r_{23}^K \\
r_{13}^K & r_{23}^K & r_{33}^K
\end{bmatrix} \begin{bmatrix}
\epsilon_{xx}^K \\
\epsilon_{yy}^K \\
\gamma_{xy}^K
\end{bmatrix},
\]

(1)

where \( r_{ij}^K \) represent the transformed stiffness, \( \sigma_{xx}^K, \sigma_{yy}^K \) are the mean stresses of \( K \) lamina on \( x \)- respective \( y \)-axis and \( \tau_{xy}^K \) represent the mean shear stress of \( K \) lamina against the \( x \)-\( y \) coordinate system. The balance equations of the laminate structure are:

\[
n_{xx} = \sum_{K=1}^{N} \left( \sigma_{xx}^K \cdot t_K \right) = \sum_{K=1}^{N} n_{xx}^K,
\]

(2)

\[
n_{yy} = \sum_{K=1}^{N} \left( \sigma_{yy}^K \cdot t_K \right) = \sum_{K=1}^{N} n_{yy}^K,
\]

(3)

\[
n_{xy} = \sum_{K=1}^{N} \left( \tau_{xy}^K \cdot t_K \right) = \sum_{K=1}^{N} n_{xy}^K,
\]

(4)
where \( n_{xx}, n_{xy} \) are the normal forces on the unit length of the laminate on \( x \)- respective \( y \)-axis and \( n_{xy} \) represents the shear force, in plane, on the unit length of the laminate against the \( x-y \) coordinate system. \( \sigma_{xx}, \sigma_{yy} \) are the normal stresses on \( x \)- respective \( y \)-axis of the laminate, \( \tau_{xy} \) represent the shear stress of the laminate against the \( x-y \) coordinate system. \( t_k, t \) represent the thickness of the \( K \) lamina respective the laminate thickness, \( n_{xxK}, n_{yyK} \) are forces on the unit length of \( K \) lamina on \( x \)- respective \( y \)-axis directions and \( n_{xyK} \) is the shear force in plane, on the unit length of \( K \) lamina against the \( x-y \) coordinate system.

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
\sum_{k=1}^{N} n_{xxk} t_k \\
\sum_{k=1}^{N} n_{xyk} t_k \\
\sum_{k=1}^{N} n_{xyk} t_k
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix},
(6)
\]

where the laminate stiffness \( r_{ij} \) are:

\[
r_{ij} = \sum_{K=1}^{N} \sum_{k=1}^{N} r_{ijK} \ \frac{t_k}{t}
\]

(7)

So, the laminate elasticity can be expressed as follows:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
\sum_{k=1}^{N} n_{xxk} t_k \\
\sum_{k=1}^{N} n_{xyk} t_k \\
\sum_{k=1}^{N} n_{xyk} t_k
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix},
(8)
\]

Computing the laminate strains as a function of stresses, the expressions (8) are:

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix},
(9)
\]
where $c_{ij}$ represents the laminate compliance tensor. This tensor can be computed as a function of elastic constants. Thus [1]:

$$
E_x = \frac{1}{\xi_{11}}; E_y = \frac{1}{\xi_{22}}; G_{xy} = \frac{1}{\xi_{33}}; \nu_{xy} = -E_x \cdot \xi_{12}. \quad (10)
$$

It is obvious that the laminate will exhibit different elastic constants if the loading system is applied at a randomly angle, $\Phi$, to the x-y coordinate system.

3 Some Advanced Composite Laminates

The laminates taken into account at elasticity evaluation are based on epoxy resin reinforced alternatively with HM carbon, HS carbon and Kevlar49 fibres.

These laminates present the following plies sequence: [0/30/-30/60]$_S$, [0/45/90]$_{2S}$ and [90/45/45/0]$_S$. Carbon fibres of type HM (high modulus) present a value of Young modulus larger than 300 GPa. High strength (HS) carbon fibre is a general purpose, cost effective carbon fibre, designed for industrial and recreational applications and is usually used for non structural components of aircrafts. Kevlar49 fibre is characterized by low density and high tensile strength and modulus. These properties are the key to its successful use as reinforcement for plastic composites in aircraft, aerospace, marine, automotive, other industrial applications, and in sports equipment. It is available in various forms of fibres. Kevlar49 is used in high performance composite applications where lightweight, high strength and stiffness, vibration damping and resistance to damage and fatigue are key properties. Reinforced composites can save up to 40% of the weight of glass fibre composites at equivalent stiffness [13], [14].

4 Results

General input data are: fibres volume fraction $\varphi = 0.5$ in all cases, laminates thickness $t = 1$ mm and off-axis loading systems varies between $0^\circ$ and $90^\circ$. For HM carbon fibres, following data have been used [15]:

- $E_M = 3.9$ GPa;
- $E_L > 300$ GPa;
- $E_T < 100$ GPa;
- $\nu_M < 0.5$;
- $\nu_T < 0.4$;
- $G_M < 25$ GPa;
- $G_T < 50$ GPa.

For HS carbon fibres the input data are:

- $E_L < 300$ GPa;
- $E_T < 80$ GPa.

For Kevlar49 fibres the following data have been used:

- $E_L < 200$ GPa;
- $E_T < 50$ GPa.

The computed elastic constants $E_{xx}$, $E_{yy}$, $G_{xy}$ and $\nu_{xy}$ are presented in figs. 3 – 6.

![Fig. 3. $E_{xx}$ distribution of some carbon and aramid fibre-reinforced epoxy based laminates](image1)

![Fig. 4. $E_{yy}$ distribution of some carbon and aramid fibre-reinforced epoxy based laminates](image2)
5 Conclusions

Under off-axis loading, normal stresses produce shear strains (and of course normal strains) and shear stresses produce normal strains (as well as shear strains). The tensile-shear interaction is also present in laminates but does not occur if the loading system is applied along the main axes of a single lamina or if a laminate is balanced.

References: