A Quantum-Inspired Hybrid Evolutionary Method

LIU ZHONGGANG, ZHOU LIANG
Glorious Sun School of Business & Management
Donghua University,
Shanghai, 200051, P.R.China
homzhou@163.com

Abstract: - The quantum evolutionary algorithm is based on the quantum superposition state and the quantum computing. Compared to the traditional evolutionary algorithm, QEA has great global search ability; however, its complexity limits the operational ability and the efficiency, at the same time, its local search ability is relatively poor. In this paper, the author presents an improved hybrid intelligent algorithm based on the quantum evolution algorithm, which is ameliorated from the mechanism of quantum evolutionary algorithm in order to construct a new improved hybrid quantum evolutionary algorithm (I-HQEA). Simulation results show that the new algorithm can heighten its efficiency and accuracy effectively.

Key-Words: - quantum evolutiaonry algorithm, quantum computing, improved hybrid quantum evolutinary algorithm, niche strategy, dynamic adjustment

1. INTRODUCTION

Quantum evolutionary algorithm (QEA) is a new optimization method which is proposed by Narayanan et al in 1996 [1−2]. QEA is based on quantum computing theory, and it uses different encoding and mutation method compared with traditional evolutionary algorithm. QEA has a lot of advantages, for example, it can avoid premature phenomena to a certain extent. However, its computing process is parallel and complexity which reduces the computing efficiency. In the 2009, Gan Changsheng, and Zhou Liang et al [3−4] point out some results in their literatures:

(1) The QEA can be improved by combining the clonal mechanism in order to keep the diversity of the swarm and overcome precocity in evolution course;

(2) The QEA can be improved by combining the Particle Swarm Optimization which is based on group brainpower in order to simplify the structure of QEA and thereby substantially increase operating efficiency of the algorithm.

However, Gan Changsheng, and Zhou Liang et al have a basic idea that they want to import the new mechanism into QEA in order to combine the advantages of the new mechanism and the QEA. The simulation results show that the new algorithm is effective when it is used in the multi-modal function optimization, however, this algorithm easily be tapped in the local optimum, especially in solving the high-dimensional multi-modal function optimization, it has poor local search ability. In this paper, a new algorithm is proposed based on the improved operation mechanism of QEA instead of importing the new mechanism into QEA (I-HQEA). Then this new algorithm is used in solving the multi-modal function optimization, the simulation result is very encouraging.

2. Quantum Evolutionary Algorithm

In Quantum Evolutionary algorithm, a qubit chromosome as a string of n qubits is defined as:

$$\begin{pmatrix} \alpha_1 & \dots & \alpha_i & \dots & \alpha_n \\ \beta_1 & \dots & \beta_i & \dots & \beta_n \end{pmatrix} \quad (1)$$

Where α_1 and β_1 are the probability amplitudes of the corresponding states, and satisfies the normalization condition:

$$|\alpha_i|^2 + |\beta_i|^2 = 1,\ i = 1, \dots, n$$

$|\alpha_i|^2$ gives the probability that the qubit will be found in the “0” state and $|\beta_i|^2$ gives the probability that the qubit will be found in the “1” state. A qubit chromosome is able to represent a linear superposition of all possible solutions. QEA adopts quantum rotation gate as evolutionary strategy to make current solution approach the best solution gradually. The better solution is increased in manner of probability, and the worse one is weakened in manner of probability. So, the direction of search is guaranteed.

3. Improved Hybrid Quantum

Evolutionary Algorithm

3.1 Description of I-HQEA

From the above description, we can know that qubit chromosome is the representation of probability. Compared with traditional evolutionary algorithm, the operation of crossover on the probability seems to convey no meaning for QEA, so the operation of mutation is the only genetic operator for the qubit chromosome. Generally, there are some types of quantum mutation as follows:

Quantum mutation 1: A. Jefferson Offutt [5] pointed out that mutation operation in the evaluation algorithm is a random change from the mechanism of algorithm, which can not use the information in the process of evaluation effectively. Especially for quantum-inspired evolutionary algorithm, it can use the current optimal solution which is mutated as a guider for quantum chromosome, and then spread randomly quantum chromosome around the current one as the new population in the next generation. This process is like probability genetic algorithm, but relatively easy to operate.

Quantum mutation 2: Ren ailian, Zhou liang et al [6] pointed out that the transfer of these quantum states is transformed by the quantum gates, and they also observed that rotation angle of quantum gates can represent the mutation operation of qubit chromosome. Mutation operator was designed as follows:

$$U(\theta)=\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$$

Generally, every individual update in QEA is rotation to the direction of optimal solution. The rotation angle $\Delta\theta$ can control the convergence speed, it can be seen: the greater $\Delta\theta$, the faster QEA tends to convergence, at the same time, it also means this algorithm may be premature; the smaller $\Delta\theta$, the slower QEA tends to convergence, at the same time, it also means this algorithm may be more complexity.

In order to prevent premature and improve the speed and accuracy of the local search, in this paper, we propose new update strategy as follows:

1. Considering the niche strategy can solve the optimization problem effectively, when the quantum chromosome is initialized, the new algorithm uses niche evolutionary strategy. That is, at the beginning of the algorithm, the probability space of quantum chromosome is divided into n sub-populations, the process is as follows:

The probability space is divided even; every sub-population is initialized to have same probability just like formula (1).

$$\begin{bmatrix}\alpha_k\\ \beta_k\end{bmatrix}=\begin{bmatrix}\sqrt{i/2n}\\ \sqrt{1-i/2n}\end{bmatrix} \qquad (1)$$

In formula (1), n is the number of sub-populations, $\begin{bmatrix}\alpha_k\\ \beta_k\end{bmatrix}$ means the initialization value of i th sub-population.

Through the division of sub-populations, the diversity of the population can increase clearly, and the ability of local optimization can increase in the multi-modal function at the same time.

2.Before updating of quantum chromosome, the new algorithm combines local updating and global updating. That is to say, every individual's updating is divided into the following two parts:

 Every individual probability vector rotates to the current optimal solution just like the traditional QEA, and the rotation angle is:

$$\theta_i = 2\times s(\alpha_i,\beta_i)\times\Delta\theta_i$$

 If the solution related to the current quantum chromosome is less than the fitness of previous individual, then the probability vector of this quantum chromosome rotate to opposite direction of its previous rotation, rotation angle is θ_i :

$$\theta_i = 2\times s(\alpha_i,\beta_i)\times\Delta\theta_i \qquad (2a)$$

If the solution related to the current quantum chromosome is higher than the fitness of previous individual, then the probability vector of this quantum chromosome rotates to same direction of its previous rotation, rotation angle is θ_i

$$\theta_i = 1/4\times s(\alpha_i,\beta_i)\times\Delta\theta_i \quad (2b)$$

3.2 I-HQEA Steps

Procedure I-HQEA

Step 1 Create $t=0$, initialize $Q(t)$, the scale of the population is N, and $Q(t)$'s initialization value is determined by formula (1);

Step 2 Observe $Q(t)$ in order to get a state: $P(t)$;

Step 3 Calculate the fitness of each individual, and save the best individual and its fitness;

Step 4 If no stopping criterion

 $t=t+1$

 Create $P(t)$ by observing $Q(t)$

 Save the best individual and its fitness in $P(t)$;

 Update $Q(t)$ according to the method which is proposed in this paper in order to get a new quantum population $Q(t)$.

4 Experiments and Results

In order to verify the validity of the I-HQEA, in this paper, three functions are chosen to be testified according to the references (3) and (4).

Sphere function is a continuous, simple single-state function, which is usually used to analyze the implementation performance of the algorithm; Rosenbrock function is a classic complex optimization problem, its global optimization is in a smooth, long and narrow parabola-shaped valley, which means it is difficult to search the global optimization, so Rosenbrock function is usually used to evaluate the implementation efficiency of the algorithm; Rastrigrin function is a typical nonlinear multi-state function, which has a wide range of search space, and a large number of local minimum point, so it is usually considered to deal with the complex multi-state issues.

sphere function

$$f_1(x) = \sum_{i=1}^{n} x_i^2 \quad x \in [-100,100]$$

Rosenbrock function

$$f_2(x) = \sum_{i=1}^{n} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$$

$x \in [-100,100]$

Rastrigrin function

$$f_3(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$$

$x \in [-10,10]$

The theory value of above functions is zero. In this paper, I-HQEA, QEA, and HQCA are selected to participate in the experiment in order to compare with each other. Population size is 50 and 100 separately, corresponding to table 1 and table 2, the largest number of iterations is 1000.

From the table 1 and the table 2, it shows that I-HQEA can get the effective result when it is used to deal with the single-apex function. When it is in dealing with other 2 complex functions, I-HQEA can get more effective results then HQCA and QEA. The reason is that I-HQEA uses a new approach of adaptive calculating rotation angle of quantum gate. All of this demonstrates I-HQEA is more suitable for dealing with the complex optimization problem than QEA, and HQCA, and shows that the performance of I-HQEA is more superior to that of QEA, and HQCA in terms of the global search capability and the ability of possessing exploration.

Table 1

Function	Algorithm	Average Value	Iteration
f_1	**I-HQEA**	0	500
	QEA	0	1000
	HQCA	0	500
f_2	**I-HQEA**	6.0221	500
	QEA	31.5779	1000
	HQCA	7.9937	500
f_3	**I-HQEA**	0.8824	500
	QEA	3.4430	1000
	HQCA	1.0071	500

Table 2

Function	Algorithm	Average Value	Iteration
f_1	**I-HQEA**	0	500
	QEA	0	1000
	HQCA	0	500
f_2	**I-HQEA**	1.2263	500
	QEA	10.9682	1000
	HQCA	2.4337	500
f_3	**I-HQEA**	0.2241	500
	QEA	3.6730	1000
	HQCA	0.8701	500

5. Conclusions

The I-HQEA presented in this paper, which core is as follows: Firstly, I-HQEA adjusts the initial value of quantum population based on niche strategies, which is a good foundation for the diversity of the population. Secondly, through dynamic adjustment of the rotation angle of quantum gates, I-HQEA can balance the global search and local optimization, which improves the efficiency of I-HQEA greatly. When this new algorithm is applied to the standard functions, the effect is admirable.

References:

[1] Narayanan A. Moore M. Quantum—inspired genetic algorithm[C]. Proc of IEEE Int Conf on Evolutionary Computation, Nogoya, 1996: 61-66.

[2] Han K H, Kim J H, Genetic quantum algorithm and its application to combinatorial optimization problems[C]. Proc of IEEE Int Conf on Evolutionary Computation, Lajolla, 2000: 1354-1360.

[3] Gan Changsheng, Zhou Liang, A New Quantum Clonal Algorithm, Proceedings of the 5th WSEAS International Conference on Mathematical Biology and Ecology, 2009:93-97

[4] Gan Changsheng, Hong Juan, Zhou Liang, A New Hybrid Quantum Evolutionary Algorithm

and Its Application, Proceedings of the 5th WSEAS International Conference on Mathematical Biology and Ecology, 2009: 98-102

[5] A. Jefferson Offutt, Stephen D. Lee, An Empirical Evaluation of Weak Mutation[J], IEEE Transactions on Software Engineering, 20(5), 1994: 337-344

[6] Ren Ailian, Zhou Liang, Qu Hongjian, An Improved Quantum Evolutionary Algorithm and Its Application, Proceedings of the 3rd International Conference on Product Innovation Management, 2008, 10: 1428-1431