Constrained Optimization Evolutionary Algorithm

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Abstract: - Constrained optimization evolutionary algorithm (COEA) is a mathematical programming problem frequently encountered in the field of engineering application. Solving constrained optimization problems by COEA has become an important research area of evolutionary computation in recent years. In this paper, the constrained optimization evolutionary algorithm is based on the quantum evolutionary algorithm. Some characteristic of QEA and lots of constrained optimization problems are taken into account and the COEA combines direct comparison method with the adaptive strategy which keeps a fixed proportion of infeasible individuals in quantum population. It has been integrated with QEA. Function simulation results show that CQEA proposed in this paper is effective.

Key-Words: - constrained optimization, evolutionary algorithm, quantum evolutionary, direct comparison, dynamic adaptive strategy

1. INTRODUCTION

Many researchers have conducted extensive research in the field of constrained optimization problems based on evolutionary algorithms in the past 10 years, and they have proposed a lot of constrained optimization evolutionary algorithms (COEAs). It should be noted that a large number of evolutionary algorithms have been applied in solving optimization problems because of EAs have a lot of advantages as follows:

(1). EAs are based on global optimization;
(2). EAs have solving ability with parallel computing;
(3). EAs are not sensitive to the characteristics of solving problems, so they have strong ability to adapt themselves when they are used for dealing with different types of optimization problems.
(4). EAs’ optimization is based on random search without complicated mathematical theory, so they are easy to use.

However, two factors influence its performance:

(1) EAs’ capability of stochastic global optimization;
(2) How to convert the objective function of optimization problems to the fitness function of EAs, because fitness function can make effective search in the whole solution area.

In this paper, the latest research results about constrained optimization problems are introduced firstly, and then the authors deeply discuss the constrained optimization evolutionary algorithm based on constrained optimization technology and QEA.

2. Constrained optimization problems and their related concepts

Generally speaking, a constrained optimization problem can be described as follows:

\[
\min f(\vec{x})
\]

subject to \( g_j(\vec{x}) \leq 0, j = 1,2,\ldots, l \)

\[
h_q(\vec{x}) = 0, q = l+1,l+2,\ldots, p
\]

Where \( \vec{x} = (x_1, x_2,\ldots, x_n) \in R^n \), and \( \vec{x} \) is a \( n \)-dimensional real vector, \( f(\vec{x}) \) is the objective function, \( g_j(\vec{x}) \leq 0 \) means \( j \) th inequality restrictions, \( h_q(\vec{x}) \) means \( q \) th equality restrictions, the limits of the value a variable \( x_i \) is as follows: \( x_i \in [x_i^l \leq x_i \leq x_i^u] \)

For the constrained optimization problem, \( S = \prod_{i=1}^{n} [x_i^l, x_i^u] \), which means search space of the solution, all the practicable solutions in \( S \) construct a feasible region: \( F \subseteq S \).
Typically, when EAs are used for dealing with COPs, penalty function method is the most commonly used method, its main idea is as follows: As for \( f(x) \), which is with penalty \( p(x) \) in order to construct a penalty fitness function \( \text{fitness}(x) \). So this will convert COPs into Non-COPs. Although the traditional penalty function method is widely used for solving COPs, it has some shortcomings such as:

1. How to set a reasonable penalty coefficient is very complex;
2. There are no fixed rules to determine the penalty function;
3. A reasonable penalty coefficient should be adjusted a lot of times.

Powell proposed a special penalty function method called separation method \(^2\), this new method could make any feasible solution better than any non-feasible solution by designing a specific penalty function. Subsequently, Deb \(^3\) combined Powell’s method with competition and choice method which is widely used in genetic algorithm, and then he proposed a new method which didn’t require punishing factors, that is, there are two given solution individuals, which are fulfill the following requirements:

1. When the two solution individuals are all feasible solutions, they can be judged by their fitness value \( \text{fitness}(x) \);
2. When there is a feasible solution and the other is not, and then the feasible solution is considered as an excellent solution categorically;
3. When the two solution individuals are all non-feasible solutions, they can be judged directly according to their corresponding penalty function. The smaller the solution violates the constraint, the better the solution is.

As a result, Deb’s method can avoid punishing factor, and this method can make any feasible solution be superior to any non-feasible solution at the same time. So Lin Dan called this method “Direct Comparison” (DC).

3. Quantum Evolutionary Algorithm concept

3.1 General Quantum Genetic Algorithm
Quantum evolutionary algorithm (QEA) is based on quantum computing theory. The smallest unit of information stored in a two-state quantum computer is called quantum bit or qubit. A qubit may be in the “1” state or “0” state, or in any superposition of the two. The state of a qubit can be represented as:

\[
|S\rangle = \alpha |0\rangle + \beta |1\rangle
\]

Where \( \alpha \) and \( \beta \) are complex numbers that specify the probability amplitudes of the corresponding states and quantum chromosome as a string is defined as follows:

\[
\begin{bmatrix}
\alpha_1 \\
\beta_1 \\
\alpha_2 \\
\beta_2 \\
\vdots \\
\alpha_m \\
\beta_m
\end{bmatrix}
\]

Where \( |\alpha_i|^2 + |\beta_i|^2 = 1 \), \( i = 1, 2, \ldots, m \).

Compared with traditional evolutionary algorithm, QEA can improve the diversity of the quantum chromosome population based on its characteristic. Subsequently, QEA uses quantum gate to mutate its quantum chromosome, and to observe the state of quantum chromosome, we can get a binary solution. Simulation results \(^7\) had proved that QEA is a good evolutionary method to solve COPs.

3.2 The steps of Quantum Genetic Algorithm
QEA is an approximate probability algorithm, it population is constructed of quantum chromosome, then the steps of QEA is as follows:

Step 1 Set \( t = 0 \);
Step 2 Initialize the population \( Q(t) \). Generally, every chromosome’s qubit is initialized as \( (1/\sqrt{2}, 1/\sqrt{2}) \), which means all of the possible states of every chromosome is added based on the same probability;
Step 3 \( Q(t) \) is observed in order to get \( P(t) \);
Step 4 Evaluate the \( P(t) \), and save the best solution;
Step 5 While not termination criterion do
   Begin
   \( t = t + 1 \);
   Make \( P(t) \) by observing \( Q(t) \) states, then make \( P(t) \) solutions;
   According to Ren ailian Zhou liang \(^7\)‘s strategy update \( Q(t) \) by using Q-gate \( U(t) \), then we will get the new population \( Q(t) \);
End
4. Constrained Optimization Problems Solving based on QEA

As for the constrained optimization problems, its optimization solution is often located in or near the restriction boundary. However, the following situations for the demand function \( f(x) \) are often occurred unavoidable:

The fitness of non-feasible solution which is near the optimal solution is often better than the fitness of feasible solution in the feasible region \( F \subseteq S \). In theory, these solutions can help us to get optimal solution, which means it is very important to keep the diversity of the population when we solve the constrained optimization problems based on QEA. At the same time, although separation method and Direct Comparison method can get better results when they are used for COPs, Lin Dan \(^4\) had pointed out that the above methods still had some disadvantages. Because if the method makes all of the individuals with feasible solution be better than the individuals with non-feasible solutions unconditionally, then it is difficult to keep the diversity of population of quantum chromosome, which means we can not access these characteristics of COPs, and use the non-feasible solution to get the optimal solution.

In this paper, we combine evolutionary algorithm with constraint technology and propose a new dynamic adaptive quantum evolutionary algorithm, which uses a comparison criteria to judge these individuals in order to achieve direct comparison, then to get the solution of COPs.

4.1 The comparison criteria between individuals

A set of penalty function \( \{f_j(x)\} \) is under consideration in this paper as follows:

\[
f_j(x) = \max \{0, g_j(x)\} \quad j = 1, 2, \ldots, l
\]

\[
f_j(x) = h_j(x) \quad q = l + 1, l + 2, \ldots, p
\]

For any individual \( x \), a formula is constructed as follows:

\[
clo(x) = \sqrt{\frac{\sum_{j=1}^{l} (f_j(x) - f_j(x^*))^2}{p}} \quad q = l + 1, l + 2, \ldots, p
\]

Where \( clo(x) \) can be described as a measure how far individual \( x \) is close to restriction boundary. Then we can get the comparison criteria between individuals:

As for the COPs (like formula (1) mentioned above), given a constant \( \varepsilon (\varepsilon > 0) \):

1. If the individual \( x \) and \( x^* \) are all feasible solutions, compare the fitness \( fitness(x) \) with \( fitness(x^*) \), and the smaller one is considered as virtual value;

2. If the individual \( x \) and \( x^* \) are all non-feasible solutions, compare \( clo(x) \) with \( clo(x^*) \).

If \( clo(x) \leq \varepsilon \) and \( clo(x^*) \leq \varepsilon \), compare the fitness \( fitness(x) \) with \( fitness(x^*) \), and the smaller one is considered as virtual value;

If only one between \( clo(x) \) and \( clo(x^*) \) is less than \( \varepsilon \), then determine the individual is the virtual value directly;

If \( clo(x) > \varepsilon \) and \( clo(x^*) > \varepsilon \), then determine the individual corresponding to smaller \( clo \) is the virtual value directly;

3. If there is only one individual: \( x \) is the feasible solution, while \( x^* \) is the non-feasible solution. Consider the circumstance include the following:

If \( clo(x) \leq \varepsilon \), compare the fitness \( fitness(x) \) with \( fitness(x^*) \), and the smaller one is considered as virtual value;

If \( clo(x^*) \geq \varepsilon \), then determine the individual \( x \) is the virtual value directly.

4.2.The new dynamic adaptive strategy

As mentioned before, we know constant \( \varepsilon \) can determine the diversity of the quantum population indirectly. In his paper, \( n \) is the scale of quantum chromosome population, \( \varepsilon (\varepsilon > 0) \) is a constant which is proposed based on the COPs, a fixed level \( p = (1 > p > 0) \) means the proportion of feasible solutions. In order to improve the diversity of quantum population and the ability of global search of the algorithm, we propose the strategy based on 4.1 as follows:

In this paper, when QEA is running from its first iteration, we will operate the following steps every \( K \) th generation:

1. Calculate the percentage shares of non-feasible solutions in the amounts of the quantum chromosome, then the result is \( r \);
(2) Compare \( r \) with \( p \), then

- If \( r = p \)
  \( \varepsilon \) is unchanged.
- End

- If \( r > p \)
  \( \varepsilon = (1 - (r - p)) \cdot \varepsilon \)
- End

- If \( r < p \)
  \( \varepsilon = (1 + (p - r)) \cdot \varepsilon \)
- End

4.3 Combining QEA with the new strategy (QS)

In this paper, QS can be described as follows:

Step 1: Initialize the parameters which include: the scale of quantum chromosome \( n \), constant \( \varepsilon \), the generation interval \( K \), the fixed level \( p \), iteration times \( t = 0 \), quantum chromosome population \( Q(t) \) which is initialized by equal probability;

Step 2: \( Q(t) \) is observed in order to get \( P(t) \);

Step 3: Evaluate \( P(t) \), calculate their fitness \( \text{fitness}(\tilde{x}), \text{clo}(\tilde{x}) \), and \( r \);

Step 4: While not termination criterion do

Step 5: \( t = t + 1 \);

Step 6: Select \( P(t) \) from \( P(t - 1) \) according to \( \text{fitness}(\tilde{x}), \text{clo}(\tilde{x}) \), and \( \varepsilon \);

Step 7: Calculate the fitness of \( P(t) \) and \( \text{clo}(\tilde{x}) \), and judge the feasible solutions and non-feasible solutions, then calculate the percentage shares of non-feasible solutions in the amounts of the quantum chromosome;

Step 8: According to the 4.2, modify the \( \varepsilon \) every \( K \) th generation;

Step 9: Update the quantum chromosome population based on the Q-gate, and go back to step 4.

5. Simulation results

A COP which is called as Keane’s bump\(^{[8]}\) aroused interest, it could be described as follows:

$$\min f(x) = \left| \sum_{i=1}^{n} \cos^2(x_i) - 2 \prod_{i=1}^{n} \cos^2(x_i) \right| \frac{\sqrt{\sum_{i=1}^{n} i x_i^2}}{\sum_{i=1}^{n} x_i}$$

s.t. \( \sum_{i=1}^{n} x_i \leq 7.5n \)

\[ \prod_{i=1}^{n} x_i \geq 0.75 \]

\( 0 \leq x_i \leq 10, \quad i = 1, \ldots, n \)

When its dimension is 20 ( \( n = 20 \) ), its optimal solution is 0.80351067 according to the reference value in the literature (8). Its optimal point is located in the boundary of \( \prod_{i=1}^{n} x_i = 0.75 \).

In this paper, we use the QS to solve the COPs, the scale of quantum chromosome population is \( n = 100 \), constant \( \varepsilon = 0.1 \), the generation interval \( K = 5 \), the fixed level \( p = 0.15 \), iteration times \( t = 2000 \).

In this paper, we compare the DCGA (Genetic algorithm based on Direct Comparison) with QS to solve the Keane’s bump function, and the results is based on ten independent trials in the table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Best Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCGA</td>
<td>0.7139</td>
<td>0.7116 0.6790 0.6640 0.6623</td>
</tr>
<tr>
<td></td>
<td>0.6617 0.6585 0.6558 0.6505 0.6371</td>
<td></td>
</tr>
<tr>
<td>QS</td>
<td>0.8002 0.79021 0.7112 0.8014 0.7881</td>
<td>0.0079</td>
</tr>
<tr>
<td></td>
<td>0.7991 0.8009 0.7663 0.7852 0.7920</td>
<td></td>
</tr>
</tbody>
</table>

From table 1, we can see that QS is compared with DCGA, QS can improve the ability to search the optimal solution, and the result 0.8014 is, in the main, satisfactory. At the same time, the results of standard deviation have show that QS is more robust than DCGA.

6. Conclusions

As for the COPs, how to deal with the constraints and design an effective algorithm to solve the COPs is very important. In this paper, we use the QEA with the help of its strong ability of global search, and then combine QEA with the new strategy (QS) to construct a new method to solve the COPs. The experimental results show that QS can get satisfactory effect.

References:


