A Simple Rate Distortion Estimation for Embedded Block Coding

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Abstract: The embedded block coding with optimized truncation (EBCOT) algorithm has been adopted by the JPEG2000 standard. In which, the post compression rate distortion (PCRD) optimization needs a large memory space to store all the code streams of code blocks; however, code blocks of less importance might not be used for the optimal decoded image at a bit rate. To avoid waste of computational power and memory space, a simple context based rate distortion estimation (CBRDE) is proposed to arrange the scanning order of code blocks in an adaptive manner. CBRDE is based on the MQ table, which is available at both encoder and decoder. Thus, there is no need to store and transmit the rate distortion information of code blocks. Experimental results show that the rate distortion curves are almost convex; this demonstrates the benefit of the proposed CBRDE.

Key-Words: image coding; wavelet transform; JPEG2000; EBCOT; CBRDE

1 Introduction

Wavelet transform offers many desirable properties, e.g. progressive transmission and embedded coding, which are suitable for image coding [1]-[3]. In general, embedded code streams of an image are organized in decreasing order of information importance. The embedded block coding with optimized truncation (EBCOT) algorithm has been adopted by JPEG2000 [4], which is preferable to the JPEG standard at the cost of increasing complexity [5]. For real-time applications, dedicated hardware is required to implement JPEG2000.

In EBCOT, the tier-1 algorithm applies embedded block coding with an arithmetic coder to each code block of the transform coefficients; the tier-2 algorithm performs rate distortion control by the post compression rate distortion (PCRD) algorithm. One of the crucial implementation issues of EBCOT is the design of PCRD, which requires a large amount of memory space to store all the code streams of code blocks with their respective rate distortion information. As all the code blocks of an image must be processed before PCRD, however, some code blocks may not be needed to reconstruct the optimal decoded image at a bit rate; this surely leads to waste of computational power. In [6], Fang et al. proposed a precompression rate distortion optimization algorithm to reduce the memory space by ignoring the unused code streams.

In this paper, we propose a context based rate distortion estimation (CBRDE) to arrange the scanning order of code blocks so that the available coding bits can be allocated for the most significant code block. The proposed CBRDE is based on the MQ table of EBCOT, which is available at both encoder and decoder. As a result, the scanning order of code blocks can be also obtained at decoder, and therefore transmission of the contributions of each code block can be omitted. The remainder of this paper proceeds as follows. In Section 2, wavelet transform and EBCOT are reviewed briefly. Section 3 presents the proposed CBRDE algorithm to arrange the code blocks of an image in an adaptive manner. Experimental results are given in Section 4, and conclusion can be found in Section 5.

2 Wavelet Transform and EBCOT

Wavelet transform provides many desirable properties such as multiresolution analysis, subband decomposition with orientation selectivity, joint space-spatial frequency localization, self similarity across subbands of the same orientation, and energy clustering within each subband. In this section, wavelet transform and the EBCOT algorithm are reviewed briefly.

2.1 Wavelet transform

The wavelet transform of a signal, \( S_\ell(n) \), at resolution \( \ell \) is as follows.

\[
S_{\ell+1}(n) = \sum_k S_\ell(k) h(2n - k)
\]

\[
D_{\ell+1}(n) = \sum_k S_\ell(k) g(2n - k)
\]

(1)
where $S_{\ell+1}(n)$ is the approximation at the next coarser resolution $\ell+1$, $D_{\ell+1}(n)$ is the detail between resolutions $\ell$ and $\ell+1$, $h(n) = \langle \phi, \phi_{\ell,n-1} \rangle$, $g(n) = \langle \psi, \phi_{\ell,n-1} \rangle$, $\langle \cdot, \cdot \rangle$ is an inner product operator, $\psi$ is a valid (mother) wavelet, $\phi$ is the corresponding scaling function and $\phi_{\ell,n}(x) = 2^{-1/2} \phi(2^{-1} x - n)$. $S_{\ell}(n)$ can be exactly reconstructed from $S_{\ell+1}(n)$ and $D_{\ell+1}(n)$ by using following inverse wavelet transform.

$$S_{\ell}(n) = \sum_k S_{\ell+1}(k) \tilde{h}(n - 2k) + \sum_k D_{\ell+1}(k) \tilde{g}(n - 2k)$$

(2)

where $\tilde{h}(n) = h(-n)$ and $\tilde{g}(n) = g(-n)$.

For image applications, 2-D wavelet transform can be obtained by the tensor product of two 1-D wavelet transforms. Similarly, 2-D inverse wavelet transform can be obtained by the tensor product of two 1-D inverse wavelet transforms.

### 2.2 The EBCOT algorithm

In many cases, the DWT-based JPEG2000 standard outperforms the DCT-based JPEG standard. The idea behind the heart of JPEG2000 known as the EBCOT algorithm is to exploit energy clustering of wavelet coefficients. EBCOT is a two-tier algorithm; tier-1 performs bit-plane coding (BPC) followed by arithmetic coding (AC); tier-2 aims for post compression rate distortion optimization. The quantized wavelet coefficients of an image are coded by a context-based arithmetic coder known as the MQ coder, in which the probability models are stored in the MQ table. More precisely, 10 context labels are defined for the significance coding operation and the cleanup coding operation, 5 context labels for the sign coding operation, and 3 context labels for the magnitude refinement coding operation.

### 3 The CBRDE Algorithm

In JPEG2000, a large image is first divided into rectangular sub-images called tiles; each tile is decomposed into subbands by wavelet transform; every subband is partitioned further into small blocks called code blocks, which are quantized to form bit-planes and then coded by EBCOT from the most significant bit-plane to the least significant bit-plane. For each bit-plane, all the code blocks of an image must be processed in the first tier of EBCOT before proceeding with the application of the PCRD algorithm in the second tier. As code blocks of less importance are not needed for the optimal decoded image at a given bit rate, waste of computational power and memory space might result. In this section, an efficient scheme is proposed for the scanning order of code blocks such that waste of computational power and memory space can be reduced.

#### 3.1 Adaptive scanning order

Recall that the code blocks of an image are coded bit-plane by bit-plane, from most to least significant, and the output bitstream can be truncated at an intermediate point between bit-planes; it raises the following interesting questions regarding the scanning order of code blocks. For each bit-plane, is there an adaptive scanning order such that the available coding bits can be allocated for the most significant code block? Is there a common piece of information available at both encoder and decoder, based on which code blocks can be arranged adaptively in decreasing order of significance? If so, there is no need to store all the code streams of code blocks and transmit the scanning order from encoder.
to decoder. Is there any simple way to solve the above-mentioned questions?

As one might expect, the first question can be solved effectively by using the rate distortion curve of code blocks. The optimal scanning strategy turns out to take the code block with the steepest rate distortion slope as the first to be coded. In other words, the available coding bits should be first allocated for the code block with the largest amount of distortion decrease per coding bit. Take the second question into consideration; estimated rate distortion slope is preferable to the true one for arranging the scanning order of code blocks. Since the MQ table of probabilities of more probable symbol (MPS) and less probable symbol (LPS) is available at both encoder and decoder, an efficient, context based rate distortion estimation (CBRDE) is therefore proposed to estimate the rate distortion slope of code blocks, which is given in the following subsection.

### 3.2 Proposed CBRDE

For each bit-plane, code blocks with significant rates of distortion decrease per coding bit should be coded as early as possible. In order to avoid transmitting the scanning order of code blocks from encoder to decoder, the estimated rate distortion slope of a code block is thus adopted, which is given by

$$ S = \frac{E[\Delta D]}{E[\Delta R]} $$

where $\Delta D$ and $\Delta R$ denote the amount of distortion decrease and the number of coding bits, respectively, and $E[\cdot]$ is an expectation operator. Though most of correlation between images pixels can be removed by wavelet transform, there may still be residual correlation between neighboring wavelet coefficients. To incorporate with the framework of EBCOT, the proposed CBRDE for code blocks is based on the MQ table, which is available at both encoder and decoder. Specifically, let $B_i$ denote the $i$th code block (in wavelet domain), $b_{ij}(m,n)$ the $j$th significant bit of a coefficient at position: $(m,n)$ in $B_i$, and $B_y = \bigcup_{m,n} \{ b_{ij}(m,n) \}$. The proposed CBRDE for $B_y$ is as follows.

$$ S_y = \sum_{m,n} \frac{\text{prob}(b_{ij}(m,n) = 1)}{\sum_{m,n} H(b_{ij}(m,n))} $$

where $H(b_{ij}(m,n))$ denotes the entropy of $b_{ij}(m,n)$ given by

$$ H(b) = -\text{prob}(b = 1) \cdot \log_2(\text{prob}(b = 1)) $$

$$ -\text{prob}(b = 0) \cdot \log_2(\text{prob}(b = 0)) $$

The numerator of equation (4) is proportional to the amount of distortion decrease; and the denominator is proportional to the number of coding bits. It is noted that probability models of $b_{ij}(m,n)$ can be obtained directly from the MQ table, and therefore the computational complexity is nothing but one lookup table operation.

Take the commonly used Lena image as an example, the true distortion: $\Delta D$ and coding bits: $\Delta R$, and their respective estimates $E[\Delta D]$ and $E[\Delta R]$ (for a given bit-plane) are shown in Fig. 2. As one can see, the nearly proportional relationship between the horizontal axis (true values) and the vertical axis (estimated values) demonstrates the potential of the proposed CBRDE.

### 3.3 Embedded block coding with CBRDE

Figure 3 depicts the proposed image encoder using embedded block coding (EBC) with adaptive block ordering (ABO). In which, the EBC algorithm is the same as the first tier of EBCOT; however, ABO with CBRDE replaces the second tier of EBCOT, i.e. PCRD. With the adaptation of the MQ table, the code blocks of an image can be arranged according to their respective estimated rate distortion slopes and then coded in an adaptive manner. Figure 4 depicts the image decoder. Recall that the rate distortion slopes of code blocks can be also estimated at decoder. Thus, the overhead of transmitting the scanning order of code blocks from encoder to decoder is unnecessary; this is beneficial to the communication applications.

### 4 Experimental Results

The proposed CBRDE for arranging the scanning order of code blocks is evaluated on grayscale images. The test images, namely Barbara and Fingerprint are shown in Fig. 5. The biorthogonal Daubechies wavelet with 9/7-tap filters is used. The compression rate is measured in bits per pixel (bpp). The distortion is defined as mean squared error (MSE). The compression rates and MSE values are used to plot the rate distortion curves. To avoid the overhead of transmitting the contributions of code blocks, the second tier of EBCOT, i.e. PCRD is replaced by the CBRDE-based ABO. For comparison, as most of images’ energies are concentrated in the low frequency subbands, the first tier of EBCOT incorporated with a fixed scanning order starting from the lowest frequency subband is evaluated.
More specifically, the wavelet subbands of an image are coded in a zigzag order from the lowest frequency subband to the highest frequency subband; the code blocks within each subband are scanned from left to right, top to bottom.

For images with textures such as Barbara, there are many significant wavelet coefficients in the middle and high frequency subbands. As shown in Fig. 6, the coding performance can be significantly improved at low bit rates. Figure 7 shows the simulation results of Fingerprint image. It is noted that the rate distortion curve obtained by the proposed image coder is almost convex; this demonstrates the benefit of the proposed CBRDE when the embedded code stream of an image is truncated at an intermediate point between bit-planes.

5 Conclusion
Wavelet transform has been adopted by JPEG2000 as the underlying method to decompose an image into subbands with orientation selectivity. It provides many desirable properties, e.g. multiresolution analysis, high correlation across wavelet subbands of the same orientation, and energy clustering within each subband; these properties are suitable for the image compression applications. In EBCOT, all the code blocks of an image are coded to generate a set of code streams with their respective contributions to the decoded image, based on which the optimal code stream can be obtained by concatenating the suitably truncated code streams through the PCRD algorithm. As some code blocks, which are less important, are not needed for the optimal decoded image at a given bit rate, waste of computational power and memory space may result. Furthermore, implementation of the PCRD algorithm is one of the crucial issues. To overcome the above-mentioned problems, a simple context-based rate distortion estimation (CBRDE) is proposed to arrange the scanning order of code blocks in an adaptive manner. To avoid transmitting the side information regarding the scanning order of code blocks from encoder to decoder, the proposed CBRDE is based on the MQ table, which is available at both encoder and decoder. The second tier of EBCOT, i.e. PCRD can therefore be replaced by the CBRDE-based adaptive block ordering (ABO). Experimental results show that the rate distortion curves are almost convex.

Recall that for each code block, the coding procedure proceeds in distinct passes. Thus, the proposed image coding with adaptive block ordering can be extended to image coding with adaptive pass ordering, however, at the cost of increasing complexity from the implementation point of view.

References:
Fig. 2 Performance of CBRDE applied to Lena image; (a) horizontal axis: true $\Delta D$, vertical axis: estimated $\hat{E}[\Delta D]$; (b) horizontal axis: true $\Delta R$, vertical axis: estimated $\hat{E}[\Delta R]$.

Fig. 3 Proposed image encoder using EBC with ABO

Fig. 4 Proposed image decoder using EBC with ABO

Fig. 5 Test images (a) Barbara (b) Fingerprint
Fig. 6 Rate distortion curves of Barbara image by EBC with the CBRDE-based ABO (dashed line) and EBC with a fixed scan order (solid line); Vertical axis: mean square error (MSE); Horizontal axis: bit rate (bpp).

Fig. 7 Rate distortion curves of Fingerprint image by EBC with the CBRDE-based ABO (dashed line) and EBC with a fixed scan order (solid line); Vertical axis: mean square error (MSE); Horizontal axis: bit rate (bpp).