Construction of Repeat Accumulate Codes with Superimposed Structured Interleavers

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Abstract: Repeat-accumulate code has been widely used in various communication systems due to its “turbo-like” structure that can be encoded with a low complexity turbo-like encoder and decoded with a high-speed parallel algorithm. In this paper, a novel RA code construction scheme is proposed. This scheme generates the parity-check matrix with superimposed structured interleavers, which thereby guaranteeing the resulting matrix to be globally optimized. Compared with the RA codes constructed with traditional structured interleavers such as $\pi$-rotation interleavers, pseudo-random interleavers and combinatorial interleavers, the codes constructed with the proposed method exhibit better error-correcting performance with the same hardware implementation complexity.

Key–Words: Repeat-accumulate (RA) codes, Superimposed Structured Interleaver (SSI), low-density parity-check (LDPC) codes

1 Introduction

Repeat accumulate (RA) code is a class of low-density parity-check (LDPC) codes with outstanding error-correcting capability and inherently parallelizable decoding scheme [1][2][3]. Moreover, the RA code has a "turbo-like" structure with two constitute codes being a repetition code and a convolutional code, the encoding of which may thereby be implemented with linear-complexity. As a result, RA code has been accepted as one of the most successful LDPC codes and are widely used in various communication systems such as DVB-S2 and WiMax.

However, the construction of an RA code still has some challenging problems. A parity-check matrix with fully random interleavers may have good error-correcting performance, while the hardware implementation complexity of which may be very high. For practical applications, the parity-check matrix should be constructed with semi-random interleavers that could obtain near Shannon limit error-correcting performance and could be implemented with complexity close to that of structured matrix. In recent years, many efficient semi-random interleavers are proposed for RA codes, such as $\pi$-rotation interleavers [4], pseudo-random interleavers [5], and combinatorial interleavers [6]. The parity-check matrixes with these interleavers are generally constructed with a concatenated scheme, which generates the parity-check matrix with two steps: first constructs a based matrix and then each zero element of the based matrix is expanded to be a zero submatrix and each nonzero element is expanded to be an interleaved identity matrix. This scheme may obtain an LDPC code with good trade-off between coding performance and hardware complexity, while the parity-check matrix is locally optimized result rather than a globally optimized one. In recent years, the construction of RA codes with globally optimized scheme is a hot topic.

In this paper, a novel scheme is proposed for constructing good RA codes. This scheme generates the parity-check matrix with superimposed structured interleavers, and thereby guaranteeing the resulting matrix globally optimized as possible. Compared with the RA codes constructed with traditional structured interleavers such as $\pi$-rotation interleavers, pseudo-random interleavers and combinatorial interleavers, the codes constructed with the proposed method exhibit better error-correcting performance with the same hardware implementation complexity.

The rest of this paper is organized as follows. In section 2, the repeat-accumulate codes and its parity-check matrix construction schemes with traditional structure interleavers are described. Then, the proposed code construction method with superimposed structured interleavers is proposed in section 3. After that, simulation results for codes generated with the proposed algorithm are presented in section 4. Moreover, the hardware implementation issue of the RA codes with superimposed structure interleavers is discussed in section 5. Finally, conclusions are drawn in section 6.
2 Traditional Construction Schemes for Repeat Accumulate Codes

RA codes can be considered as a special class of Irregular Low Density Parity Check (LDPC) codes [7].

The RA code parity matrix is composed of two sub-matrices, $H = [H_m, H_c]$, where $H_m$ is an $M$ by $M$ dual-diagonal square matrix and $H_c$ is an $M$ by $N - M$ matrix, where $N = K(1 + l/a), M = Kl/a$. An example of $H_m$ is shown as follows.

$$H_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

It is clear that $H_m$ is constant for given $M$ and thereby the crucial issue is to optimize the interleavers to improve the coding performance. It equals to optimize $H_c$.

It has been proved that RA codes with randomly chosen interleavers may obtain good performance. Thereby most RA research has only considered the error rate result of RA codes pseudo-randomly interleavers [1], [8], [9]. However, randomly constructed interleavers may have implementation challenges. Many researchers focus on the trade-off between performance and implementation complexity, and many such interleavers have been proposed.

2.1 RA Code Construction Scheme with π-rotation Interleaver

The algorithm constructs $H_c$ as a composition of $m \times m$ circularly shifted identity matrices, which can be described by a $1 \times m$ permutation vector. We label the single sub-matrices $\pi_A, \pi_B, \pi_C$, and $\pi_D$, which rotates $\pi/2$ counterclockwise each other. For example, using the $(m = 3)$ permutation vector of [2 3 1], indicating the position of the nonzero element in each row-counting from the left. Thereby we obtain the following four $\pi$-rotations.

$$\pi_A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \pi_D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\pi_B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \pi_C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Then we can create a rate 1/2 $H_c$ from $\pi_A, \pi_B, \pi_C$ and $\pi_D$ by arranging them as follows, obviously symmetrically.

$$H_c = \begin{bmatrix} \pi_A & \pi_B & \pi_C & \pi_D \\ \pi_B & \pi_C & \pi_D & \pi_A \\ \pi_C & \pi_D & \pi_A & \pi_B \\ \pi_D & \pi_A & \pi_B & \pi_C \end{bmatrix}$$

2.2 RA Code Construction Scheme with Pseudo-Random Interleaver

The construction contains two steps: First, construct a base parity-check matrix with a random generation or PEG algorithm. Second, each nonzero element of the base matrix is expanded to be an $L$ by $L$ sub-matrix, denotes $\pi(i, j)$, in whose $k$-th row, the element of the $f(\alpha^i, (\alpha^j)^k)$ column is 1, others 0, for $0 \leq k \leq L - 1$. $L = 2^p - 1$, $\alpha$ is the primitive element in Galois Field $GF(2^p)$ and its corresponding value denotes $f(\alpha)$, where $p$ is a prime integer. For example, we set the parameter $p = 3, i = 1, j = 2$, then we get,

$$\pi(1, 2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The constructed interleaver is called pseudo-random interleaver and the sub-matrix is structured and determined by $i$ and $j$. The pseudo-random interleaver can be as follows:

$$H_c = \begin{bmatrix} \pi(i_0, j_0) & 0 & \cdots & \pi(i_4, j_4) \\ \pi(i_1, j_1) & \cdots & \pi(i_3, j_3) & 0 \\ \cdots & \pi(i_2, j_2) & \cdots & \pi(i_5, j_5) \\ \pi(i_7, j_7) & \cdots & \pi(i_6, j_6) & \cdots \end{bmatrix}$$

2.3 RA Code Construction Scheme with Combinatorial Interleaver

In [4], it has been proved that for every possible Steiner 2-design there exists a row and column permutation that maps the incidence matrix of the design into an RA code interleaver and accumulator, thus producing a high rate RA code which is 4-cycle free and the combinatorial interleaver can be implemented easily. But construction 1 and 3 introduced in [4] only construct the RA codes with $q = 3$. Although construction 2 can construct regular RA codes with any $a$ and $q$, it can not make the irregular RA codes. These
three construction methods are not flexible to design a variety of RA codes. Readers can refer to it in details.

The above optimization of the RA code interleavers always contains the following thought: construct an \( M_b \) by \( N_b - M_b \) base matrix, where \( M = M_b L \), \( N = N_b L \) and \( L \) is an integer; each non-zero element of the basic matrix is expanded to be an \( L \) by \( L \) sub-matrix, which is always a circulant permutation matrix distinct from identity matrix or random matrix, otherwise zero matrix. The good basic matrix and sub-matrices can be optimized respectively, but it is noted that they could be optimal locally and may be not the global optimal solutions.

3 Proposed RA Code Construction Scheme

In [10], parallel edges are permitted in a protograph, whose Tanner graph \( G = (V, C, E) \) consists of a set of variable nodes \( V \), a set of check nodes \( C \), and a set of edges \( E \), the mapping \( e \rightarrow (v_e, c_e) \) is not necessarily 1:1. The statement shows the sub-matrices can be superimposed in the same location. The above ideas motivate us to come up with designing better interleavers for RA codes from the global view, which we call superimposed structured interleavers (SSI).

Before starting designing SSI, some signs are introduced for convenient description. For simplicity, \((i, j, l)\) means the non-zero element in row \( l \) in \( \pi(i, j) \). Clearly, the sub-matrix \( \pi(i, j) \) is structured and determined by \( i \) and \( j \). After introducing the signs, the construction of SSI is proposed as follows:

Step1: According to the code length, and rate, choose the suitable expansion factor \( L = 2^p - 1 \), where \( p \) is a prime integer.

Step2: If all column-blocks are filled, algorithm exits. Otherwise, if weights of all rows are greater than the maximum, go to step7; else, randomly select one row under the upper bound.

Step3: If all \( \pi(i, j) \) do not meet the requirements, go to step2: Otherwise select one.

Step4: If no sub-matrix has been inserted, go to Step5: Otherwise compare with the inserted sub-matrix, if nonzero elements overlap, go to Step3.

Step5: Given \( G \), if all the nonzero elements satisfy: for any \( k \) non-zero elements in \( H, 1 \leq k \leq G/2 \), denoting \((i_0, j_0, l_0), (i_1, j_1, l_1) \ldots (i_2k-1, j_2k-1, l_2k-1)\),

\[
\begin{align*}
& \left\{ \begin{array}{l}
(l_{2k-1} = l_{2k-1} \\
(f(\alpha^{2k-2} \cdot (\alpha^{j_{2k-2}})^{j_{2k-2}}) = f(\alpha^{j_{2k-1}} \cdot (\alpha^{j_{2k-1}})^{j_{2k-1}})
\end{array} \right.
\end{align*}
\]

insert the sub-matrix; otherwise go to Step3.

Step6: If all the sub-matrices have been inserted completely in this column-block, then insert the next, go to Step2: otherwise directly go to Step2.

Step7: delete the formerly inserted sub-matrix, go to Step2.

After that, slightly coordinate the sub-matrices to meet the row-block requirements if necessary, then the formula on behalf of superimposed structured interleaver could be shown in equation (1).

4 Simulations and Numeric Results

With the SSI, a rate 0.5, code length 2032 RA code is constructed with \( L=127 \), and the column-weight density function and row-weight density function are

\[
\lambda(x) = 0.5x + 0.5x^3 \quad (2)
\]

and

\[
\rho(x) = 0.125x^4 + 0.875x^5 \quad (3)
\]

The error correction performance of the code is presented in Figure 1. We are aware of one combinatorial interleaver for RA codes from [4], named modified construction 2. The performance curve of the code with the same degree distribution and rate constructed is plotted, length 2022. The performance of the RA code with \( \pi \)-rotation interleaver and the same parameters is presented, length 2000. In Figure 2, we demonstrate the performance of the RA code with SSI compared to that with pseudo-random interleaver, plotted by round and square lines respectively.

It is shown in Figure 1 that the performance of the code generated with the proposed method is much better than that generated with the combinatorial algorithm. The performance improvement is about 0.78 dB for output bit error rate of \( 10^{-7} \). It is observed that the code generated with the proposed method outperforms the \( \pi \)-rotation RA code about 0.1 dB and has a lower error floor. It is also seen from Figure 2 that
the RA code with SSI significantly outperforms the code with pseudo-random interleaver and exhibit improved error-floor performance. Compared with them, the RA codes with SSI are capable of outperforming the RA codes with three above interleavers, while the code length is almost the same.

5 Implementation Architecture

5.1 Encoder Architecture

RA codes can also be seen as a class of “Turbo-like” codes. The encoding schedule is shown in Figure 3. The \( K \) information bits \( m = [m_1 \ldots m_K] \) are repeated \( l \) times, showed as follows:

\[
\overrightarrow{b} = [m_1^{(1)}, m_1^{(2)}, \ldots, m_1^{(l)}, \ldots, m_K^{(1)}, m_K^{(2)}, \ldots, m_K^{(l)}].
\]

Then, the output sequence \( \overrightarrow{b} \) is permuted with interleaver \( \Pi \), and the result of which is \( \overrightarrow{d} \), where

\[
\Pi = [\pi_1, \pi_2, \ldots, \pi_n],
\]

and

\[
\overrightarrow{d} = [b_{\pi_1}, b_{\pi_2}, \ldots, b_{\pi_n}].
\]

After that the parity bits \( \overrightarrow{p} \) may be calculated by equation (6), for \( 1 \leq i \leq Kl/a,

\[
\begin{align*}
g_i &= d_{(i-1)a+1} \oplus d_{(i-1)a+2} \oplus \cdots \oplus d_{ia} \\
p_i &= p_{i-1} \oplus g_i
\end{align*}
\]

where \( a \) is the parameter of the combiner, which deals with the message before entering into the accumulator. Then the final systematic codeword is

\[
\overrightarrow{c} = [\overrightarrow{m}, \overrightarrow{p}]
\]

Thus an RA code with length \( N = K(1 + l/a) \) and rate \( R = a/(a + l) \) is obtained.

5.2 Decoder Architecture

Thereby we propose an RA code encoder implementation architecture as shown in Figure 4. It consists of an array of memory blocks corresponding to the RA interleaver, denoted as MEMI, and a parity-check-bit-generating unit, denoted as PCBGU[6]. As shown in section 3, the SSI is structured and MEMI is associated with nonzero elements of the interleaver. The output ports of the MEMI are connected to input ports of PCBGU, which generate one parity-check bit each clock cycle. With this encoder, the whole encoding procedure could cost \( N_b \cdot L \) clock cycles. Thereby in the first \( (N_b - M_b) \cdot L \) clock cycle, source bits are output, and simultaneously operated in the MEMI. In the
$M_b \cdot L$ clock cycles, the parity-check bits are generated by PCBGU. It is clearly shown that the implementation complexity of the RA code with SSI is probably the same as that of the RA code with the pseudo-random interleaver.

5.2 Decoder Architecture

RA codes can be described by a Tanner graph, as shown in Figure 5. The regular RA code described by a Tanner graph in Figure 5 has the same parameters in Figure 3, where the message bit nodes are presented at the top of the figure and the parity check bit nodes at the bottom.

As described in section 2, the RA codeword vector $\mathbf{c}$ is divided into $\mathbf{m}$ and $\mathbf{p}$, where $\mathbf{m}$ are the information bits and $\mathbf{p}$ the parity check bits. Corresponding to $H_c^T = \mathbf{0}$, then

$$H_m \cdot \mathbf{p} = H_c \cdot \mathbf{m} \quad (8)$$

Then we have,

$$\mathbf{p} = H_m^{-1} \cdot H_c \cdot \mathbf{m} \quad (9)$$

where $H_m^{-1}$ is the lower triangular matrix. We can calculate $\mathbf{p}$ with equation (9) in any arbitrary information vector $\mathbf{m}$.

Thereby the RA code can be seen as an LDPC code and the LDPC high-speed parallel decoder architecture could be adopted.

Due to its “turbo-like” structure that can be encoded with a low complexity turbo-like encoder and decoded with a high-speed parallel algorithm, these advantages make the RA codes with SSI valuable for practical communication systems.

6 Conclusions

In this paper, a novel scheme of constructing RA codes is proposed. This scheme generates the parity-check matrix with superimposed structured interleavers, guaranteeing it close to global optimum as possible. Compared with the RA codes constructed with traditional structured interleavers such as pi-rotation interleavers, pseudo-random interleavers and combinatorial interleavers, the codes constructed with SSI exhibit better error-correcting performance with the same hardware implementation complexity.

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