Optimal Control of Electric Drives Acceleration with Static Torque with Constant and Speed Proportional Component and Heating Outline

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Abstract: - In this paper it is considered an electric drive with static torque with constant component and speed proportional component. Using the classic calculus of variations is determined the extremal control and trajectory and the overheating that ensures maximum exploitation of the system resources represented by the achievement of a maximum variation of speed in the acceleration processes.

Key-Words: - analytic and numerical model, extremal trajectory, extremal control, optimal control, overheating.

1 Introduction
In the case of electric drives working in continuous duty type (S1), it is necessary to perform the start-up process and in the case of those electric drives working in continuous duty type with periodical change of speed (S8), it is necessary to perform changes of speed.

To estimate the heating process at the drive system acceleration, as performance number can be adopted the maximum exploitation of the system resources. Using the classic calculus of variations can be solved this optimization problem.

2 Problem Formulation
Please It is considered an electric drive with static torque with constant component, speed and square speed proportional component [1], [2].

\[ M_s = M_0 + k_1 \omega \omega + k_2 \omega \omega^2 \]

or

\[ F_s = F_0 + k_1 v + k_2 v^2 \]

Neglecting the electromagnetic inertia in respect of the mechanics inertia, supposing a constant inertia moment, the electric drive will be described by the general movement equation

\[ M = M_s + J \frac{d \omega}{dt}, \quad \text{or} \quad F = F_s + J \frac{dv}{dt} \]

and by the dependence between speed and acceleration.

\[ \omega = \int \omega dt \quad \text{or} \quad v = \int \omega dt. \]

To expand the interpretations and the conclusions, with and for the restraint of the value intervals, will be introduced relative coordinates. In this sense, considering as a reference for time the mechanical constant of time

\[ T = \frac{J \omega_n}{M_n} \]

and for electricity, couple and speed, their nominal values will be obtained the relative values

\[ \tau = \frac{t}{T}, \quad \mu = \frac{M}{M_n}, \quad F = \frac{F_s}{F_n}, \quad v = \frac{\omega}{\omega_n} = \frac{v}{v_n}, \]

\[ \mu_s = \frac{M_s}{M_n}, \quad \mu_0 = \frac{M_0}{M_n}, \quad F_0 = \frac{F_0}{F_n} \]

and for relative acceleration there will be the relation

\[ \frac{\dot{v}}{v_n/T} = \frac{\dot{v}}{v_n/T}. \]

In the hypothesis of proportionality between the electromagnetic couple and the burden power, the equations (1), (2) and (3) in the relative coordinates it becomes [1, 4]

\[ \mu_s = \mu_0 + k_1 v + k_2 v^2 \]

\[ \mu_0 + k_1 + k_2 = 0, \quad v = \int \dot{v} dt \]

\[ i = \mu_s + \dot{v} = \mu_0 + k_1 v + k_2 v^2 + \dot{v}, \]

with the initial and fixed conditions

\[ \tau = \tau_j, \quad v(\tau_j) = v_j, \quad \tau = \tau_j, \quad v(\tau_j) = v_j. \]

The multitude of the conclusions admitted and the multitude of the trajectories that will be admitted will be

ISSN: 1790-5117
considered marginal and open multitudes. If from electromechanical point of view, the drive is described by the general movement equation (7) and by the dependence between acceleration and speed given by (8), then from heating point of view, considering that the driving motor is a homogenous object and all the motor points have the same temperature in the same time, based on the infinitesimal heat balance results:

\[ Q d \theta + q \theta dt = \Delta P dt, \quad (10) \]

where \( \theta \) is the motor heating with respect to the temperature of the environment, \( Q \) is the quantity of heat necessary to rise with one degree the drive temperature, \( q \) is the quantity of heat yielded to the environment in time unit and at a temperature difference of one degree and \( \Delta P \) is the motor power loss which is transformed in heat.

The differential equation of the driving motor heating is

\[ T_0 \frac{d \theta}{dt} + \theta = \frac{\Delta P}{q}, \quad (11) \]

where by \( T_0 \) (heating time constant) it is noted the ratio \( Q / q \). Taking into account only the heat determined by the load current through Joule effect and relating the equation with the nominal heating \( \theta / \theta_m = \theta \), and the time with the time mechanics constant

\[ \frac{\Delta P}{q} = \frac{R_i^2}{q} = i^2, \]

the differential equation in relatives coordinates becomes

\[ \frac{T_0}{T} \frac{d \theta}{d \tau} + \theta = i^2, \quad (12) \]

It is noted with \( m \) the ratio between the time mechanics constant that takes values of seconds or seconds fractions size and the heating constant that takes values of tenth of minutes size and taking into account the equation (7), the heating differential equation becomes

\[ \frac{\Delta P}{q} = \frac{R_i^2}{q} = i^2, \quad (13) \]

The set of accepted controls and trajectories are considered as open and bounded sets. To use the driving motor at its whole capacity, the set of heating trajectories is considered as close and bounded set, that means will exists the heating upper restriction

\[ \theta \leq \theta_{\max} \leq 1. \quad (14) \]

3 The Optimization Criterion

Please To estimate the drive system working, a maximum exploitation criterion of the system resources can be adopted. This criterion is represented by the achievement of a maximum variation of speed and is expressed by the integral

\[ J[v(\tau)] = \Delta v = v_2 - v_1 = \int_{\tau_1}^{\tau_2} v d\tau, \quad (15) \]

4 Formulation of Optimization Problem

The optimization problem consists in determining the admitted optimal control \( i^*(\tau) \) or \( \mu^*(\tau) \), that assures the system evolution from the initial conditions \( (\tau_1, v(\tau_1), \theta(\tau_1)) \) to final conditions \( (\tau_2, v(\tau_2), \theta(\tau_2)) \), on an admitted trajectories represented by the speed extremal \( v^*(\tau) \) and by the motor overheating extremal \( \theta^*(\tau) \), so that is obtained the maximizing of the speed variation that is the maximizing of the criterion functional

\[ J[v(\tau)] = \Delta v = v_2 - v_1 = \int_{\tau_1}^{\tau_2} v d\tau = \text{max}. \quad (16) \]

for a given value of the time interval expressed by

\[ \tau_2 - \tau_1 = \int_{\tau_1}^{\tau_2} d\tau = \text{min} \]

satisfying the differential link (13), the initial and final conditions and the temperature restriction (14). To solve the isometric extreme problem it is necessary to reduce it to an unconditional extreme problem, by determining Lagrange auxiliary function with the help of Lagrange multiplier \( \lambda(\tau) \)

\[ L = \dot{v} + \lambda(\tau) [\dot{\theta} / m + \theta - (\mu_0 + k_1 v + k_2 v^2 + v)^2], \quad (18) \]

and determining the unconditional extreme with the following functional [1]

\[ J[v(\tau), \theta(\tau)] = \int_{\tau_1}^{\tau_2} L \left[ \theta(\tau), \dot{\theta}(\tau), v(\tau) \right] d\tau = \text{min} \]

5 The Extrem Condition

The low extreme necessary condition is expressed by Euler-Lagrange equation [5]

\[ \frac{\partial L}{\partial \theta} - \frac{d}{d \tau} \frac{\partial L}{\partial \dot{\theta}} = 0, \quad (20) \]

where, having

\[ \frac{\partial L}{\partial \theta} = \lambda, \quad \frac{\partial L}{\partial \dot{\theta}} = \frac{1}{m} \lambda \]

and

\[ \frac{d}{d \tau} \frac{\partial L}{\partial \dot{\theta}} = \frac{1}{m} \lambda \]

results the homogeneous differential equation in \( \lambda \)

\[ \dot{\lambda} - m \lambda = 0, \quad (22) \]

and from Euler-Lagrange equation for the function \( v(\tau) \)

\[ \frac{d}{d \tau} \frac{\partial L}{\partial v} = \frac{d}{d \tau} \frac{\partial L}{\partial \dot{v}} = 0, \quad (23) \]

Where

\[ \frac{\partial L}{\partial v} = -2 \lambda \left( \mu_0 + k_1 v + k_2 v^2 + v \right), \quad (24) \]
\[
\frac{d}{d\tau} \frac{\partial L}{\partial \dot{\nu}} = -2 \lambda \left( \mu_0 + k_1 \nu + k_2 \nu^2 \right) - 2 \lambda \left( k_1 \dot{\nu} + 2 k_2 \nu \ddot{\nu} + \dddot{\nu} \right)
\]
results the linear differential equation of the second order
\[
\lambda \left( k_1 \dot{\nu} + 2 k_2 \nu \ddot{\nu} + \dddot{\nu} \right) + \lambda \left( \mu_0 + k_1 \nu + k_2 \nu^2 + \nu \right) = 0.
\]
(25)

6 The Optimal Solution

Cas the constant static torque, considering the particularizations
\[
\mu_0 > 0, \quad k_1 = 0, \quad \mu_s = \mu_0,
\]
the condition of extremum, expressed by the differential equation (23), becomes
\[
\lambda \left( \mu_0 + \nu \right) = C_1
\]
(27)
The characteristic equation attached to the differential equation (20) [5]
\[
r - m = 0, \quad r = m
\]
has the general solution
\[
\lambda = C_2 e^{mr}
\]
(29)
Using the overheating initial condition, it is determined the arbitrary constant
\[
\tau_1 = 0, \quad \phi(0) = \phi_1, \quad -i'(0) + C = \phi_1 \Rightarrow C = i'(0) + \phi_1
\]
and the overheating extremal trajectory
\[
\phi' \left( \tau \right) = i'(0) \left( e^{-m\tau} - e^{-mt\tau} \right) + \phi_1 e^{m\tau}
\]
(31)
For evolution current temperature expression haven maximal value (fig.1).

\[
\text{Fig.1} \quad \text{Temperature evolution for initial current}
\]
and \( \phi_1 = 0 \)

The overheat maximal condition is given by the equation resulted by cancelling the overheat derivate.
\[
i'(0) \left( -1 + 2 e^{-m\tau} \right) - \phi_1 = 0
\]
(32)
from which the critical time value of this maxim is obtained
\[
m \tau_{cr} = \ln \frac{2}{1 + \phi_1} = \ln 2 - \ln \left( 1 + \frac{\phi_1}{i'(0)} \right)
\]
(33)
For the use of the driving motor at its entire capacity under thermal aspect, the equality sign from \( \phi \leq 1 \). It is imposed to be obtained in the final moment \( \tau = \tau_2 \), that is
\[
\phi(\tau_2) = i'(0) \left( e^{-m\tau_2} - e^{-mt\tau_2} \right) + \phi_1 e^{m\tau_2} = 1
\]
(34)
from where the initial value of optimal current can be computed (fig.2)
\[
i(0) = \frac{1 - \phi_1 e^{-m\tau_2}}{e^{-m\tau_2} - e^{-mt\tau_2}}
\]
(35)

\[
\text{Fig.2. Extreme trajectory of current and the temperature} \quad (\phi_1 = 0, \quad \tau_2 > \tau_{cr})
\]

For the case when the final time overflow the critical time \( \tau_2 > \tau_{cr} \), the overheat maximum is realized inside the period \([0, \tau_2]\), and the functional extreme, that means the maximum variation of the speed, is obtained on the trajectory composed by extremala \( \phi'(T) \) for \( T \in [0, \tau_{cr}] \) and from domain boundary \( \phi = 1 \) for \( T \in [\tau_{cr}, \tau_2] \).
In case having \( \mu_0 \neq 0, \quad k_1 \neq 0, \quad k_2 \neq 0 \) and \( \mu_0 + k_1 + k_2 = 1 \), the extreme necessary condition expressed by the differential equation (25) becomes
\[
\lambda \left( k_1 \dot{\nu} + \nu \right) + \lambda \left( \mu_0 + k_1 \nu + \nu \right) - k_1 \lambda \left( \mu_0 + k_1 \nu + \nu \right) = 0
\]
(36)
The characteristic equation attached to the differential equation (20) [5]
\[
r - m = 0, \quad r = m
\]
has the general solution
\[
\lambda = C_0 e^{mr}
\]
(38)
Replacing $\lambda$ and its derivative in the differential equation (23), it is obtained

$$C_0e^{mr}(k_1\dot{v} + \ddot{v}) + mC_0e^{mr}(\mu_0 + k_1v + \dot{v}) - k_1Ce^{mr}(\mu_0 + k_1v + \dot{v}) = 0 \tag{39}$$
and then the differential equation for speed

$$\ddot{v} + m\ddot{v} - k_1(k_1 - m)v = \mu_0(k_1 - m). \tag{40}$$

Having the particular solution of the heterogeneous differential equation

$$r^2 + m\dot{r} - k_1(k_1 - m) = 0 \tag{41}$$
with

$$r_1 = k_1 - m, \quad r_2 = -k_1$$
with the general solution of the homogeneous differential equation

$$v_g = C_1e^{(k_1-m)r} + C_2e^{-k_1r} \tag{42}$$
will results the general solution of the heterogeneous differential equation, that is the speed trajectories family

$$v = -\frac{\mu_0}{k_1} + C_1e^{(k_1-m)r} + C_2e^{-k_1r} \tag{43}$$

Using the speed initial condition

$$\tau_1 = 0, \quad v(0) = v_1, \quad -\frac{\mu_0}{k_1} + C_1 + C_2 = v \tag{44}$$

obtained

$$C_2 = \frac{\mu_0}{k_1} - C_1 + v_1 \tag{45}$$
the speed trajectories are

$$v = -\frac{\mu_0}{k_1}(1 - e^{-k_1}) + C_1\left(\frac{e^{(k_1-m)r} - e^{-k_1r}}{k_1}\right) + v_1e^{-k_1r} \tag{46}$$
The acceleration is determined by speed differentiation

$$\dot{v} = -\mu_0e^{k_1r} + C_1\left[(k_1-m)e^{(k_1-m)r} + k_1e^{-k_1r}\right] - k_1v_1e^{k_1r} \tag{47}$$
Corresponding to the movement general equation, the current (the torque) is

$$i = \mu_0 - \mu_0\left(1 - e^{-k_1\tau}\right) + \frac{k_1C_1}{k_1}\left[\frac{e^{(k_1-m)r} - e^{-k_1r}}{k_1}\right] + \frac{k_1\dot{v}e^{k_1r}}{k_1} + C_1\left[(k_1-m)e^{(k_1-m)r} + k_1e^{-k_1r}\right] - k_1v_1e^{k_1r} \tag{48}$$
Considering the current initial condition $\tau_1 = 0$

$$i(0) = C_1(2k_1 - m) \Rightarrow C_1 = \frac{i(0)}{2k_1 - m} \tag{49}$$
the optimal current (torque) has the exponential expression (fig.3)

$$i'(\tau) = \mu'(\tau) = i(0)e^{(k_1-m)\tau} \tag{50}$$
The arbitrary constant $C_1$ being determined, the speed and acceleration extremals are (fig.3)

$$v^*(\tau) = \frac{i(0)}{2k_1 - m}\left[\left(k_1 - m\right)e^{(k_1-m)\tau} - e^{-k_1\tau}\right] - \frac{\mu_0}{k_1} + \frac{\mu_0 + v_1}{k_1}e^{-k_1\tau} \tag{51}$$
$$\ddot{v}^*(\tau) = \frac{i(0)}{2k_1 - m}\left[k_1\left(k_1-m\right)e^{(k_1-m)\tau} + k_1e^{-k_1\tau}\right] - (\mu_0 + k_1v_1)e^{-k_1\tau} \tag{52}$$
Imposing the condition like the speed has the value $v_2$ at the end of the acceleration interval

$$v(\tau_2) = \frac{i(0)}{2k_1 - m}\left[(k_1-m)e^{(k_1-m)\tau} - e^{-k_1\tau}\right] \tag{53}$$

$$\frac{\mu_0}{k_1} + \frac{\mu_0 + v_1}{k_1}e^{-k_1\tau} = v_2$$
it can be determined the initial value of the acceleration current

$$i(0) = \frac{(2k_1 - m)}{k_1}\left[\left(k_1 - m\right)e^{(k_1-m)\tau} - \frac{\mu_0}{k_1} + v_1\right] \tag{54}$$
that assures the requested speed variation.

The performance number, that is the maximum speed variation has the value

$$J(\nu) = \Delta \nu = v_2 - v_1 = \frac{i(0)}{2k_1 - m}\left[(2k_1 - m)e^{(k_1-m)\tau} - e^{-k_1\tau}\right]$$
where

$$\frac{\mu_0}{k_1} + \frac{\mu_0 + v_1}{k_1}$$
being of type

$$\dot{y} + P_0y = Q_0(x) \tag{57}$$
with the general solution

$$y = e^{-P_0\tau}\left[\int Q_0e^{P_0\tau} + C\right]. \tag{58}$$
Having \( P_0 = m \), \( \int P_0 \, dx = \int m \, d\tau = m \, \tau \), and
\[
\int q_0 \, (P_0 \, dx) = \int m^2 (0) e^{2(k_1-m)\tau} e^{\tau} \, d\tau = \frac{m}{2k_1 - m} \int m^2 (0) e^{2(k_1-m)\tau} \, d\tau
\]
the overheating will be
\[
\theta = e^{-m \tau} \left( \frac{m}{2k_1 - m} - \frac{i^2}{2} (0) e^{(2k_1-m)\tau} + C \right) = \frac{m}{2k_1 - m} i^2 (0) e^{2(k_1-m)\tau} + Ce^{-m \tau}
\]
(60)

Using the overheating initial condition, it is determined the arbitrary constant
\[
\tau_i = 0, \theta(0) = \theta_i, \theta_i = \frac{m}{2k_1 - m} i^2 (0) + C
\]
\[
\Rightarrow C = \theta_i - \frac{m}{2k_1 - m} i^2 (0)
\]
(61)

and the overheating extremal trajectory
\[
\theta^*(\tau) = \frac{m}{2k_1 - m} i^2 (0) e^{2(k_1-m)\tau} + \theta_i e^{-m \tau}
\]
(62)

This expression is consistent as time \( \theta \leq 1 \). The acceleration current (torque) for \( m > k_1 \) (excepting the case \( m=2k_1 \)) is time decreasing (fig. 4 and fig. 5), for \( m=k_1 \) is constant (fig. 6), and for \( m<k_1 \) (fig. 7) is time increasing.

Fig. 4 Speed, acceleration, temperature and current extremals in the case of acceleration \( (\mu=0.9, \, k_1=0.1, \, m=0.15, \, \nu_1=0.2, \, \nu_2=0.2, \, k_1<m<2k_1) \)

Fig. 5 Speed, acceleration, temperature and current extremals in the case of acceleration \( (\mu=0.9, \, k_1=0.1, \, m=0.25, \, \nu_1=0.2, \, \nu_2=0.2, \, k_1<k_1) \)

Fig. 6 Speed, acceleration, temperature and current extremals in the case of acceleration \( (\mu=0.9, \, k_1=0.1, \, m=0.1, \, \nu_1=0.2, \, \nu_2=0.2, \, m=k_1) \)

Fig. 7 Extremals in the case of acceleration \( (\mu=0.9, \, k_1=0.1, \, m=0.05, \, \nu_1=0.2, \, \nu_2=0.2, \, 0<m<k_1) \)

Initial values of the acceleration current \( i(0) \) and of the acceleration \( \nu(0) \), that exceed the admissible values because of the big heating time constant, result by calculating the static current (torque) initial value from the final condition of heating \( \theta(\tau) = 1 \), at reduced values of the initial heating and specially at reduced values of the acceleration time interval. The speed exceeds the necessary values at the end of the acceleration interval. Such situations determine a bigger
heating and power loss. For this reason, it is imposed to calculate the initial value of the acceleration current \( i(0) \) both from the final speed condition \( v(t_f) = v_2 \) and from the overheating final condition \( \theta(t_f) = 1 \) and to use the least value for the other expressions (current, acceleration, speed and overheating).

### 7 Conclusion

The obtained results expressed throw the extremal control and trajectory can be used both in design and in optimal control of electric drive systems with static torque speed dependent working in continuous duty or in continuous duty with periodical change of speed. An increase of the quality and of the efficiency of those electric drive systems is obtained due to these results. Thermovision/thermography in infrared is a recent technique in the domain of modern methods of diagnosis in industry, and it offers high precision results which reduce the time to detect faults and which evaluate very precisely the state of equipments during work, without their stop or their removal and transportation to a diagnosis centre.

In some industrial processes there are systems or parts of the process which don’t need to be permanently observed and diagnosed, but which regularly need a kind of inspection or analysis based on previous behavior within the process. Usually, when a problem appears at a part of a working system, it overheats. It emits more heat than before, in normal functioning conditions. There are equipments which are more resistant in time, but still have a point where they fail. This failure point can be predicted with a lot of time before the actual failure happens. For this kind of equipments or systems we thought up a system capable of extracting the useful data about it’s temperature in infrared, administrate it over a period of time which is not critical in case of fault appearance, and process it at a certain point, established from the beginning.

The result would be whether there is a change compared to the previous check-up, and if there is, establish the cause for that, and of course, acting to stop the problem to appear. In the power production, transport, distribution and use of electric energy installations, the unprogrammed stops may lead to significant increase of the exploitation costs.


