Parallel Alternating Group Explicit Iterative Method For Convection-Diffusion Equations

Qinghua Feng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
fqhua@sina.com

Bin Zheng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
zhengbin2601@126.com

Abstract: In this paper, based on an unconditionally stable finite difference implicit scheme, we present a concept of deriving a class of effective alternating group explicit iterative method (AGEI) for periodic boundary value problem of convection-diffusion equations, and then give two AGEI methods. The AGEI methods are verified to be convergent, and have the property of parallelism. Results of numerical experiments show that the methods are of higher accuracy than the original AGE method in [1], and will not lead to numerical instability in convection dominant case.

Key–Words: iterative method, iterative method, parallel computing, alternating group, parabolic equations

1 Introduction

In this paper, we will consider the following time-dependent periodic initial boundary value problem of convection-diffusion equations:

\[
\begin{align*}
\frac{\partial u}{\partial t} + k \frac{\partial u}{\partial x} &= \varepsilon \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq t \leq T \\
u(x, 0) &= f(x), \\
u(x, t) &= \nu(x + 1, t).
\end{align*}
\]

(1)

In scientific and engineering computation, with the development of parallel computer technology, researches on parallel finite difference methods are getting more and more popular. As we all know, Most of explicit methods are short in stability and accuracy, while implicit methods usually have good stability, but are complex in computing, and need to solve large equation set in the cost of large memory spaces and CPU cycles. Thus it is necessary to construct methods with the advantages of explicit methods and implicit methods, that is, simple for computation and good stability. Many parallel numerical methods have been presented so far for parabolic partial differential equations, in which a class of alternating group explicit method (AGE) presented in [1-3] is of special meaning for its parallelism and absolute stability. The AGE method is derived by a special composition of two asymmetry schemes, therefore the truncation error can be counteracted much, which leads to high accuracy. Besides the above, In solving large equation set, all the work in the whole domain can be decomposed to many sub-domains for the AGE method. The disadvantage of the original AGE method is that numerical vibration will appear in the case of convection dominant convection-diffusion equations. Based on the original AGE method, many alternating group methods have been presented such as in [4-7]. Rohallah Tavakoli derived a class of domain-split method for diffusion equations in [8-9]. Most of the methods inherit the advantages of the AGE method, that is, parallelism and absolute stability. But we notice researches on alternating group iterative methods are also scarcely presented, and effective methods for convection dominant problems have been scarcely constructed.

We will try to establish a class of parallel unconditionally stable alternating group explicit iterative method for solving (1). The rest of this paper will be organized as follows:

In section 2, we will get the integral conservative form of (1) by a kind of exponential type transformation [7]. Then a symmetry implicit finite difference scheme based on the form will be presented. Based on the scheme we give four asymmetry iterative schemes, and then construct a class of alternating group explicit iterative method (AGEI). In section 3, we will apply the concept in section 2 to construct another four order AGEI method. In section 4, convergence analysis and stability analysis are given. In section 5, results of several numerical examples are presented.
2 The Parallel AGE Iterative Method

The domain $\Omega : (0, 1) \times (0, T)$ will be divided into $(m \times N)$ meshes with spatial step size $h = \frac{1}{m}$ in $x$ direction and the time step size $\tau = \frac{T}{N}$. Grid points are denoted by $(x_i, t_n)$ or $(i, n)$, $x_i = ih(i = 0, 1, \cdots, m)$, $t_n = n\tau(n = 0, 1, \cdots, N)$. The numerical solution of (1) is denoted by $u^n_i$, while the exact solution $u(x_i, t_n)$. In this paper we let $U^n = (u^n_1, u^n_2, \cdots, u^n_m)^T$.

The purpose of this paper is to get the solution of $(n + 1) - th$ time level with the solution of $n-th$ time level known. We notice that the equation (1) is equivalent to $e^{-k^n \frac{\partial}{\partial t}} u^n_i = \frac{\partial}{\partial t}(e^{-k^n \frac{\partial}{\partial x}} u^n_i)$. Integral from $x_{i-\frac{1}{2}}$ to $x_{i+\frac{1}{2}}$ we have

\[ \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} e^{-k^n \frac{\partial}{\partial x}} u^n_i \, dx \approx \varepsilon [e^{-k^n \frac{\partial}{\partial x}} u^n_i (x_{i+\frac{1}{2}}) - e^{-k^n \frac{\partial}{\partial x}} u^n_i (x_{i-\frac{1}{2}})]. \]

We can derive an implicit scheme for solving (1) as below:

\[ \left( e^{\frac{k^n h}{\tau}} - e^{\frac{k^n h}{\tau}} \right) u^n_{i+1} - u^n_i = \frac{k^n}{h} \left( e^{-\frac{k^n h}{\tau}} u^n_{i+1} - u^n_i \right) \]

\[ + \frac{u^n_{i+1} - u^n_i}{2} - e^{-\frac{k^n h}{\tau}} u^n_{i+1} \left( u^n_{i+1} - u^n_{i-1} \right) \frac{u^n_i - u^n_{i-1}}{2} \]

Applying Taylor’s formula to the scheme at $(x_i, t^{n+\frac{1}{2}})$, we can easily have that the truncation error of the scheme is $O(\tau^2 + h^2)$.

Let $p = e^{\frac{k^n h}{\tau}}$, $q = e^{\frac{k^n h}{\tau}}$, $r = \frac{kr}{h^{p-q}}$, then we have

\[ -\frac{r}{2} u^n_{i+1} + [1 + \frac{r}{2}(p + q)] u^n_{i+1} - \frac{r}{2} u^n_{i+1} \]

\[ = \frac{r}{2} u^n_{i+1} + [1 - \frac{r}{2}(p + q)] u^n_{i+1} + \frac{r}{2} u^n_{i+1} \]

We denote it as $AU^{n+1} = F^n$. Here $F^n = (2I - A)U^n$.

In order to solve $U^{n+1}$, we have to solve an implicit equation set, which is complex in composition. Then we will try to construct an alternating group explicit iterative method instead in the following.

First we will present four asymmetry iterative schemes to solve $u^n_{i+1(k)}$ with the value at $k$ known. Here $k$ denotes the iterative number.

\[ [1 + \frac{r}{2}(p + q)] u^n_{i+1(k+1)} - \frac{r}{2} u^n_{i+1(k+1)} \]

\[ = -\frac{r}{2} u^n_{i-1(k)} + [1 + \frac{r}{2}(p + q)] u^n_{i+1(k)} \]

\[ -\frac{r}{2} u^n_{i-1(k+1)} + [1 + \frac{r}{2}(p + q)] u^n_{i+1(k)} - \frac{r}{2} u^n_{i+1(k+1)} \]

We denote it as $AU^{n+1} = B_{i-1}^{-1}(C_iU^{n+1}) + D_i$.

Here $D_1 = C_1U^{n+1} + B_1$, which shows the values of $(u^n_{1(k+1)}, u^n_{2(k+1)}, \cdots, u^n_{m(k+1)})^T$ can be worked out in one group explicitly.

\[ L_{k+1} = \left( \begin{array}{ccc} 1 & \frac{r}{2} & 0 \\ \frac{r}{2} & 1 + \frac{r}{2}(p + q) & \frac{r}{2} \\ 0 & 0 & 1 + \frac{r}{2}(p + q) \end{array} \right) \]

Then $U^{n+1}_{k+1} = B_{i+1}^{-1}(C_iU^{n+1}_{i+1}) + D_i$, which shows the values of $(u^n_{1(k+1)}, u^n_{2(k+1)}, \cdots, u^n_{m(k+1)})^T$ can be worked out in one group explicitly.

\[ L_{k+1} = \left( \begin{array}{ccc} 1 & \frac{r}{2} & 0 \\ \frac{r}{2} & 1 + \frac{r}{2}(p + q) & \frac{r}{2} \\ 0 & 0 & 1 + \frac{r}{2}(p + q) \end{array} \right) \]

Then $U^{n+1}_{k+1} = B_{i+1}^{-1}(C_iU^{n+1}_{i+1}) + D_i$, which shows the values of $(u^n_{1(k+1)}, u^n_{2(k+1)}, \cdots, u^n_{m(k+1)})^T$ can be worked out in one group explicitly.

\[ L_{k+1} = \left( \begin{array}{ccc} 1 & \frac{r}{2} & 0 \\ \frac{r}{2} & 1 + \frac{r}{2}(p + q) & \frac{r}{2} \\ 0 & 0 & 1 + \frac{r}{2}(p + q) \end{array} \right) \]

Then $U^{n+1}_{k+1} = B_{i+1}^{-1}(C_iU^{n+1}_{i+1}) + D_i$, which shows the values of $(u^n_{1(k+1)}, u^n_{2(k+1)}, \cdots, u^n_{m(k+1)})^T$ can be worked out in one group explicitly.

\[ L_{k+1} = \left( \begin{array}{ccc} 1 & \frac{r}{2} & 0 \\ \frac{r}{2} & 1 + \frac{r}{2}(p + q) & \frac{r}{2} \\ 0 & 0 & 1 + \frac{r}{2}(p + q) \end{array} \right) \]
On the other hand, grouping explicit computation can be obviously obtained. Thus computing in the whole domain can be split into many sub-domains, and can be worked out with several parallel computers. So the method has the obvious property of parallelism.

We denote the alternating group explicit iterative method I described above as below:

\[
\begin{align*}
\{ (pI + G_1)\tilde{U}_{k+1}^{n+1} &= (pI - G_2)\tilde{U}_{k}^{n+1} + \tilde{F}_n, \\
(pI + G_2)\tilde{U}_{k+2}^{n+1} &= (pI - G_1)\tilde{U}_{k+1}^{n+1} + \tilde{F}_n \}
k = 0, 1, \ldots
\end{align*}
\]

Here \( \tilde{F}_n = 2F^n \), \( \rho \) is an iterative parameter.

\[
G_1 = \left( \begin{array}{ccc}
B_1 & \cdots & \cdots \\
\cdots & B_1 & \cdots \\
\cdots & \cdots & B_1 \\
\end{array} \right)_{m \times m}
\]

\[
G_2 = \left( \begin{array}{ccc}
B_2 & B_1 & \tilde{C} \\
\tilde{C} & B_1 & B_2 \\
\end{array} \right)_{m \times m}
\]

\[
B_2 = \frac{1 + \frac{1}{2}(p + q)}{-\frac{rq}{2}} \left( \begin{array}{ccc}
1 + \frac{1}{2}(p + q) & -\frac{rp}{2} \\
-\frac{rp}{2} & 1 + \frac{1}{2}(p + q) \\
\end{array} \right)
\]

\[
\tilde{C} = \left( \begin{array}{cc}
0 & -rq \\
0 & 0 \\
\end{array} \right), \ \tilde{C} = \left( \begin{array}{cc}
0 & 0 \\
0 & -rp \\
0 & 0 \\
\end{array} \right)
\]

3 The Four Order Alternating Group Iterative Method

In section 2, we present a class of alternating group explicit iterative method with intrinsic parallelism. The method is based on an \( O(\tau^2 + h^4) \) order implicit scheme, which is of absolute stability. Since the construction of the method is universal, of course we can establish another alternating group iterative method based on another high order implicit scheme.

We present another implicit scheme with truncation error \( O(\tau^2 + h^4) \) for solving (1) as below:

\[
-\epsilon^k \frac{1}{2h^4} \left( \begin{array}{ccc}
\frac{-u_{i+1}^{n+1} - u_{i-1}^{n+1}}{h} - \frac{u_{i+1}^{n+1} - 3u_{i+1}^{n+1} + 3u_{i-1}^{n+1} - u_{i-2}^{n+1}}{h} \\
\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{h} - \frac{u_{i+1}^{n+1} - 3u_{i+1}^{n+1} + 3u_{i-1}^{n+1} - u_{i-2}^{n+1}}{h} \\
\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{h} - \frac{u_{i+1}^{n+1} - 3u_{i+1}^{n+1} + 3u_{i-1}^{n+1} - u_{i-2}^{n+1}}{h} \\
\end{array} \right)
\]

\[
\frac{1}{h} \left( \begin{array}{ccc}
\frac{-u_{i+1}^{n+1} - u_{i-1}^{n+1}}{h} - \frac{u_{i+1}^{n+1} - 3u_{i+1}^{n+1} + 3u_{i-1}^{n+1} - u_{i-2}^{n+1}}{h} \\
\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{h} - \frac{u_{i+1}^{n+1} - 3u_{i+1}^{n+1} + 3u_{i-1}^{n+1} - u_{i-2}^{n+1}}{h} \\
\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{h} - \frac{u_{i+1}^{n+1} - 3u_{i+1}^{n+1} + 3u_{i-1}^{n+1} - u_{i-2}^{n+1}}{h} \\
\end{array} \right)
\]

that is,

\[
rq u_{i-2}^{n+1} - (q + 27p)ru_{i+1}^{n+1} + [1 + 27(p + q)r]u_{i+1}^{n+1}
\]

\[
- (q + 27p)u_{i+1}^{n+1} + rpu_{i+1}^{n+1} = -rq u_{i-2}^{n+1} + (p + 27q)r u_{i+1}^{n+1}
\]

\[
+ [1 + 27(p + q)r]u_{i+1}^{n+1} + (q + 27p)r u_{i+1}^{n+1} - rpu_{i+1}^{n+1}
\]

We denote (8) as \( \mathcal{AU}^{n+1} = \mathcal{F}^n \), here

\[
\mathcal{F}^n = (2I - \tilde{A})U^n
\]

Let

\[
\tilde{A} = \frac{1}{2}(G_1 + G_2)
\]

here \( \tilde{G}_1 = \text{diag}(G_{11}, \cdots, G_{11})_{m \times m} \),

\[
G_2 = \frac{1}{2} 
\]

\[
G_1 = \left( \begin{array}{cccc}
G_{11} & G_{112} & \cdots & G_{11n} \\
G_{113} & G_{114} & \cdots & G_{11n} \\
\vdots & \vdots & \ddots & \vdots \\
G_{11n} & \cdots & \cdots & G_{11n} \\
\end{array} \right)_{m \times m}
\]

\[
G_{11} = \left( \begin{array}{cccc}
1 + 27(p + q)r & \cdots & \cdots & \cdots \\
\cdots & 1 + 27(p + q)r & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & 1 + 27(p + q)r \\
\end{array} \right)
\]

\[
G_{112} = \left( \begin{array}{cccc}
0 & \cdots & \cdots & \cdots \\
\cdots & 0 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & 0 \\
\end{array} \right)
\]

\[
G_{113} = \left( \begin{array}{cccc}
0 & \cdots & \cdots & \cdots \\
\cdots & 0 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & 0 \\
\end{array} \right)
\]

\[
G_{114} = \left( \begin{array}{cccc}
0 & \cdots & \cdots & \cdots \\
\cdots & 0 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & 0 \\
\end{array} \right)
\]
\[ G_{114} = \begin{pmatrix} 1 + 27(p + q)r & -q(p + 27q)r & 27(p + q)r & -q(p + 27q)r \\ -(p + 27q)r & 1 + 27(p + q)r & -q(p + 27q)r & 27(p + q)r \\ -q & 1 + 27(p + q)r & -q(p + 27q)r & 27(p + q)r \\ r & -q & 1 + 27(p + q)r & -q(p + 27q)r \end{pmatrix} \]

Then the alternating group explicit iterative method II can be derived as below:

\[
\begin{align*}
(pI + G_1)U_{k+1}^{n+1} &= (pI - G_2)U_k^n + \tilde{F}^n \\
(pI + G_2)U_{k+2}^{n+1} &= (pI - G_1)U_{k+1}^{n+1} + \tilde{F}^n 
\end{align*}
\tag{9}
\]

Here \( \tilde{F}^n = 2F^n \), \( \rho \) is an iterative parameter.

4 Convergence Analysis and Stability Analysis

**Lemma 1**[10] Let \( \theta > 0 \), and \( G + G^T \) is nonnegative, then \( (\theta I + G)^{-1} \) exists, and

\[
\begin{align*}
\| (\theta I + G)^{-1} \|_2 &\leq \theta^{-1} \\
\| (\theta I - G)(\theta I + G)^{-1} \|_2 &\leq 1
\end{align*}
\tag{10}
\]

**Theorem 1** The alternating group explicit iterative method I given by (7) is convergent.

**proof:** From the construction of the matrixes we can see \( G_1, G_2, (G_1 + G_1^T), (G_2 + G_2^T) \) are all nonnegative matrices. Then we have \( \| (pI - G_1)(pI + G_1)^{-1} \|_2 \leq 1 \), \( \| (pI - G_2)(pI + G_2)^{-1} \|_2 \leq 1 \).

From (7), we have \( U_{k+1}^{n+1} = GU_k^n + 2(pI + G_2)^{-1}(pI - G_1)(pI + G_1)^{-1}F_n + \tilde{F}_n \), \( G = (pI + G_2)^{-1}(pI - G_1)(pI + G_1)^{-1} \) is the growth matrix.

Let \( \tilde{G} = (pI + G_2)G(pI + G_2)^{-1} = (pI - G_1)(pI + G_1)^{-1}(pI - G_2)(pI + G_2)^{-1} \), then \( \rho(\tilde{G}) = \rho(\tilde{G}) \leq \| \tilde{G} \|_2 \leq 1 \), which shows the alternating group method I given by (7) is convergent.

Analogously we have:

**Theorem 2** The four order alternating group explicit iterative method II given by (9) is convergent.

In order to analyze the stability of (2) we will use the Fourier method. Let \( u^n_0 = \tilde{u}^n e^{ixy} \), \( x = \frac{2}{\pi}(p + q) - \frac{2}{\pi}(p + q)\cos(\alpha h) \), \( y = \frac{2}{\pi}(p - q)\sin(\alpha h) \), then from (2) we have

\[
\tilde{u}^{n+1} = \tilde{u}^n \frac{1 - x + iy}{1 + x - iy} \tag{11}
\]

Considering \( x \geq 0 \), it follows that

\[
\frac{1 - x + iy}{1 + x - iy} = \frac{(1 - x)^2 + y^2}{(1 + x)^2 + y^2} \leq 1
\]

So we have:

**Theorem 3** The scheme (2) is unconditionally stable.

In order to analyze the stability of (8) Let \( u^n_0 = V^n e^{i\alpha x} \), \( \tilde{x} = 27(p + q)r + r(p + q)\cos(2\alpha h) - 28(p + q)\cos(\alpha h) \), \( \tilde{y} = (p - q)\sin(2\alpha h) - 26(p - q)\sin(\alpha h) \), then from (8) we have

\[
V^{n+1} = \frac{1 - \tilde{x} + i\tilde{y}}{1 + \tilde{x} - i\tilde{y}} V^n \tag{12}
\]

Considering \( \tilde{x} \geq 0 \), it follows that

\[
\frac{1 - \tilde{x} + i\tilde{y}}{1 + \tilde{x} - i\tilde{y}} = 1 - \frac{\tilde{x}^2 + \tilde{y}^2}{(1 + \tilde{x})^2 + \tilde{y}^2} \leq 1
\]

So we have:

**Theorem 4** The scheme (8) is unconditionally stable.

5 Numerical Experiments

We consider the following problem:

\[
\begin{align*}
\frac{\partial u}{\partial t} + k \frac{\partial u}{\partial x} &= \varepsilon^2 \frac{u}{x^2}, & 0 \leq t \leq T \\
u(x, t) &= u(x, 1, t) \\
u(x, 0) &= \sin(2\pi x), & u(X, t) = u(x + 1, t)
\end{align*}
\tag{13}
\]

The exact solution of the problem above is denoted as below:

\[
u(x, t) = e^{-4\pi^2 t} \sin[2\pi(x - kt)]
\]

Let A.E.I denotes maximum absolute error of the alternating group explicit iterative method I, while P.E.I denotes maximum relevant error. Let A.E.II denotes maximum absolute error of the alternating group explicit iterative method II, while P.E.II denotes maximum relevant error. A.E.\([u^n_i - u(x_i, t_n)] \), P.E.\([u^n_i - u(x_i, t_n)] / u(x_i, t_n) \]. In the numerical experiments we let \( \rho = 1 \). Using the iterative error \( 1 \times 10^{-10} \) to control the process of iterativeness, We compare the numerical results by the presented method in the paper with the results in [1] in the following tables:

Table 1: Results at \( n = 28, k = \varepsilon = 1 \)

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>10^{-5}</th>
<th>10^{-6}</th>
<th>10^{-7}</th>
<th>10^{-8}</th>
<th>10^{-9}</th>
<th>10^{-10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.E.I</td>
<td>1.624×10^{-5}</td>
<td>1.132×10^{-4}</td>
<td>8.499×10^{-6}</td>
<td>5.953×10^{-5}</td>
<td>8.879×10^{-2}</td>
<td>9.876×10^{-2}</td>
</tr>
<tr>
<td>A.E.II</td>
<td>8.499×10^{-6}</td>
<td>5.953×10^{-5}</td>
<td>8.879×10^{-2}</td>
<td>9.876×10^{-2}</td>
<td>6.221×10^{-3}</td>
<td>6.761×10^{-2}</td>
</tr>
<tr>
<td>A.E.[1]</td>
<td>8.879×10^{-2}</td>
<td>9.876×10^{-2}</td>
<td>6.221×10^{-3}</td>
<td>6.761×10^{-2}</td>
<td>5.05×10^{-2}</td>
<td>5.05×10^{-2}</td>
</tr>
<tr>
<td>P.E.I</td>
<td>8.211×10^{-3}</td>
<td>6.761×10^{-2}</td>
<td>5.05×10^{-2}</td>
<td>5.05×10^{-2}</td>
<td>27.757</td>
<td></td>
</tr>
<tr>
<td>P.E.II</td>
<td>8.211×10^{-3}</td>
<td>6.761×10^{-2}</td>
<td>5.05×10^{-2}</td>
<td>5.05×10^{-2}</td>
<td>27.757</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Results at $m = 32, k = \varepsilon = 1$

<table>
<thead>
<tr>
<th>Method</th>
<th>$\tau = 10^{-5}, t = 100\tau$</th>
<th>$\tau = 10^{-5}, t = 1000\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.E.I.</td>
<td>$1.244 \times 10^{-5}$</td>
<td>$8.743 \times 10^{-5}$</td>
</tr>
<tr>
<td>A.E.II.</td>
<td>$6.487 \times 10^{-6}$</td>
<td>$4.558 \times 10^{-5}$</td>
</tr>
<tr>
<td>A.E.[1]</td>
<td>$8.837 \times 10^{-2}$</td>
<td>$8.424 \times 10^{-2}$</td>
</tr>
<tr>
<td>P.E.I.</td>
<td>$6.290 \times 10^{-3}$</td>
<td>$5.177 \times 10^{-2}$</td>
</tr>
<tr>
<td>P.E.II.</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.214 \times 10^{-2}$</td>
</tr>
<tr>
<td>P.E.[1]</td>
<td>$1.059$</td>
<td>$3.496$</td>
</tr>
</tbody>
</table>

Table 3: Results at $m = 28, k = 1, \varepsilon = 0.01$

<table>
<thead>
<tr>
<th>Method</th>
<th>$\tau = 10^{-4}, t = 100\tau$</th>
<th>$\tau = 10^{-4}, t = 1000\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.E.I.</td>
<td>$3.517 \times 10^{-4}$</td>
<td>$6.305 \times 10^{-3}$</td>
</tr>
<tr>
<td>A.E.II.</td>
<td>$3.492 \times 10^{-4}$</td>
<td>$3.315 \times 10^{-3}$</td>
</tr>
<tr>
<td>A.E.[1]</td>
<td>$1.138 \times 10^{-2}$</td>
<td>$3.780 \times 10^{-1}$</td>
</tr>
<tr>
<td>P.E.I.</td>
<td>$1.180 \times 10^{-1}$</td>
<td>$7.892 \times 10^{-1}$</td>
</tr>
<tr>
<td>P.E.II.</td>
<td>$9.757 \times 10^{-2}$</td>
<td>$5.541 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

6 Conclusions

In this paper, based on two unconditionally stable implicit schemes, we present a process of constructing a class of alternating group iterative method (AGEI), and two methods are derived. The AGEI method has the property of intrinsic parallelism, and is verified to be convergent. Results of Table 1 and Table 2 show that the AGEI method is of higher accuracy than the original AGE method in [1]. Results of Table 3 shows the AGEI method can obtain high accuracy even in the convection dominant case. Considering the construction of the AGEI method mentioned in this paper is a universal process, the concept can also be applied to other problems.

References:


