Active-Tunable Inductor Effected by a Novel Impedance Synthesis Circuit

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Abstract: This paper describes a novel impedance synthesis (IS) circuit: the impedance of an inductor is made tunable as a function of the frequency by means of a transconductance function $g_m(s)$ that is located in the feedback loop. In order to show the potential of this novel IS circuit, the effect of several basic transconductance functions onto the synthesized impedance is presented. The specific case of $g_m(s) \sim s^{-1}$ brings up the 'coil enhancement' feature of this IS circuit: the synthesized impedance is purely inductive with an actual boosted inductance level compared to the inductance of the physical inductor. Based upon this 'coil enhancement' property, the IS circuit is incorporated in new 'plain old telephone service' (POTS) low-pass filter which is fully compliant to the 'TS 101 952-1-1 V1.2.1' standard\(^1\) of the European Telecommunications Standards Institute (ETSI).

Key–Words: inductor, transformers, passive filter, active filter, feedback

1 Introduction

Passive electronic filters have been used for decades and are still preferable over other filter topologies in the lower frequency range. These cost-effective passive filters offer several advantages such as guaranteed stability, zero power consumption and good large signal endurance. However, for very demanding applications (low passband ripple and high stopband attenuation with a small pass- to stopband window) higher order and complex passive filters are required which are notoriously area-consuming.

The opposite evolution, namely area-reduction, is found in the continuous miniaturisation of passive components. However, not all passive elements follow the same scaling rate: package sizes of resistors and capacitors have dropped drastically over the last two decades, whereas package sizes of inductors (and transformers) seem to have reached a lower limit and stagnate. This difference in size between the inductors and the other passive elements turns the inductors into the area-qualifying factor in passive filter design.

It is, hence, not surprising that several techniques exist for reducing or circumventing the use of inductors. For the very small inductance values, one may consider a monolithic integration of the inductor\(^2\). But for the majority of inductance values, the widespread method of actively generating the inductor behaviour is applicable. Popular circuits that are selected for such inductor emulation are based upon CMOS active inductors\(^3\)\(^-\)\(^4\), or gyrator implementations\(^5\)\(^-\)\(^6\) or rely on current conveyor combinations\(^7\)\(^-\)\(^8\). However, filters that require a certain robustness against overvoltages (e.g. 1.5kV), interchanging all inductors in a filter with their emulation circuit is not possible. The circuit implementations that can handle such voltages will not be area-saving (and cost-efficient).

In this paper, a new interesting area-saving option is elaborated: associating extra functionality with the inductors. Instead of the linearly rising impedance versus frequency characteristic of inductors, one may benefit from a more complex frequency domain behaviour. This adaptable inductor approach can reduce the amount of filter stages at the cost of a few extra components. In the following, a circuit for generating such an adaptable inductor is discussed in detail and it’s employment in a POTS low-pass filter is presented.
2 Concept

A single inductor is electrically represented by its inductance \( L \). In the Laplace domain, the inductor’s impedance is formulated as:

\[
\frac{v_L}{i_L} = sL
\]  

where \( s \) embodies the complex frequency. The inductor’s reactance \( X_L = \omega \cdot L = 2\pi \cdot f \cdot L \) is proportional to the signal frequency \( f \) and the inductance \( L \). Hence, once a specific value for \( L \) is chosen, the inductor’s impedance for all frequencies is predetermined.

In order to obtain more flexibility in impedance control, the circuit from fig. 1 is developed. This so-called IS circuit comprises a pair of coupled inductors \( L_1 \)-\( L_2 \) and a transconductance amplifier. The first inductor \( L_1 \) is placed directly in-between the input terminal node \( t_1 \) and the output terminal node \( t_2 \). The second inductor \( L_2 \) is wound onto the same core as the first inductor \( L_1 \) and forms the transformer combination \( L_1 \)-\( L_2 \). One terminal of \( L_2 \) is connected to ground, whereas the other end of \( L_2 \) is attached to the input of a transconductance amplifier. The output of the transconductance amplifier is fed back to the second inductor \( L_2 \). The combination of the second inductor \( L_2 \) with the transconductance amplifier constitutes a feedback circuit.

The electrical behaviour of the IS circuit of fig. 1 relies on the expressions of the coupled inductors \( L_1 \)-\( L_2 \) (in the s-domain):

\[
\begin{align*}
    v_{L_1} &= (sL_{11}) \cdot i_{L_1} + (sM_{12}) \cdot i_{L_2} \\
    v_{L_2} &= (sL_{22}) \cdot i_{L_2} + (sM_{12}) \cdot i_{L_1}
\end{align*}
\]  

where \( L_{11} \) and \( L_{22} \) are the self-inductances of the inductors \( L_1 \) and \( L_2 \) respectively and where \( M_{12} \) describes the mutual inductance between \( L_1 \) and \( L_2 \). The voltage \( v_{L_2} \) over \( L_2 \) is sensed by the transconductance amplifier and converted into a current \( i_{L_2} \) through some transconductance function \( g_m(s) \). This configuration is mathematically expressed as:

\[
i_{L_2} = g_m(s) \cdot v_{L_2}
\]  

Note that no current flows through the high-ohmic inputs of the transconductance amplifier. Combining the equations (2) and (3), further adopting a lossless coupling between \( L_1 \) and \( L_2 \) \((M_{12}=\sqrt{L_{11} L_{22}})\), leads to the following expression for the synthesized impedance \( Z_{\text{syn}} \) between the electrical terminals \( t_1 \)-\( t_2 \):

\[
Z_{\text{syn}} = \frac{v_{t_1}}{i_{L_1}} = sL_{11} \cdot \frac{1}{1 - g_m(s) \cdot sL_{22}}
\]  

This equation (4) reports that the synthesized impedance of the IS circuit of fig. 1 equals a product of two factors. The first factor matches the impedance of a single inductor \( L_1 \) without feedback circuit and is to be considered as a reference impedance \( Z_{\text{ref}} \). The second factor \((1 - g_m(s) \cdot sL_{22})^{-1}\) in (4) originates from the feedback circuit and hands out an instrument for altering the synthesized impedance between the output terminals \( t_1 \)-\( t_2 \) from the reference impedance level.

The feedback inside the IS circuit draws extra attention to the stability analysis. Whenever some filter is designed with the IS circuit, the standard stability criteria are to be met onto the open loop gain (OLG):

\[
\text{OLG} = g_m(s) \cdot \left( \frac{n_2}{n_1} \right)^2 \cdot (sL_{11} // Z_{\text{ext}})
\]  

in which \( // \) symbolizes a parallel connection, \( n_1 \) and \( n_2 \) the number of windings on the transformer of the coils \( L_1 \) and \( L_2 \) respectively, and where \( Z_{\text{ext}} \) represents all the external impedances seen at the output terminals \( t_1 \)-\( t_2 \).

3 Impedance synthesis

In contrast with the single inductor reference \( Z_{\text{ref}} = sL_{11} \), the IS circuit of fig. 1 introduces multiple degrees of freedom for shaping the frequency behaviour, allowing higher order filters based on a single inductor. Depending on the transconductance function \( g_m(s) \), the frequency behaviour of the synthesized impedance \( Z_{\text{syn}} \) is anything but the frequency behaviour of the reference. Table 1 presents several elementary examples of possible transconductance functions and the corresponding synthesized impedances. The transconductance function parameters \( k, z_1 \) and

![Figure 1: The impedance synthesis (IS) circuit: coupled inductors with v-sensing and current driving on secondary](image-url)
p₁ are supposed to be real arbitrary constants. And keeping the stability of the complete filter in mind, the parameter p₁ must be furthermore restricted to exclusively positive real values.

An extra tuning option for the synthesized impedance arises when the transconductance function itself becomes steerable through external inputs: each set of inputs defines a different transconductance function. This extra measure of freedom increases the variety of realisable impedances of the IS circuit even more and can be utilized for fine-tuning purposes or for integration of two or more filters in one compact filter set-up.

## 4 Coil enhancement [9]

One of the major advantages of the IS circuit is the ability of realizing purely inductive impedances between the output terminals t₁-t₂ which have amplitude levels above the reference |sL₁₁|. The terms for an increased inductive Zₚₛᵧₚ follow from:

\[
\begin{align*}
&\left|1 - gₘ(s) \cdot sL₂₂\right| < 1, \\
&\angle (1 - gₘ(s) \cdot sL₂₂) = 0 \degree 
\end{align*}
\]

(6)

The first condition |1 - gₘ(s) \cdot sL₂₂| < 1 by itself determines transconductance functions that lead to impedances Zₚₛᵧₛ with amplitudes larger than |sL₁₁|, but the realized Zₚₛᵧₛ is not necessarily inductive any more. The situations at which the Zₚₛᵧₛ is not only increased in amplitude but also purely inductive have to meet both requirements of (6).

One case that is able to satisfy the ‘enhanced coil’ conditions is the pure integrator transconductance function \(gₘ(s) = k \cdot s^{-1}\) in which k is a real constant (see also Table 1). Such a transconductance function is similar to placing a (positive or negative) inductive load in parallel with Zₚₛᵧₛ. The synthesized impedance Zₚₛᵧₛ is equivalent to Zₚₛᵧₛ apart from a frequency-independent scaling factor \((1 - kL₂₂)^{-1}\). Four different k-zones can be identified:

\[
\begin{align*}
k < 0 & \implies \left|Zₚₛᵧₛ\right| < \left|Zₚₛᵧₛ\right|, \quad \angle Zₚₛᵧₛ = 90\degree, \\
0 < k < \frac{1}{L₂₂} & \implies \left|Zₚₛᵧₛ\right| > \left|Zₚₛᵧₛ\right|, \quad \angle Zₚₛᵧₛ = 90\degree, \\
\frac{1}{L₂₂} < k < \frac{2}{L₂₂} & \implies \left|Zₚₛᵧₛ\right| > \left|Zₚₛᵧₛ\right|, \quad \angle Zₚₛᵧₛ = -90\degree, \\
k > \frac{2}{L₂₂} & \implies \left|Zₚₛᵧₛ\right| < \left|Zₚₛᵧₛ\right|, \quad \angle Zₚₛᵧₛ = -90\degree,
\end{align*}
\]

Whereas generally spoken the product \(gₘ(s) \cdot sL₂₂\) is some arbitrary vector in the complex plane. That shifts with changing frequencies, this product becomes purely real and constant for all frequencies. The specification \(|1 - gₘ(s) \cdot sL₂₂| < 1\) can therefore be translated into a limiting range for the parameter k, namely \(0 < k < 2 \cdot L₂₂^{-1}\). The phase requirement \(\angle (1 - gₘ(s) \cdot sL₂₂) = 0\degree\) also produces a bounding scope for k, namely \(k < L₂₂^{-1}\). The intersection of both k ranges leads to \(0 < k < L₂₂^{-1}\) and consequently defines the limiting k-range for obtaining ‘coil enhancement’.

Another, more realistic, situation that results in ‘coil enhancement’ occurs when a multiple pole-zero transconductance function \(gₘ(s)\) behaves proportional to \(s^{-1}\) in a frequency range \(\{f₁₁,f₂₂\}\). This is the case when each pole or zero in \(gₘ(s)\), represented by the factor \((s + c)\) in the transfer function, can be approximated as either s or c (with c is an arbitrary complex number) for frequencies in-between \(\{f₁₁,f₂₂\}\). In other words, the roll over frequencies of the zeros and poles are located far away of the frequency range \(\{f₁₁,f₂₂\}\). One example is presented below in order to demonstrate such an approximation operation on more complicated \(gₘ(s)\) function.

The example starts from a third order transconductance function 7 with three real zeros and three real negative poles. This indicates that \(z₁, z₂, z₃\) are real constants, that \(p₁, p₂, p₃\) are real positive con-

<table>
<thead>
<tr>
<th>(gₘ) function</th>
<th>(Zₚₛᵧₛ)</th>
<th>(Zₚₛᵧₛ)</th>
<th>(Zₚₛᵧₛ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k)</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
</tr>
<tr>
<td>(k \cdot s)</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
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</tr>
<tr>
<td>(k \cdot (s + z₁))</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
</tr>
<tr>
<td>(k \cdot s²)</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
</tr>
<tr>
<td>(k) (\frac{s + p₁}{sL₁₁})</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
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<tr>
<td>(k) (\frac{s²}{sL₁₁})</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
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</tr>
<tr>
<td>(k) (\frac{(s + z₁)}{sL₁₁})</td>
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<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
</tr>
<tr>
<td>(k) (\frac{s}{s + p₁})</td>
<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
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<td>(\frac{sL₁₁}{1 - kL₂₂})</td>
<td>(sL₁₁)</td>
<td>(- \frac{L₁₁}{kL₂₂})</td>
</tr>
</tbody>
</table>

Table 1: Examples of the synthesized impedance \(Zₚₛᵧₛ\) corresponding with some basic transconductance functions
stant and that K is a real frequency independent factor. Boundary conditions that are further presupposed for frequencies in-between \([f_1, f_2]\) are \(s \gg z_1, s \ll z_2, s \ll z_3, s \gg p_1, s \gg p_2\) and \(s \ll p_3\).

\[
g_m(s) = K \frac{(s + z_1) \cdot (s + z_2) \cdot (s + z_3)}{(s + p_1) \cdot (s + p_2) \cdot (s + p_3)}
\]

\[
\cong K \frac{s \cdot z_2 \cdot z_3}{s \cdot s \cdot p_3}
\]

for \(s \in [2\pi f_1, 2\pi f_2]\)

The major advantage of the ‘coil enhancement’ circuit of fig. 1 is already described in previous section: the series resistance \(R_{s2}\) of the transconductance amplifier. The exact contribution of \(R_{s2}\) on the generated current \(i_{L_2}\) can be adapted, by altering the gain of the feedback circuit. Another benefit that can be attributed to the ‘coil enhancement’ circuit is a good performance for common mode rejection: common mode signals from the feedback circuit will not affect the main circuit branch \(t_1-t_2\) (and visa versa) due to the presence of the common mode blocking \(L_1-L_2\) transformer. Figure 2 shows the frequency plot according to equation (9) for three different \(k_{L_2}\) values: 0.9, 0.99 and 0.999.

Note that \(R_{s2}\) is not exactly known as it is a parasitic of the transformer \(L_1-L_2\) and is hence difficult to compensate for.

A more comprehensive interpretation of the influence of the series resistance \(R_{s2}\) of inductor \(L_2\) on \(Z_{syn}\) is obtained when constricting equation (8) to the ‘coil enhancement’ situation. The substitution of \(g_m = k \cdot s^{-1}\) yields the following formula:

\[
Z_{syn} = \frac{v_{L_1}}{i_{L_1}} = sL_{11} \cdot \frac{1}{1 - k_{L_22}} \cdot \frac{s - 2\pi f_z}{s - 2\pi f_p}
\]

with \(f_z = \frac{k \cdot R_{s2}}{2\pi}\) and \(f_p = \frac{k \cdot R_{s2}}{2\pi \cdot (1 - k_{L_2})}\)

One of the major issues that blocks the circuit of fig. 1 from being ready-to-use, is the influence of the series resistance \(R_{s2}\) of inductor \(L_2\). This winding resistance \(R_{s2}\) which is not shown on the circuit schematic, creates an additional voltage \(v_{L_2}\) that also contributes to the voltage \(v_{L_1}\). This gives an error on the generated current \(i_{L_2}\) at the output of the transconductance amplifier. The exact contribution of this series resistance \(R_{s2}\) to the synthesized impedance comes to the surface from the extended expression of \(Z_{syn}\):

\[
Z_{syn} = \frac{v_{L_1}}{i_{L_1}} = sL_{11} \cdot \frac{1}{1 - \frac{g_m(s)}{1 - \frac{R_{s2}}{Z_{syn}}}}
\]

If the net effect of the simplification for the frequency range \([f_1, f_2]\) yields a \(g_m\) function of the form \(k \cdot s^{-1}\) and if \(0 < k < L_2^{-1}\), then a ‘coil enhancement’ situation occurs. Concretised into the example (7), ‘coil enhancement’ arises if the constant factor \(K \cdot z_2 \cdot z_3 \cdot p_3^{-1}\) is positive and less than \(L_2^{-1}\).

5 Improvement

The major advantage of the ‘coil enhancement’ circuit of fig. 1 is already described in previous section: the value of the inductor \(L_1\) can be adapted, by altering the gain of the feedback circuit. Another benefit that can be attributed to the ‘coil enhancement’ circuit is a good performance for common mode rejection: common mode signals from the feedback circuit will not affect the main circuit branch \(t_1-t_2\) (and visa versa) due to the presence of the common mode blocking \(L_1-L_2\) transformer. Next to that, the circuit is by nature protected against large overvoltages and overcurrents (e.g. nearby lightning strikes): only a fraction of the large overvoltages/overcurrents will arrive at \(L_1-L_2\) transformer’s secondary side as the feedback circuit becomes isolated from the transformer’s primary side in saturated conditions. The small overvoltages that eventually appear at the secondary transformer side can be dealt with by a simple voltage-clamping protection.

One of the major issues that blocks the circuit of fig. 1 from being ready-to-use, is the influence of the series resistance \(R_{s2}\) of inductor \(L_2\). This winding resistance \(R_{s2}\) which is not shown on the circuit schematic, creates an additional voltage \(v_{L_2}\) that also contributes to the voltage \(v_{L_1}\). This gives an error on the generated current \(i_{L_2}\) at the output of the transconductance amplifier. The exact contribution of this series resistance \(R_{s2}\) to the synthesized impedance comes to the surface from the extended expression of \(Z_{syn}\):

\[
Z_{syn} = \frac{v_{L_1}}{i_{L_1}} = sL_{11} \cdot \frac{1}{1 - \frac{g_m(s)}{1 - \frac{R_{s2}}{Z_{syn}}}}
\]
former combination \(L_1-L_2-L_3\). One terminal of \(L_2\) is connected to ground, whereas the other terminal of \(L_2\) is attached to the input of a transconductance amplifier. Finally, one terminal of the third inductor \(L_3\) is attached to the output of the transconductance amplifier, whereas the other terminal of \(L_3\) is connected to ground. In the improved IS circuit, the feedback circuit is implemented by the combination of the second inductor \(L_2\), the third inductor \(L_3\) and the transconductance amplifier.

The extended IS circuit of fig. 3 fulfills the same functionality as the basic version of fig. 1 but the voltage sense and current drive action are now separated:

\[ i_{L_3} = g_m(s) \cdot v_{L2} \]

Note that \(i_{L2} \approx 0\) due to the high-ohmic input of the \(g_m\)-amplifier and, hence, the effect of the series resistance \(R_{S2}\) is ruled out. The synthesized impedance \(Z_{Syn}\) realized by the improved IS circuit is expressed as:

\[
Z_{Syn} = \frac{v_{L1}}{i_{L1}} = sL_{11} \cdot \frac{1}{1 - \frac{n_1}{n_2} \cdot \frac{g_m(s)}{sL_{22}} \cdot \sqrt{L_{22} \cdot L_{33}}}
\]

\[
= sL_{11} \cdot \frac{1}{1 - \frac{n_1}{n_2} \cdot \frac{g_m(s)}{sL_{22}}}
\]

\[
= sL_{11} \cdot \frac{1}{1 - \frac{n_1}{n_2} \cdot \frac{g_m(s)}{sL_{11}}}
\]

in which \(n_1\), \(n_2\) and \(n_3\) are the number of windings of inductors \(L_1\), \(L_2\) and \(L_3\) on the transformer core respectively that comply to \(L_{11} = (n_1/n_2)^2 \cdot L_{22}\), \(L_{11} = (n_1/n_3)^2 \cdot L_{33}\) and \(L_{22} = (n_2/n_3)^2 \cdot L_{33}\).

Figure 4: POTS low-pass filter circuit topologies: (a) the passive version, (b) the ‘active’ version

6 Application

The single ended circuit topology of a typical, fully passive, POTS low-pass filter is shown in fig. 4(a) and is connected to the telephone exchange by means of the ‘pots’ terminal and is attached to the line (=twisted pair) through the ‘line’ node. The filter consists of three stages and three inductors: \(L_1\), \(L_2\) and \(L_3\). Figure 4(b), however, presents the circuit topology of a new ‘active’ POTS low-pass filter that is based upon the improved IS circuit of fig. 3. This ‘active’ filter version consists of only two stages and one inductor \(L_4\) and one transformer \(L_5-L_5s-L_5d\).

The dashed lines on the fig. 4 mark out which impedances in both filters correspond: the series connection of \(L_2//R_2\) with \(L_3//C_3//R_3\) in the passive POTS filter is implemented by the \(L_5-L_5s-L_5d-g_m(s)//C_5\)-combination in the ‘active’ POTS filter. And as \(L_3=L_5\), the feedback loop of the IS circuit has to realise (in a certain frequency window) an inductive value of \(L_2 + L_3\) through ‘coil enhancement’.

Figure 5 shows the top and bottom side of a PCB with four ETSI ‘TS 101 952-1-1 V1.2.1’-compliant ‘active’ POTS low-pass filters. The top side of the panel is reserved for the passive elements of the forward loop (inductors/transformers, resistors and capacitors). The bottom side of the board, on the other hand, is filled with the feedback loop elements that generate the required \(g_m(s)\) and some voltage-clamping protections. This passive-to-active switch-
**Figure 5:** Top (left) and bottom (right) view of 4 active POTS low-pass filters over in filter topology caused an area profit of 40%.

7 Conclusions

In this paper, a novel IS circuit is elaborated that creates a dirigible synthesized impedance through a transconductance function $g_m(s)$ in the feedback loop. An overview of several transconductance functions and their resulting synthesized impedances is presented. One very interesting characteristic of this IS circuit is its ability for realizing ‘coil enhancement’: the original inductor is boosted. The conditions for reaching ‘coil enhancement’ are derived and discussed.

The practical use of the IS circuit is blocked by the detrimental impact of the series resistance of the second inductor. A work-around for this issue is given through the use of a third coupled inductor.

The IS circuit, in ‘coil enhancement’ mode, is incorporated in a newly developed ‘active’ POTS low-pass filter. This ‘active’ POTS filter originates from a fully passive version in which two of the inductors are merged together into one ‘active’ inductor. The ‘active’ POTS filters are compliant to the ETSI ‘TS 101 952-1-1 V1.2.1’ standard. The passive-to-active conversion leads to a significant 40% area reduction.

References:


