Generalized Methodology for Circuit Optimal Design

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Abstract: - The design process for analog network design is formulated on the basis of the optimum control theory. A special control vector is defined to redistribute the compute expensive between a network analysis and a parametric optimization. This redistribution permits the minimization of a computer time. The problem of the minimal-time network design can be formulated in this case as a classical problem of the optimal control for some functional minimization. The principal difference between the new approach and before elaborated generalized methodology is presented. This difference is based on a higher level of the problem generalization. In this case the structural basis of different design strategies is more complete and this circumstance gives possibility to obtain a great value of computer time gain. Numerical results demonstrate the efficiency and perspective of the proposed approach.

Key-Words: - Time-optimal design algorithm, control theory formulation, general methodology.

1 Introduction

The computer time reduction of a large system design is one of the sources of the total quality design improvement. This problem has a great significance because it has a lot of applications, for example on VLSI electronic circuit design. Any traditional system design strategy includes two main parts: the mathematical model of the physical system that can be defined by the algebraic equations or differential-integral equations and optimization procedure that achieves the optimum point of the design objective function. In limits of this conception it is possible to change optimization strategy and use the different models and different methods of analysis but in each step of the circuit optimization process there are a fixed number of the equations of the mathematical model and a fixed number of the independent parameters of the optimization procedure.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose \cite{1}-\cite{2}. Other approach to reduce the amount of computational required for both linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in \cite{3} and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation \cite{5}. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in \cite{6} for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization \cite{7} and will be improved later on. The practical aspects of these methods were developed for the electronic circuits design with the different optimization criterions \cite{8}-\cite{9}. The fundamental problems of the development, structure elaboration, and adaptation of the automation design systems have been examine in some papers \cite{10}-\cite{11}.

The above described system design ideas can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed on heuristic level some decades ago \cite{12}. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the
circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [13] and for the synthesis of high-performance analog circuits [14] in extremely case, when the total system model was eliminated. The authors of the last papers affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy.

Nevertheless all these ideas can be generalized to reduce the total computer design time for the system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [15]. The number of the different design strategies, which appear in the generalized theory, is equal to $2^M$ for the constant value of all the control functions, where $M$ is the number of dependent parameters. These strategies serve as the structural basis for more strategies construction with the variable control functions. The main problem of this new formulation is the unknown optimal dependency of the control function vector that satisfies to the time-optimal design algorithm.

However, the developed theory [15] is not the most general. In the limits of this approach only initially dependent system parameters can be transformed to the independent but the inverse transformation is not supposed. The next more general approach for the system design supposes that initially independent and dependent system parameters are completely equal in rights, i.e. any system parameter can be defined as independent or dependent one. In this case we have more vast set of the design strategies that compose the structural basis and more possibility to the optimal design strategy construct.

### 2 Problem Formulation

In accordance with the new system design methodology [15] the design process can be defined as the problem of the cost function $C(X)$ minimization for $X \in R^N$ by the optimization procedure and by the analysis of the modified electronic system model. The optimization procedure can be determined in continuous form as:

$$\frac{dx_i}{dt} = f_i(X, U), \quad i = 1, 2, ..., N \quad (1)$$

The modified electronic system model can be expressed in the next form:

$$\left(1 - u_j\right)g_j(X) = 0, \quad j = 1, 2, ..., M \quad (2)$$

where $N=K+M$, $K$ is the number of independent system parameters, $M$ is the number of dependent system parameters, $X$ is the vector of all variables $X = \{x_1, x_2, ..., x_K, x_{K+1}, x_{K+2}, ..., x_N\}$; $U$ is the vector of control variables $U = \{u_1, u_2, ..., u_M\}$; $u_j \in \Omega$; $\Omega = \{0; 1\}$.

The functions of the right hand part of the system (1) depend on the concrete optimization algorithm and, for instance, for the gradient method are determined as:

$$f_i(X, U) = -b \frac{\delta}{\delta x_i} \left[ C(X) + \frac{1}{\epsilon} \sum_{j=1}^{M} u_j g_j^2(X) \right]$$

for $i = 1, 2, ..., K$,

$$f_i(X, U) = -b \cdot u_{-k} \frac{\delta}{\delta x_i} \left[ C(X) + \frac{1}{\epsilon} \sum_{j=1}^{M} u_j g_j^2(X) \right]$$

$$+ \frac{1 - u_{-k}}{dt} \left[ -x_i + \eta_i(X) \right]$$

for $i = K + 1, K + 2, ..., N$,

where $b$ is the iteration parameter; the operator $\frac{\delta}{\delta x_i}$ hear and below means

$$\frac{\delta}{\delta x_i} \phi(X) = \frac{\partial \phi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M} \frac{\partial \phi(X)}{\partial x_p} \frac{\partial x_p}{\partial x_i},$$

$x_i$ is equal to $x_i(t - dt)$; $\eta_i(X)$ is the implicit function $(x_i = \eta_i(X))$ that is determined by the system (2), $C(X)$ is the cost function of the design process.

The problem of the optimal design algorithm searching is determined now as the typical problem of the functional minimization of the control theory. The total computer design time serves as the necessary functional in this case. The optimal or quasi-optimal problem solution can be obtained on the basis of analytical [16] or numerical [17]-[20] methods. By this formulation the initially dependent parameters for $i = K + 1, K + 2, ..., N$ can be
transformed to the independent ones when \( u_j = 1 \)
and it is dependent when \( u_j = 0 \). On the other hand
the initially independent parameters for \( i = 1, 2, \ldots, K \),
are independent ones always.

We have been developed in the present paper
the new approach that permits to generalize more
the above described design methodology. We
suppose now that all of the system parameters can
be independent or dependent ones. In this case we
need to change the equation (2) for the system
model definition and the equation (3) for the right
parts description.

The equation (2) defines the system model and
is transformed now to the next one:

\[
(1 - u_j)g_j(X) = 0
\]  

where \( J \) is the index set for all those functions
\( g_j(X) \) for which \( u_j = 0, J = \{j_1, j_2, \ldots, j_l\}, j_s \in \Pi \)
with \( s = 1, 2, \ldots, Z \). \( \Pi \) is the set of the indexes from
1 to \( M \), \( \Pi = \{1, 2, \ldots, M\} \). \( Z \) is the number of the equations
that will be left in the system (4), \( Z \in \{0, 1, \ldots, M\} \).

The right hand side of the system (1) is defined
now as:

\[
f_i(X, U) = -b \cdot u_i \frac{\partial}{\partial x_i} F(X, U) + \frac{(1 - u_i)}{dt} \{- x_i (t - dt) + \eta(X)\}
\]  

The nonlinear element has the following
dependency: \( y_{ni} = y_0 + b(V_i - V_j)^2 \). The vector \( X \)
includes five components: \( x_1 = y_1, x_2 = y_2, x_3 = y_3, x_4 = V_1, x_5 = V_2 \). The redefinition of the independent
variables \( x_1, x_2, x_3 \) by squares gives us the possibility
to solve the problem of feasibility. The model (4) of
the circuit includes two equations \( M = 2 \). The functions
\( g_i(X) \) are defined by the next formulas:

\[
g_1(X) = (1 - x_2)x_1^2 - (x_4 - x_5)(y_0 + a(x_4 - x_3)^2) - x_4x_3^2 = 0
\]
\[
g_2(X) = (x_4 - x_5)(y_0 + a(x_4 - x_3)^2) - x_4x_3^2 = 0
\]  

The optimization procedure (1), (5) includes five
equations. The structural basis includes four design
strategies according to the generalized methodology
of the first level. Nevertheless, in the limits of the
second level of generalization, the total structural
basis contains 16 different design strategies
\( \sum_{i=0}^{M} C_{K+i} = 16 \). The system (4) is solved by the
Newton-Raphson method. The cost function \( C(X) \)
is defined by the formula \( C(X) = (x_1 - k_1)^2 + (x_5 - k_2)^2 \).

3 Numerical Results

New generalized methodology has been used for
some non-linear electronic circuit optimization. The
numerical results correspond to the integration of
the system (1) with variable optimized step. The
cost function \( C(X) \) has been defined as a sum of
squares of differences between before defined and
current value of some node voltages.

3.1 Example 1

A simple two-node nonlinear passive circuit is
presented in Fig. 1. The design procedure was
realized by means of the new generalized
methodology.
The design results for some strategies of full structural basis are presented in Table 1.

Table 1. Some strategies of the structural basis for two-node circuit.

<table>
<thead>
<tr>
<th>N</th>
<th>Control functions vector U (u1, u2, u3, u4, u5)</th>
<th>Calculation results</th>
<th>Iterations number</th>
<th>Total design time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0 1 0 1 1)</td>
<td></td>
<td>5</td>
<td>0.000851</td>
</tr>
<tr>
<td>2</td>
<td>(0 1 1 1 1)</td>
<td></td>
<td>178</td>
<td>0.016671</td>
</tr>
<tr>
<td>3</td>
<td>(1 0 0 1 1)</td>
<td></td>
<td>201</td>
<td>0.026235</td>
</tr>
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<td>4</td>
<td>(1 0 1 1 1)</td>
<td></td>
<td>3162</td>
<td>0.300012</td>
</tr>
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<td>5</td>
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<td></td>
<td>23</td>
<td>0.002205</td>
</tr>
<tr>
<td>6</td>
<td>(1 1 0 1 0)</td>
<td></td>
<td>49</td>
<td>0.002405</td>
</tr>
<tr>
<td>7</td>
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<td></td>
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<td>0.002205</td>
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<td>107</td>
<td>0.010365</td>
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<td>9</td>
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<td>1063</td>
<td>0.170011</td>
</tr>
<tr>
<td>10</td>
<td>(1 1 1 1 0)</td>
<td></td>
<td>143</td>
<td>0.013115</td>
</tr>
<tr>
<td>11</td>
<td>(1 1 1 1 1)</td>
<td></td>
<td>243</td>
<td>0.006215</td>
</tr>
</tbody>
</table>

Four last strategies of the table are the same that had been defined inside the previously (first level) formulated methodology. We can name these strategies as the “old” ones. It is very interesting that some new strategies have the computer time significantly lesser than all the “old” strategies.

The strategy number 1 with the control vector (01011) has the minimal computer time among all the strategies and it has the maximum time gain 12.2 with respect to the traditional design strategy (TDS) number 8 that corresponds to the control vector (11100). At the same time the modified traditional design strategy (MTDS) that corresponds to the control vector (11111) is the best among all of the “old” strategies and has the time gain 1.67 only. So, strategy 1 has an additional time gain 7.3 times.

3.2 Example 2

This example corresponds to the network in Fig.2.

Figure 2. One-stage transistor amplifier.

The Ebers-Moll static model of transistor has been used [21]. The vector \( X \) includes six components: \( x_1^2 = y_1, \ x_2^2 = y_2, \ x_3^2 = y_3, \ x_4 = V_1, \ x_5 = V_2, \ x_6 = V_3 \). The model (4) of this network includes three equations \((M=3)\), the optimization procedure (1) includes six equations \((K+M=6)\). The total “old” structural basis contains eight different design strategies. The total number of the different design strategies that compose the new structural basis of the second level of generalized theory is equal \( \sum_{i=0}^{3} C_i^3 = 42 \). The strategy that has the control vector (111000) is the TDS in terms of the first level of generalized methodology. In this case only three first equations of the system (1) are included in optimization procedure to minimize the generalized cost function \( F(X, U) \). The model of the circuit includes three equations too. The cost function \( C(X) \) was defined by the formula

\[
C(X) = [x_1 - x_2 - m_1]^2 + [x_3 - x_2 - m_2]^2
\]

where \( m_1, m_2 \) are the necessary, before defined voltages on transistor junctions.

Some design strategies of full structural basis are presented in Table 2.

Table 2. Some strategies of the structural basis for one-stage transistor amplifier.

<table>
<thead>
<tr>
<th>N</th>
<th>Control functions vector U (u1, u2, u3, u4, u5, u6)</th>
<th>Calculation results</th>
<th>Iterations number</th>
<th>Total design time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0 1 1 1 0 0)</td>
<td>12850</td>
<td>6650</td>
<td>10862.33</td>
</tr>
<tr>
<td>2</td>
<td>(0 1 1 1 0 1)</td>
<td>47</td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>(0 1 1 1 1 0)</td>
<td>30015</td>
<td>1036</td>
<td>10996.24</td>
</tr>
<tr>
<td>4</td>
<td>(1 0 1 1 1 0)</td>
<td>59962</td>
<td>25094</td>
<td>25094.21</td>
</tr>
<tr>
<td>5</td>
<td>(1 0 1 1 1 1)</td>
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<td>170</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(1 1 0 0 1 1)</td>
<td>174</td>
<td>60.01</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(1 1 0 1 0 1)</td>
<td>606</td>
<td>220.21</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(1 1 0 1 1 1)</td>
<td>778</td>
<td>139.11</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(1 1 1 0 0 0)</td>
<td>9311</td>
<td>7977.01</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(1 1 1 0 0 1)</td>
<td>7514</td>
<td>4989.11</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(1 1 1 0 1 0)</td>
<td>75635</td>
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<td></td>
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<td>12</td>
<td>(1 1 1 0 1 1)</td>
<td>324</td>
<td>60.11</td>
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<td>10232</td>
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<td>16</td>
<td>(1 1 1 1 1 1)</td>
<td>2418</td>
<td>196.21</td>
<td></td>
</tr>
</tbody>
</table>

The strategy 16 that corresponds to the control vector (111111) is the MTDS. All six equations of system (1) are involved in the optimization procedure, but the model (2) has been vanished in this case. Other strategies can be divided in two parts. The strategies that have units for three first components of the control vector define the subset of “old” strategies in limits of the first level of
3.3 Example 3

The last example corresponds to the three-stage transistor amplifier in Fig.3.

![Three-stage transistor amplifier](image)

In this case the vector \( X \) includes 14 components. Seven components define the independent parameters \( x_1 = y_1, x_2 = y_2, x_3 = y_3, x_4 = y_4, x_5 = y_5, x_6 = y_6 \) and other seven components \( x_7 = V_1, x_8 = V_2, x_9 = V_3, x_{10} = V_4, x_{11} = V_5, x_{12} = V_6, x_{13} = V_7 \) define the dependent parameters in accordance with the traditional approach. The cost function \( C(X) \) for the design problem was defined by the formula similar to the previous examples.

The structural basis consists of 128 different design strategies according to the first level of generalization. On the other hand the structural basis of the second level of generalization is equal to

\[
\sum_{i=0}^{14} C_{i4} = 9908.
\]

Once again we have very broaden structural basis in the second case. The results of the analysis of some design strategies for this network are presented in Table 3.

The design strategies numbered from 15 to 28 belong to the subset that appears in limits of the first level of design methodology generalization. The strategy 15 that corresponds to the control vector \((11111100000000)\) is the traditional design strategy. The strategy 22 that corresponds to the control vector \((11111110111111)\) has the minimum computer time among all the strategies of this subset. The time gain in this case is equal to 368 times. The strategies from 1 to 14 belong to the subset of new design strategies. Six strategies of this subset have the design time lesser than the best strategy of the “old” structural basis. The best strategy among new structural basis has the time gain 11715 times with respect to the traditional design strategy and has an additional time gain 31.8 times with respect to the better “old” strategy.

Table 3. Some strategies of the structural basis for three-stage transistor amplifier.

<table>
<thead>
<tr>
<th>N</th>
<th>Control function vector ( U = (u_1, u_2, \ldots, u_{14}) )</th>
<th>Calculation results</th>
<th>Iterations number</th>
<th>Total design time (sec)</th>
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</table>

So, taking into consideration the obtained results we can state that the second level of the design methodology generalization gives the possibility to improve all characteristics of the generalized design theory. Further analysis may be focused on the problem of the optimal design strategy searching by means of the control vector manipulation. It is intuitively clear that we can obtain very great time gain by means of the new structural basis.
4 Conclusion

The traditional approach for the analog circuit design is not time-optimal. The problem of the optimum algorithm construction can be solved more adequately on the basis of the optimal control theory application. The time-optimal design algorithm is formulated as the problem of the functional minimization of the optimal control theory. In this case it is necessary to select one optimal trajectory from the quasi-infinite number of the different design strategies, which are produced. The new and more complete approach to the electronic network design methodology has been developed now. This approach generates structural basis of the different design strategies that is more broadened than for the previous developed methodology. The total number of the different design strategies, which compose the structural basis by this approach, is equal \( \sum_{i=0}^{M} C_{K+M} \).

This new structural basis serves as the necessary set for searching the optimal design strategy. This approach can reduce considerably the total computer time for the system design. Analysis of the different problems of the electronic system design shows a significant potential of the new level of generalized design methodology. The potential computer time gain that can be obtain on the basis of new approach is significantly more than for the previous developed methodology.

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References: